Astronomical Papers
by
Joseph Morrison
Vol. 33

(The Computation
of
Ephemerides.)

Contents

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Preface

These Papers bound in the rough were intended to form part of a Work on Spherical and Practical Astronomy, but fate decided otherwise. The first and last were lost; the former contained a History of Ephemerides from the earliest records to the issuance of the "French Connaissance des Temps", and the latter of the transformation of Polar Coordinate Ephemeris of Saturn's Rings etc. now partially supplied in MS.

The author hopes that they will assist some struggling Student to gain a knowledge of the grandest, the noblest and the most sublime of all the physical Sciences.

J.M.

San Antonio, Tex.
Feby 22, 1927.

stellar velocities. But great as are these velocities they have brought no star to us and carried none from us enough seriously to change its brightness, even since the earliest ages; as a little figuring will show.

Let a star be a close binary, with equal components, so revolving that we are in or near the plane of its orbit. When the line through the two components is directed toward us neither component is approaching us or receding from us. One-fourth of a revolution later the line through the components is perpendicular to the line from us to the star; one is then moving toward us in its orbital revolution, the other away from us. At this time the lines in the spectrum due to one component are shifted toward the blue, those due to the other component toward the red; that is, if the orbital motion is great enough, the lines of the star's spectrum are doubled. Now, this periodical doubling, indicating a "spectroscopic binary," has been observed in Spica, β Aurigæ, and Mizar (ζ Ursæ Majoris). The orbital period of such a binary is measured in days, and the two components must be so close that no telescope will ever separate them.

Let the variable star Algol have a smaller dark component which partially eclipses the bright. At eclipse neither is moving toward us nor from us. One-fourth revolution after eclipse the dark body is moving from us, and Algol toward us. One-fourth revolution before eclipse the dark body should move toward us, Algol from us. This motion of Algol from us before eclipse and toward us after eclipse should be seen in the displacement of lines in the spectrum. And this displacement has been detected. The spectroscope so seems to confirm the eclipse theory of stars of the Algol type. From this is derived our first serious estimate of the diameter of a fixed star. Assuming only that we are nearly in the plane of the orbit so that the eclipse is central, the measured velocities give:—for the diameter of Algol proper 1,000,000 miles; for the diameter of the dark companion 800,000 miles; for the distance between the centres of Algol and companion 3,200,000; orbital velocity of Algol 25 miles a second; orbital velocity of companion 54 miles a second; mass of Algol four-ninths that of the Sun; of the component two-ninths that of the Sun. This indicates that in magnitude Algol at least is of the same order as our Sun; and Algol is a Sirian star.

In these articles the attempt has been to state some of the results of spectroscopic work. Such outline must indicate the greatness of spectroscopic phenomena; but not their surpassing

beauty. And many of the most beautiful are well within the ability, experimental and financial, of the amateur.

In procuring a telescope, except where special work is selected for it, I think it sound judgment to sacrifice a trifle in size (not in perfection) of the telescope to procure spectroscopic accessories. Any equatorial telescope with clock-work can properly carry a solar spectroscope; and any one who can manage and care for such a telescope is equal to the manipulation of a solar spectroscope. The hardest part is the original adjusting, for which the beginner should insist on careful instructions from the maker. The cost will be about $200 or $250 for a small good one—avoid poor ones. The beautiful and varying solar prominences are better seen through such a spectroscope than through a very large one. And even the solar spectrum considered alone is a thing of beauty.

Any telescope, with clock-work or without, can have a small ocular star spectroscope sufficient to show the different types of stellar spectra magnificently.

As an adjunct a pocket terrestrial spectroscope is a most useful and delightful instrument, as well as cheap and portable (size of a cigar). With it you may examine anything in sight:—lightning, aurora, flames from volcanos, fire flies, the electric arc light, electric sparks with any electrodes, flame of Bunsen burner, and with some special forms of instrument metors if you are quick enough—here some valuable work should be done by amateurs as well as anybody else, in the meteoric showers soon due.

THE SOLAR EPHEMERIS.

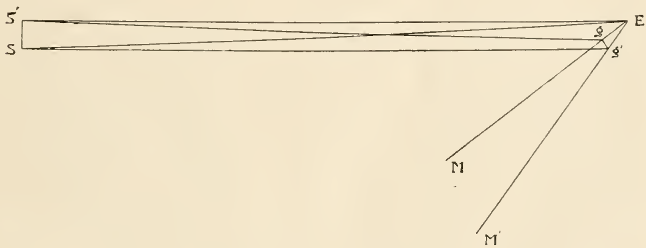
BY J. MORRISON, M. A., M. B., PH. D.

FOR POPULAR ASTRONOMY.

In the preparation of an ephemeris of the celestial bodies, that of the Sun holds the first place and has the most extensive application. Such an ephemeris is necessary for the determination of latitude and time at sea, as well as on land, and for the calculation of eclipses and of the local rising and setting of the Sun. The Sun's place in the celestial vault from day to day, is expressed in our Nautical Almanacs in two systems of coördinates, polar and rectangular. On page III of each month in the American Ephemeris and Nautical Almanac are given the Sun's longitude,

latitude and radius vector constituting a system of polar coördinates referred to the ecliptic as a fundamental plane and to two origins, viz., one the mean equinox of Jan. 1.0 which remains fixed throughout the year, and the other to the true equinox of date. The former is found to be most convenient in some of the more refined calculations such as the computation of orbits and perturbations. These coördinates are obtained directly from the solar tables for every fourth day and interpolated for intermediate dates, are fundamental in the solar ephemeris and constitute the basis for all subsequent computations relating to the Sun. The Sun's latitude is always very small; it is not deducible by direct observation but rests on theory; it is a function of, and has the same sign as, the Moon's latitude. It may be computed in the following manner:—

The Earth and Moon revolve around their common centre of gravity which point is situated within the Earth and always in the plane of the ecliptic. Conceive two planes one passing through the centre of the Sun and the centre of gravity of the



Earth and Moon and the other parallel to it and passing through the centre of the Earth. In the diagram let S', E and M' represent the Sun, Earth and Moon, g' the centre of gravity of the last two and g its projection on the latter plane, then we shall have

$$\sin \odot\text{'s lat.} = \frac{SS'}{ES} = \frac{gg'}{ES}, \text{ approximately}$$

$$= \frac{Eg \sin \text{☽'s lat.}}{ES} = \frac{M}{E+M} \cdot \frac{EM}{ES} \cdot \sin \text{☽'s lat.}$$

in which M and E denote the masses of the Moon and Earth respectively. But

$$\frac{EM}{ES} = \frac{\pi'}{\pi}$$

(π and π' representing the parallaxes of the Moon and Sun) and since

$$M = \frac{E}{81},$$

we shall have finally

$$\sin \odot\text{'s lat.} = \frac{1}{82} \cdot \frac{\pi'}{\pi} \cdot \sin ☽\text{'s lat.}$$

or expressing it in seconds of arc, we have approximately

$$\odot\text{'s lat.} = \frac{1}{82 \cdot \sin 1''} \cdot \frac{\pi'}{\pi} \cdot \sin ☽\text{'s lat.} \qquad (1)$$

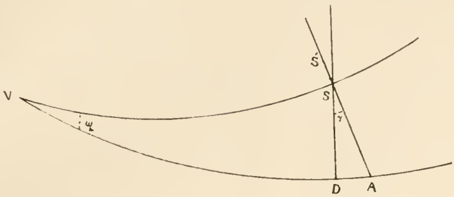
This formula will never vary more than 0″.05 from the truth and is accurate enough for all purposes. The effect of the planets is not here considered.

On pages I and II of each month we have another system of polar coördinates of the Sun referred to the equinoctial as the fixed plane and to the true equinox of date as the initial line; those on page II being given at equal intervals (Greenwich mean noon) and those on page I at unequal intervals; or at the moment of transit across the meridian of Greenwich. The latter are used at sea for determining the latitude. The Sun's R. A. and Decl. as given on pages I and II are deduced from the longitude and latitude given on page III, but most solar tables give an auxiliary table from which the R. A. and Decl. for both mean and apparent noon are given directly. The American Almanac gives the Sun's *true* longitude and the *apparent* R. A. and Decl. and therefore the former must be reduced to the apparent longitude before computing the R. A. and Decl. therefrom, by applying the aberration which is given on page 278, for every 10th day throughout the year. The true longitude always exceeds the apparent by the amount of the aberration, because we see the Sun not where he actually is but where he was some 8 minutes and 18 seconds ago. In the right angled spherical triangle formed by the apparent longitude (λ), the R. A. (α), the declination (δ), we shall have (ω being the apparent obliquity of the ecliptic) the following relations:

$$\begin{aligned} \tan \alpha &= \cos \omega \tan \lambda \\ \tan \delta &= \sin \alpha \tan \omega \\ \cos \lambda &= \cos \alpha \cos \delta \\ \sin \delta &= \sin \omega \sin \lambda \end{aligned} \qquad (2)$$

The second of the above is always to be preferred to the fourth for the reason that an angle is always determined more accurately from its tangent than from its sine. In these formulæ, the latitude has been neglected. When it is desired to take it into account, it can be done by first computing the angle between the circle of latitude and declination passing through the Sun's centre.

This angle varies from ω (the obliquity) at the equinoxes, to $0°$ at the solstices, and is computed thus: Let AV and VS represent arcs of the equinoctial and ecliptic respectively, V the vernal equinox,



SD and SA circles of declination and latitude passing through S the Sun's centre. Denoting the Sun's latitude by β, his longitude by λ, the obliquity by ω and the angle ASD by γ, we have from the right angled spherical triangle VSD

$$\cos VS = \cot\omega \cot VSD$$

or

$$\cos\lambda = \cot\omega \tan\gamma \tag{3}$$

whence

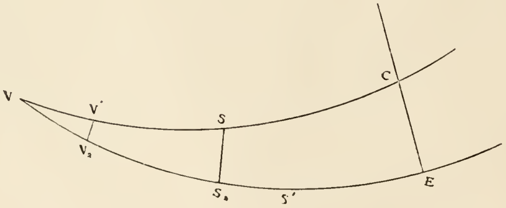
$$\tan\gamma = \cos\lambda \tan\omega$$

Now if the Sun has the small latitude $S'S$, then it is evident that the projection of $S'S$ or β on the circle of declination DS, or $\beta\cos\gamma$ is the correction to the declination, and of course $\frac{1}{15}\beta\sin\gamma\cos\delta$ will be the correction to the R. A., which however is almost inappreciable. Computors form a table giving the value of γ for every degree of longitude. In the application of these formulæ seven figure tables will be required to give the utmost degree of accuracy. The Sun's R. A. and Decl. for apparent noon, given on page I of each month, may be computed from those of mean noon by means of the equation of time as will be easily seen from the following example. Thus on Jan. 18, 1896, the equation of time at Greenwich mean noon is — 10 min. 34.47 sec., that is, apparent noon takes place 10 min. 34.47 sec. after mean noon. We must now find how much the Sun's R. A. and Decl. have changed during this interval and apply it with the proper signs to the R. A. and Decl. at mean noon; but the equation of time itself has changed slightly during these 10 min. 34.47 sec. and therefore we must first correct it and then compute the change in the R. A. and Decl. The variation of the equation of time in one hour on Jan. 18 is + 0.802 sec., hence the variation in 10 min. 34.47 sec. is + 0.141 sec., therefore the equation of time at apparent noon is + 10 min. 34.61 sec., which is tabulated on page

I in the column "Equation of Time, etc." During this time the R. A. increased by 1.88 sec., which added to the R. A. at mean noon gives $20^h\ 0^m\ 13.28^s$ the R. A. at *apparent* noon. The Decl. at apparent noon is found in the same manner.

The sidereal time at mean noon is found directly from the solar tables but it may be readily computed from the data already found. An example will best illustrate it. Thus, at apparent noon the Sun's R. A. is the sidereal time *at that instant*, and on the date we have selected, viz. Jan. 18, 1896, we have just seen that mean noon occurred 10 min. 34.61 sec. *before* apparent noon, therefore it is plain that if we convert 10 min. 34.61 sec. into *sidereal* time and subtract it from the R. A. at apparent noon we shall have the sidereal time at mean noon. By table III appendix we find 10 min. 34.61 sec. of mean time equal to 10 min. 36.35 sec. sidereal time, which subtract from $20^h\ 0^m\ 13.28^s$ the R. A. at apparent noon and we have $19^h\ 49^m\ 36.93^s$ which is the sidereal time at mean noon and tabulated opposite Jan. 18.

The Equation of Time is derived directly from the solar tables; it is given for both mean and apparent noon and is the connecting link between mean and apparent time. It arises from two causes, 1st, the unequal motion of the Sun in longitude and 2nd, the



obliquity of the ecliptic, each of which may be considered separately. In the diagram let VE be an arc of the equinoctial, VC of the ecliptic, V the *true*, V' the *mean* and V_2 the *reduced* place of the mean equinox, S the *true* and S' the mean Sun and CE an arc of a meridian of any place, then ES' (expressed in time of course) is the *mean* and ES_2 the *apparent* solar time, VS_2 is the right ascension of the true Sun, V_2S' the mean longitude (or R. A.) of the Sun and VV_2 "the equation of the equinoxes in R. A." The arc $S'S_2$ is the equation of time and from the figure we have

$$S'S_2 = VS_2 - VV_2 - V_2S'$$

or

$$S'S_2 = VS_2 - (V_2S' + VV_2)$$

that is, the equation of time is equal to the Sun's true right ascension minus the algebraic sum of the Sun's mean longitude (or R. A.) and the "equation of the equinoxes in R. A."

The mean longitude, which is of course equal to the mean R. A., is given in the solar tables and the "equation of the equinoxes in R. A." is given on page 278.

The equation of time is given only for apparent and mean noon, and for any other time we must take a proportional part, using second differences when the equation varies rapidly and when great accuracy is required. The part of the equation of time which is due to the 1st cause, vanishes twice a year, viz., when the Sun is in perigee or apogee, for then the *mean* and true longitudes agree; and the part arising from the 2d cause vanishes four times in the year, viz., at the equinoxes and solstices, for at these dates the difference between the Sun's longitude and right ascension is zero. By combining the effects of the two causes, it is found that the equation of time vanishes four times in the year, but these dates vary continually by reason of the change in the positions of the equinox and Sun's perigee which are separating from each other at the rate of about $61''.5$ per year. The equation of time can be expressed in a series of sines of multiple arcs of the Sun's true longitude, from which series it can be easily computed. Thus if E denotes the equation of time in arc, λ the Sun's true longitude, p the longitude of the Sun's perigee and Ω the longitude of the Moon's ascending node, the following series has been deduced:

$$\begin{aligned} E'' = {} & 461''.786 \sin (\lambda - p) - 593''.146 \sin 2\lambda \\ & - 2.907 \sin 2(\lambda - p) + 12.793 \sin 4\lambda \\ & + 0.022 \sin 3(\lambda - p) - 0.368 \sin 6\lambda \\ & + 0.099 \sin \Omega + \text{planetary perturbations.} \end{aligned}$$

The coefficients are functions of the obliquity and the eccentricity of the Earth's orbit, both of which vary. The demonstration of this series is too difficult for insertion here and must be taken on credit, for we are writing not for the professed mathematician and astronomer but for students, amateurs and others who desire to extend their knowledge of astronomy, to gain admission to its sanctuary and to participate to some extent in the feelings and enjoyments of its votaries.

Before proceeding further we must digress to speak very briefly of the "Equation of the Equinoxes," which has been alluded to in the preceding pages, for it is of the utmost importance to the student that he acquire at the outset a clear and definite concep-

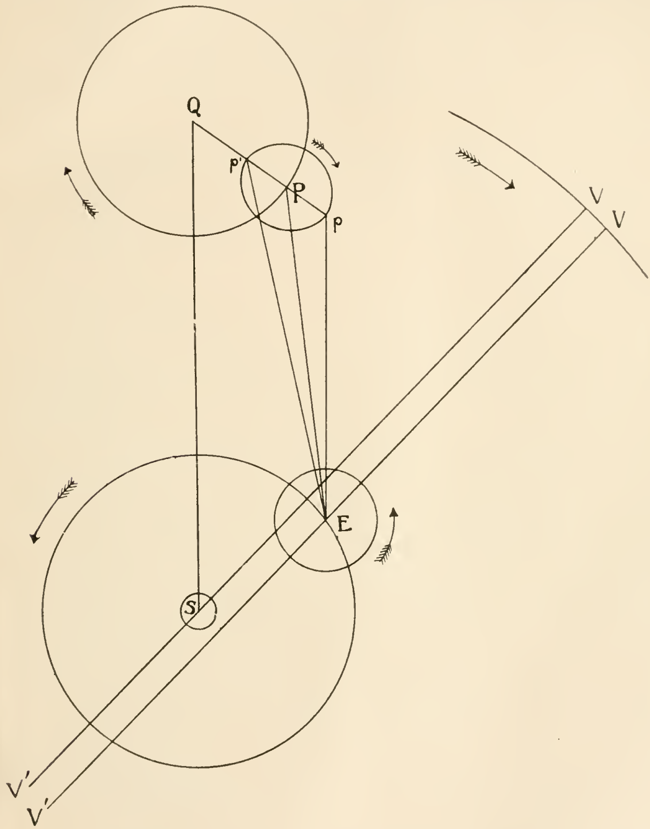
tion of the nature and use of the *five* astronomical corrections, viz., precession, nutation, aberration, parallax and refraction. The last one forms no factor in the construction of an ephemeris and as to the fourth, only the equatorial horizontal parallax of the Sun, Moon and planets is given. It is the basis for the computation of the parallax in altitude or zenith distance.

Precession and Nutation.

The longitude of the stars slowly increases from year to year while the latitude remains sensibly constant. This is due to a slow, gyratory motion of the Earth's polar axis in a retrograde direction and is caused by the rotary motion of the Earth and the combined action of the Moon, Sun and planets on the ring of equatorial matter in excess of the sphere of which the polar axis is the diameter. It is thus the resultant of two component motions. By the first of these the polar axis would describe in every 18.6 years an acute conical surface whose vertex is at the centre of the Earth, and whose intersections with the celestial sphere, form two equal ellipses whose transverse and conjugate axis are 18″.5 and 13″.7 respectively, the former being always directed to the pole of the ecliptic. By the second, the centres of these ellipses are carried uniformly around the poles of the ecliptic in a retrograde direction or from east to west in circles, whose radii are equal to the obliquity (about 23° 28′) and at a rate of 50″.2 in the interval of time between two consecutive returns of the Sun to the mean vernal equinox—which interval is the tropical year upon which our seasons depend.

In the diagram *S* represents the Sun, *E* the Earth, *p* the true and *P* the mean north pole of the equinoctial and *Q* the north pole of the ecliptic. The curve about *S* represents the Earth's orbit, and that about *E* the Earth's equator whose plane is always perpendicular to *Ep*; *EV* is the line of instersection of the planes of the equinoctial and ecliptic or the direction of the vernal equinox. The small circle about *Q* has a radius equal to the obliquity (23° 28′ nearly) and the curve about *P* is the elliptical path which the true pole *p* describes about *P* the mean pole. The transverse axis *pp′*, of this little ellipse is always directed to *Q* the pole of the ecliptic. The arrows show the direction of motion. Let us suppose for the sake of illustration that the second component force ceases to act, then by the first acting alone, the true pole *p* would describe the small ellipse whose centre is *P* in every 18.6 years, the time required by the Moon's nodes to make one entire revolution; and again, if the first component

ceases to act, then by virtue of the second the mean pole P would be carried around Q in the direction of the arrow at the rate of $50''.2$ per year. The change due to the first component or to the motion of the true pole in the small ellipse is called *nutation* and



that due to the second component, or the motion of P in the small circle about Q, is called *precession*. By the combined motions of nutation and precession the true pole p is carried with a variable motion along a gently waving curve whose undulations lie both within and without the circle described by P the mean pole, and which it intersects at points whose angular dis-

tances as seen from the centre of the celestial sphere are evidently equal to $9.3 \times 50''.2$ sin obliquity.

The equinoxes V, V' must always conform to the positions of the equinoctial poles and therefore have a slow, *irregular* and continuous retrograde motion.

The position of the equinox *without* nutation is the *mean* equinox; with nutation, the *true* or *apparent* equinox.

The inclination of the equinoctial to the ecliptic *without* nutation is called the *mean* obliquity; with nutation the *true* or *apparent* obliquity.

The difference between the *mean* and *true* or *apparent* equinox is the *nutation in longitude* and the difference between the mean and apparent obliquity is called the *nutation* of the obliquity.

The nutation is thus seen to be a correction to be applied to the *mean* equinox or obliquity to obtain the *true* or apparent equinox or obliquity.

In the *American Ephemeris* the nutation is called by the antiquated name of "the equation of the equinoxes in longitude." The *British Almanac* uses the proper and modern term "nutation" in longitude R. A., etc. The *American Almanac* does not give the nutation of the obliquity but gives at once (for every 10th day of the year on page 278) the apparent or true obliquity, *i. e.*, the mean obliquity corrected for nutation.

The nutation in R. A. is simply that of the longitude projected on the equinoctial and is found by multiplying the nutation in longitude by the cosine of the apparent obliquity. From the diagram it is easily seen that the apparent equinox deviates from the mean by a distance equal to the half of $13''.7$ which it reaches at the points where the little ellipse intersects the circle about Q and when the mean and apparent obliquity are equal; and the apparent obliquity varies on either side of the mean from zero to the half of $18''.5$ the maxima being reached at the points p, p' where the mean and apparent equinoxes coincide.

The changes here referred to are due to the action of the Sun and Moon and when estimated along the ecliptic are called lunisolar precession in longitude. By the action of the planets the Earth is slightly deflected from the path it would describe by the action of the Sun alone and the plane of the ecliptic is therefore changed but the amount of this change is exceedingly small, being only $46''$ in a century. At present its effect is to diminish the mean obliquity and this will continue for ages. The combined effect of all these bodies acting in the same direction, is the *general* precession in longitude.

[TO BE CONTINUED.]

PROBLEMS.

Solutions of the following problems will be acknowledged in subsequent numbers. They may be sent in on or before June 1st, to Professor Payne or to Dr. Morrison, Washington, D. C.:

1. If λ denote the Sun's longitude and E, the equation of time (in angle) due to the obliquity of the ecliptic, (ω) alone show that

$$\cot E = \cot 2\lambda - \cot^2 \frac{\omega}{2} \operatorname{cosec} 2\lambda$$

2. Show that when that portion of the equation of time due to the obliquity (ω) alone, is a maximum

$$\cos^2\delta = \cos\omega$$

where δ is the declination.

3. The times of Sun rise and Sun set on a certain day, are $6^h\ 54^m$ and $4^h\ 30^m$ respectively; find approximately the equation of time.
4. If the right ascension of a star is equal to its latitude, show that its declination must be equal to its longitude.
5. If E_m, in arc, denote the maximum value of the equation of time due to the obliquity alone, show that

$$\sin E_m = \tan^2 \frac{\omega}{2}.$$

6. Find the Sun's true longitude when that portion of the equation of time due to the eccentricity of the Earth's orbit alone is a maximum, the eccentricity being 0.016771.
7. The latitudes of two places on the Earth are complementary to each other and on a given day the Sun was found to rise 45 minutes earlier at one place than at the other, what is the latitude of each place?

LONG PERIOD VARIABLES.

J. A. PARKHURST.

FOR POPULAR ASTRONOMY.

SUGGESTIONS FOR SPRING AND SUMMER WORK.

Several observers have informed me by private letters that they intend to begin work on variables this spring. I will therefore review the forty-two variables already charted in POPULAR ASTRONOMY, indicating the stars which will be available, the aperture which will be required to deal with them, and the particular ones in need of observation. The third column gives the number of POPULAR ASTRONOMY in which the variable is charted.

No.	Name.	Pop. Ast. No.	Aperture required inches.	Remarks.
107	T Cassiopeæ	5	2 or 3	Max. due in Aug. Change slow.
320	U Cephei	7	2	Minima Apr. 10th, 15^h; 15th, 14^h; 20th, 14^h; 25th, 14^h; 30th, 13^h. May 5th, 13^h, 90th meridian time.
432	S Cassiopeæ	5	9	Min. due in Sept. Change slow.
814	S Persei	5	5 or 6	Min. due in May or June. Change slow.
243	U Cassiopeæ	17	3	Max. due in April. Quite red.
678	U Persei	15	6	Max. just past, star low at next min.
906	R Trianguli	15	6	Max. just past, star low at next min.
1113	U Arietis	4	5-7	Min. in morning twilight in June?
2478	R Lyncis	5	3	Max. due in July.
1805	V Orionis	15		Now near min. Next max. too near the Sun to be visible.
1855	R Aurigæ.	6	2-4	Now faint. Max. due in Aug. or Sept. when it will require morning observation.
2815	U Geminorum	7 and 9	3	Next max. too near the Sun.
3170	S Hydræ	17	6	Next min. too near the Sun.
3825	R Urs. Maj.	5	2 or 3	Now brightening. Max. due in June.
3890	W Leonis	9	4-6	Max. due in Nov. Will require morning observation in October and November.
4300	X Virginis	9	3	My obs. 1894 Mar.-June show no change.
4315	R Comæ	8	4-6	Max. due in July or early in Aug. Rapid rise. Becomes visible with 6 inches 6 weeks before max. Period uncertain.
4511	T Ursæ Maj.	5	2-3	Max. due in Apr. In good position for obs.
4557	S Ursæ Maj.	5	2-3	Max. due in July. In good position for obs.
4596	U Virginis	8	3	Max. due in May.
4805	W Virginis	7	4-5	Interesting. Short period. Obs. needed.
4816	V "	7	9-10	Min. due in May or June. Should be observed. No min. on record.
4492	Y Virginis	18	6	See next page.
4847	S "	8	2-3	Max. due in May. Obs. should begin at once.
5157	S Boötis	6	6	Min due in July. In good position for obs.
5190	R Camelopard	6	3	Max. due in June. In good position.
5194	V Boötis	8	3	Min. due in Aug. Should be observed.
5338	U Boötis	10	5	Max. in Mar. and Sept. Min. in June and Nov. Should be observed. Period changing?
5675	V Coronæ	7	3	Max. due in May. Very red. Change slow.
5955	R Draconis	6	3	Max. due in June. In good position.
6207	Z Ophiuchi	8	5	= B.D. + 1°.3417. Now near max. Min. in Sept? Should be observed.
7085	R I Cygni	10	3-4	Now near min. Change rapid. Should be observed.
7118	X Aquilæ	18	3	See next page.
7220	S Cygni	6	5	Max. due in July. Secondary maxima?
7609	T Cephei	6	2 3	Max. due in Sept. Change slow.
7779	S Cephei	6	4-5	Now near min. Very red. Difficult.
8324	V Cassiopeæ	15	6	Now near max. Next min. in July, in good position. Should be observed.
8600	R Cassiopeæ	6	2-4	Max. due in Aug. In good position at max.

We have considered the Sun as a homogeneous body, but it is almost certainly denser in the central part.

It seems clear that this increase of density would result in an increase of potential, and hence in an increase in radiation. The numbers in the third column of our table would thus all be increased, but whether the comparative radiation in the beginning would be so exceedingly small is more uncertain.

Since at first the nebula was probably nearly homogeneous, perhaps the first few numbers are not far from the truth. But as soon as the increased density toward the centre begins to be considerable, we should add to the corresponding values of t the energy given up in transferring various particles from their position in a homogeneous sphere to their more central position in a heterogeneous sphere of the same radius.

This might perhaps make the values of t descend more rapidly.

It is an easy and very interesting matter to apply formula (5) to investigate the future solar radiation, and it may also be applied to the planets. In the latter case the heat given up will be relatively insignificant. Even in the case of Jupiter the whole heat radiated cannot exceed $\frac{1}{100000}$th part of that which the Sun has given up, and the heat of all the planets with their satellites is comparatively insensible.

University of Chicago, July 15, 1895.

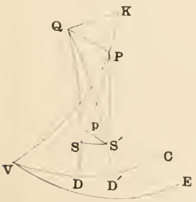
THE SOLAR EPHEMERIS.*

J. MORRISON, M. A., M. B., PH. D.

For Popular Astronomy.

The retrograde motions of the equinoxes must of course affect the right ascension and declination of a celestial body. By the aid of the diagrams the corrections for both precession and nutation may be easily found as follows:

Precession in R. A. and Decl.



Let P be the pole of the equator VE, Q of the ecliptic VC, S and S' two apparent consecutive positions of a body after an interval of t years; then $SPS' = a$, is the precession in R. A., and $Sp = b$, the precession in Decl., $S'p$ being perpendicular to PS. The co-latitude QS, QS' remaining unchanged, the precession in longitude is $SQS' = 50''.2t$. Produce SP to K so that $SK = 90°$. PK will then be the declination δ, join QK; then we shall have

* Continued from page 414.

$$a \sin PS' = S'p = SS' \sin S'Sp, \text{ but } SS' = 50.''2\ t \sin QS,$$
$$= 50''.2\ t \sin QS \cos PSQ,$$
and since $\cos KQ = \cos KS \cos QS + \sin KS \sin QS \cos KSQ,$
$$= \sin QS \cos KSQ,$$
therefore $a \sin PS = 50''.2\ t \cos KQ,$
$$= 50''.2\ t\ (\cos KP \cos PQ + \sin KP \sin PQ \cos KPQ).$$
$$= 50''.2\ t\ (\cos \delta \cos \omega + \sin \delta \sin \omega \sin \alpha).$$
because $PS' = PS$ very nearly and therefore we have
$$a \cos \delta = 50''.2\ t\ (\cos \delta \cos \omega + \sin \delta \sin \omega \sin \alpha)$$
or
$$a = 50''.2\ t\ (\cos \omega + \sin \omega \tan \delta \sin \alpha). \qquad (4)$$

Again Sp or $b = S'S \cos S'S\,p = 50''.2\,t \sin QS \sin PSQ$, approximately, the angles PSS' and PSQ being very nearly complementary,

But, $\sin QS \sin PSQ = \sin PQ \sin QPS$
therefore $b = 50''.2\ t \sin PQ \sin QPS$
$$= 50''.2\ t \sin \omega \sin (90° + \alpha)$$
$$= 50''.2\ t \sin \omega \cos \alpha \qquad (5)$$

If we make $t = 1$ year we shall have

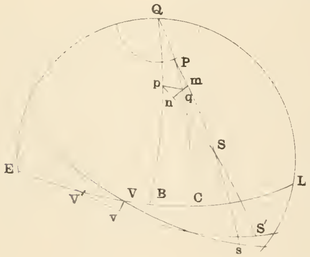
Annual precession in R. A., $\Delta\alpha = m + n \sin \alpha \tan \delta$

Annual precession in Decl., $\Delta\delta = n \cos \alpha$

where $m = 50''.2 \cos \omega,$ and $n = 50''.2 \sin \omega$
$= 46''.0256,$ $= 20''.0426,$

approximately. (See expression for a and a', *Am. Ephem.* page 280). The quantities m and n involve neither the R. A. nor the Decl. The term $50''.2\ t \cos \omega$ is expressive of that part of the precession in R. A. which is common to all stars. If the star's R. A. exceeds 12 hours, $\sin \alpha$ is negative and consequently the second term of (4) becomes negative and if it exceeds the first term, the precession in R. A. will be negative which happens with β Ursæ Minoris and others.

Nutation in R. A. and Decl.



Let Q be the pole of the ecliptic, ECL, P the pole of the mean equator Vs, and p the place of the pole of the true equator $V'S'$. Now QP and Qp can differ only by the half of $18''.5$ or $9''.25$ at most; $QP = \omega$ then $Qp = \omega + \Delta\omega$, and let $pQP = \Delta l$, where $\Delta\omega$ and Δl denote the nutation in the obliquity and longitude respectively. Let S be the position of a celestial body, then Vs is its mean R. A. and Ss

its mean Decl.; $V'S'$ its true R. A. and pS' its true Decl.; both *R. A. and Decl. as affected by nutation.*

Draw pq perpendicular to OP, nqm perpendicular to PS or pS and Vv perpendicular to $V'S'$: then we shall have

Nut. in R. A. $= V'S' - Vs = V'v + DS'$* approximately.

But $V'v = VV' \cos \omega = \Delta l \cos \omega$, and from the right-angled triangles DSS' and mnS we have

$$\cos \delta = \tan mn \cot S$$

and

$$\sin \delta = \tan DS' \cot S$$

hence

$$\tan \delta = \frac{\tan DS'}{\tan mn} = \frac{DS'}{mn}$$

since we may here use DS' and mn for their tangents

therefore $$DS' = mn \tan \delta$$

and Nut. in R. A.

$$= \Delta l \cos \omega + mn \tan \delta$$
$$= \Delta l \cos \omega + (mq + nq) \tan \delta$$
$$= \Delta l \cos \omega + (-Pq \cos \alpha + pq \sin \alpha) \tan \delta$$
$$= \Delta l \cos \omega - \Delta \omega \cos \alpha \tan \delta + \Delta l \sin \alpha \sin \omega \tan \delta$$
$$= \Delta l (\cos \omega + \sin \alpha \sin \omega \tan \delta) - \Delta \omega \cos \alpha \tan \delta \quad (6)$$

Again in the triangle Opq, we have (using pq and Δl for their sines), $pq = \Delta l \sin \omega$ and $Pq = \Delta \omega$ very nearly.

Then Nut. in Decl. $= PS - pS = Pm - pn$, approximately,

$$= Pq \cos (\alpha - 90^\circ) - pq \sin (\alpha - 90^\circ)$$
$$= \Delta \omega \sin \alpha + \Delta l \cos \alpha \sin \omega \quad (7)$$

Investigations in Physical Astronomy have shown that

$$\Delta l = -17''.2524 \sin \Omega + 0''.2063 \sin 2\Omega - 1''.2691 \sin 2\odot$$

and

$$\Delta \omega = \quad 9''.2237 \cos \Omega - 0''.0895 \cos 2\Omega + 0''.5507 \cos 2\odot$$

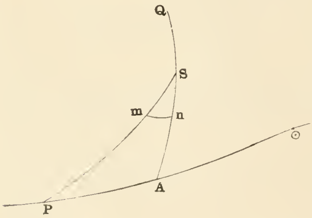
where Ω is the mean longitude of the Moon's ascending node and $\odot$ the true longitude of the Sun.

Aberration.

The next correction to be considered is aberration which is one of the most pleasing and refined subjects in astronomy. It furnishes us with a direct proof of the orbital motion of the earth and is due to the fact that light is not propagated instantaneously, but its velocity bears a sensible ratio to that of the spectator, resulting from the motion of the Earth in her orbit. In consequence of aberration the celestial bodies are not seen in the positions they really occupy, but are made visible by light which left them some

* The letter D was omitted in the copy for the cut and it was not noticed until after the cut was made. D should be at the intersection of lines Ss and $V'S'$.—*Ed.*

time previously during which both we and they have changed our position in space. Aberration takes place in the plane passing through the body, the Earth and the tangent to the Earth's orbit, and is always in the direction to which the Earth is moving, that is, to the point of the ecliptic 90° behind the Sun's place, and consequently the body is elevated or depressed with regard to the direction of the Earth's motion, according as the Earth is receding from or approaching the body. If we denote the angle which a line joining the Earth and body makes with the direction of the Earth's motion by $\oplus$, which is technically called the angle of the "Earth's way," then it is very easy to show that the aberration $= \dfrac{\text{velocity of Earth}}{\text{velocity of light}} \cdot \sin \oplus = h \sin \oplus$, where h is a constant and is found as follows: If R denote the radius of Earth's orbit and L the velocity of light per second, then $\dfrac{R}{L}$ = number of seconds required for light to pass from the Sun to the Earth = 498 seconds, during which time the Sun (or rather the Earth) will move through $\dfrac{498}{365.256d} \times 2\pi R$, or in seconds of arc, $20''.445 = h$. This mean value of h is subject to slight variations, not much exceeding $0''.33$, in consequence of the variable motion of the Earth in its orbit. As the plane in which aberration takes place is always changing its position in space, the effect will be to make the bodies when referred to the celestial sphere, describe curves about their true place



Thus, let $PA\odot$ be an arc of the ecliptic, P the point to which the Earth is moving, $\odot$ the Sun, 90° in advance, Q the pole of the ecliptic and S a star, then SP measures the angle of the "Earth's way," and SA the star's latitude. In SP take $Sm = h \sin PS$, m will then be the star's *apparent* place or its place as affected by aberration. Draw mn perpendicular to AQ and let $mn = x$ and $Sn = y$, then we have:

$$x = mn = Sm \sin mSn = h \sin PS \sin mSn = h \sin AP$$
$$y = Sn = Sm \cos mSn = h \sin PS \cos mSn = h \sin AS \cos AP.$$

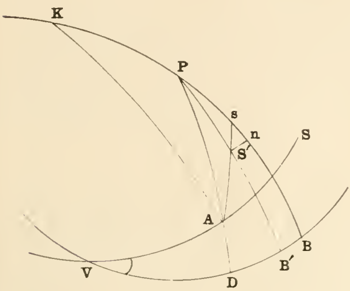
Dividing each of these equations by h, squaring and adding we have

$$\left(\frac{x}{h}\right)^2 + \left(\frac{y}{h \sin AS}\right)^2 = 1$$

which is the equation of an ellipse whose semi-axes are h and $h \sin AS$ or $20''.445$ and $20''.445$ sin lat., the major axis being parallel to the ecliptic.

If the lat. $= 90°$, or the body is at the pole of the ecliptic, then the ellipse becomes a circle and if lat. $= 0$, then we have $x = h \sin AP = h \cos (\lambda - \odot)$ where λ and $\odot$ are the longitudes, of the star and Sun respectively. This is the equation of a straight line and therefore the body will appear to vibrate about its mean place.

Aberration in R. A. and Decl.



In the diagram let VS be the ecliptic, VB the equator, P its pole, s the position of a star and A a point $90°$ behind S, the Sun, and S' its apparent position.

Draw $S'n$ perpendicular to PB and take $s\,K = 90°$, then BB' or BPB' is the aberration in R. A. and sn in Decl., VB the star's R. A. $= \alpha$ and sB its Decl $= \delta$.

We have

$$\begin{aligned} S'n &= S's \sin S'sn \\ &= h \sin As \sin S'sn \\ &= h \sin AP \sin APs \\ &= h \cos AD \sin DB \\ &= h \cos AD \sin (\alpha - VD) \\ &= h \cos AD \sin \alpha \cos VD - h \cos AD \cos \alpha \sin VD \\ &= h \sin \alpha \cos VA - h \cos \alpha \cos \omega^* \sin VA \end{aligned}$$

but $VA = \odot - 90°$, $\odot$ being the Sun's longitude, therefore

$$S'n = h (\sin \alpha \sin \odot + \cos \alpha \cos \omega \cos \odot)$$

and the aberration in R. A.

$$\begin{aligned} &= - S'n \sec \delta \\ &= - h \sec \delta \,(\sin \alpha \sin \odot + \cos \alpha \cos \omega \cos \odot) \qquad (8) \\ &= \text{apparent R. A.} - \text{true R. A.} \end{aligned}$$

The aberration in declination is $- sn$,

* Angle $SVB = \omega$

$$
\begin{aligned}
\text{but } -sn &= -sS' \cos S'sn \\
&= h \sin As \cos AsP \\
&= h \cos AK \\
&= h \cos AP \cos PK - \sin AP \sin PK \cos APK \\
&= h\,(\sin AD \cos sB - \cos AD \sin sB \cos(\alpha - VD)) \\
&= h\,(\sin(\odot - 90^\circ) \sin\omega \cos\delta - \sin\delta \cos AD \cos\alpha \cos VD \\
&\qquad\qquad - \sin\delta \cos AD \sin\alpha \sin VD) \\
&= h\,(-\cos\odot \sin\omega \cos\delta - \sin\delta \cos\alpha \cos VA \\
&\qquad\qquad - \sin\delta \sin\alpha \cos\omega \sin VA)
\end{aligned}
$$

therefore the aberration in declination

$$
\begin{aligned}
&= -h\,(\cos\odot \sin\omega \cos\delta + \sin\delta \cos\alpha \sin\odot \\
&\qquad\qquad - \sin\delta \sin\alpha \cos\omega \cos\odot) \qquad (9) \\
&= \text{apparent Decl.} - \text{true Decl.}
\end{aligned}
$$

In the case of the Sun, Moon and planets the aberration is easily found thus: Let Δ denote the distance of the body from the Earth expressed in terms of the Earth's mean distance from the Sun regarded as unity, and m'' the *geocentric* motion of the body in *one* second of time, we shall evidently have

$$\text{the aberration} = 498\Delta m \text{ (in seconds of arc)} \qquad (10)$$

This quantity can of course be resolved in any direction and will preserve the same ratio to the body's geocentric motion when resolved in the same directions, hence it follows that if T denote the true and A the apparent longitude, latitude, R. A. or Decl. of a celestial body and m the geocentric motion of the same quantities in one second, we shall have

$$A - T = 498''\Delta m \qquad (11)$$

For the Sun we always have

$$\text{Apparent long.} = \text{true long.} - 20''.445\, R$$

where R denotes the radius vector.

In the case of the Moon whose distance from the Earth is about $\frac{1}{400}$ of that of the Sun the aberration will always be about

$$0''.5 \text{ or } \frac{20''.445}{400}$$

The student should make himself perfectly familiar with these formulæ which find an extensive application in the computation of the apparent places of the fixed stars, Sun and planets.

Before dismissing the consideration of these astronomical corrections we may remark that, in the inequalities of precession, aberration and nutation, observation preceded theory. They

were first detected as phenomena and their physical cause discovered afterwards, except in the case of that portion of the nutation which depends on the Sun. From its extreme minuteness it could never be perceived as a phenomenon but was first conjectured to exist from analogy. The inequality of the Moon's force in generating precession being found to produce a lunar nutation, that of the Sun was also presumed to cause a solar nutation resembling the lunar. Its law has been investigated and its quantity computed and added to the lunar portion thus making what is called the luni-solar nutation.

After this long digression we will now resume the consideration of the solar ephemeris and will commence with the column headed "Sidereal time of semi-diameter passing the meridian."

Let m = the increase of the Sun's R. A. in one mean solar second, then since any meridian moves $15''$ in one sidereal second, $15''\mu - m''$ = the gain of the meridian on the Sun, in the same interval, where $\mu = 1.00273791$ the ratio of a mean solar to a sidereal day.

If d denote the Sun's semi-diameter, then $\frac{d}{15\mu - m}$ = number of mean solar seconds in which the Sun's semi-diameter crosses the meridian, when the Sun is on the equator, and for any declination δ, it will be $\frac{d}{(15\mu - m)\cos\delta}$ in mean time or, if t_s denote the sidereal interval, we shall have

$$t_s = \frac{d\mu}{(15\mu - m)\cos\delta}$$

$$= \frac{d}{(15 - .9972696m)\cos\delta} \qquad (12)$$

The value of m can be found from the tabulated R. A. or the hourly variation of the same in the adjoining column.

The sidereal time occupied by the Sun's semi-diameter in crossing the meridian can also be found without using the value of m as follows:

Let R denote the Earth's radius vector and T the length of a tropical year in days, then the increase of the Sun's longitude in one sidereal day $= \frac{360^\circ}{R^2(T+1)}$, by the principles of elliptic motion, and when this is reduced to the equator the increase in R. A. $= \frac{360^\circ}{R^2(T+1)} \cdot \frac{\cos\omega}{\cos^2\delta}$, for in the right spherical triangle formed by the longitude (l), the R. A. (α), and Decl. (δ), we have

$$\tan \alpha = \cos \omega, \tan l$$

and
$$\frac{\frac{d\alpha}{dt}}{\cos^2 \alpha} = \frac{\cos \omega \frac{dl}{dt}}{\cos^2 l} = \frac{\cos \omega \frac{dl}{dt}}{\cos^2 \alpha \cos^2 \delta}$$

therefore
$$\frac{d\alpha}{dt} = \frac{dl}{dt} \,.\, \cos \omega \sec^2 \delta$$

that is, the Sun's increment in longitude is reduced to that in R. A. By multiplying the former by $\cos \omega \sec^2 \delta$.

[If $d\alpha = dl$, *i. e.* when $(l - \alpha)$ is a maximum, the above equation furnishes a solution of Problem 2.]

Hence it follows that in one sidereal second the Sun's centre recedes from the equinoctial point by $\dfrac{15''}{R^2 (T + 1)} \dfrac{\cos \omega}{\cos^2 \delta}$, and in the same time the meridian recedes from the same by 15″, consequently the gain of the meridian on that passing through the Sun's centre is

$$15'' \left(1 - \frac{\cos \omega}{R^2 \, (T + 1) \, \cos^2 \delta}\right)$$

The Sun's semi-diameter at mean distance is 16′ 2″ = 962″ and at distance R it is $\dfrac{962''}{R} = d$ and the angle which this subtends at the pole of the equator is $\dfrac{d}{\cos \delta}$, hence we have

$$15''\left(1 - \frac{\cos \omega}{R^2 \, (T + 1) \cos^2 \delta}\right) t_8 = \frac{d}{\cos \delta}$$

whence
$$t_8 = \frac{64''.13 \, R^2 \, (T + 1) \cos \delta}{R^2 \, (T+1) \cos^2 \delta - \cos \omega} \qquad (13)$$

which may be easily adapted to logarithmic computation, the factor $64''.13 \; (T + 1)$ being constant and ω the apparent obliquity of the ecliptic.

[TO BE CONTINUED.]

Problems.

8. Show that when the equation of the centre is a maximum,

$$\cos E = \frac{1 - (1 - e^2)^{\frac{1}{4}}}{e}$$

where E is the eccentric anomaly and e the eccentricity.

9. Find the R.A. of a star for which the aberration in R.A. vanishes at the summer solstice.

10. Suppose the difference between the declination of a star at the pole of the ecliptic in March 21 and Sept. 21, is 41″, find the velocity of light, the Sun's distance being 92,500,000 miles and the year of 365.256 days.

11. If the aberration of a star in long. be equal to that in lat. prove that

$$\sin 2\beta = 2 \cot (\odot - l)$$

where l and β are the long. and lat. of the star and $\odot$ the long. of the Sun.

12. Is there a time when all stars which lie in a certain great circle, *have no* aberration? And if not, why not?

13. If $\Delta\omega$ denote the change of the obliquity due to nutation show that the changes in the R. A. and Decl. are

$$\Delta\alpha = -\tan\delta\ \Delta\omega$$
$$\Delta\delta = \sin\alpha\,\Delta\omega$$

14. The zenith distances of a star when it crosses the meridian and prime vertical are z and z', and if δ be the Decl. and φ the lat. of the place show that

$$\cot\delta = \operatorname{cosec} z \sec z' - \cot z$$

and

$$\cot\varphi = \cot z - \operatorname{cosec} z \cos z'$$

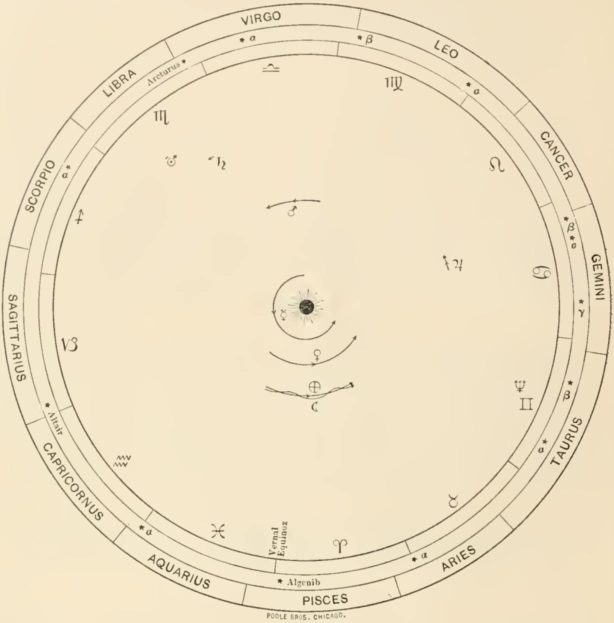
PLANET NOTES FOR SEPTEMBER AND OCTOBER.

H. C. WILSON.

Mercury, *Venus* and *Mars* begin the month of September nearly in line and all quite close to the Sun, as seen from the Earth, Mercury and Mars on the farther side and Venus this side of the Sun. Mercury and Mars will be in conjunction in right ascension on the morning of Sept. 1 and only 1′ apart in declination. Mercury and Venus will be in conjunction, but about 10° apart in declination on the evening of Sept. 5. Venus and Mars will be in conjunction, 10° apart in declination on the evening of Sept. 9. By referring to the diagram one will easily see the difference in their movements during the two months. Mars, moving forward more slowly than the Earth, falls behind the Sun, reaching conjunction on the morning of Oct. 11. Mercury moving more rapidly comes out from behind the Sun; is at greatest eastern elongation Oct. 1; passes quickly between the Earth and Sun and is at inferior conjunction Oct. 25. Venus, moving between the Sun and Earth catches up with the latter,—is at inferior conjunction,—at midnight Sept. 18, and thereafter becomes morning planet.

Jupiter is coming into position for observation in the morning. He is in the constellation Cancer, southeast from Castor and Pollux. He will be at quadrature, 90° west from the Sun on the last day of October. From the 1st to the 20th of October Jupiter will be just south of the Præsepe cluster of stars, but not close enough to occult any of them.

Saturn and Uranus are approaching conjunction with the Sun and so are not in favorable position for observation.



The Planets and the Zodiac for September and October.

Neptune is in Taurus about 1° southwest of the star *n*. This planet may be observed in the morning, but only with telescopes of considerable power.

Planet Tables for September and October.

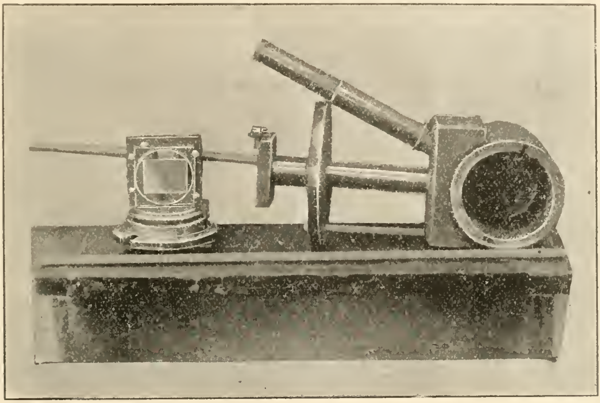
[The times given are local time for Northfield. To obtain Standard Times for places in approximately the same latitude, add the difference between Standard and Local Time if west of the Standard Meridian or subtract if east.]

MERCURY.

Date.	R. A. h m	Decl. ° ′	Rises. h m	Transits. h m	Sets. h m
Sept. 5	11 56.9	+ 0 44	6 52 A. M.	12 58.4 P. M.	7 5 P. M.
15	12 50.9	− 6 27	7 35 "	1 12.8 "	6 51 "
25	13 38.4	− 12 35	8 7 "	1 21.0 "	6 35 "
Oct. 5	14 16.1	− 17 0	8 26 "	1 19.4 "	6 13 "
15	14 30.0	− 17 58	7 59 "	12 48.9 "	5 38 "
25	13 55.4	− 12 27	5 23 "	11 36.1 A. M.	4 49 "

slit is fitted to the collimator and a negative eye-piece magnifying about six or eight diameters used on the observing telescope.

A five inch circle of hard wood one-half inch in thickness is fitted near the slit end of the collimator, through the center of which the tube passes; holes are also bored for the brass rods, which are fastened to it with screws the observing telescope is also attached to the circle so that the entire instrument is light, strong and steady and easily adjusted to the large telescope.



The cut shows the spectroscope with one end of the grating box removed, to this is attached the T piece holding the grating which is rotated by means of a couple of geared wheels.

To attach the spectroscope to the telescope is but the work of a minute or so, simply inserting the rods into the collar the proper distance to secure a sharp image of the Sun or other object, on the slit plate, and clamping. Of course some adjustments are necessary with both collimator and observing telescope, for different orders of spectra, varying conditions of the atmosphere, etc.

As to the cost, the material for box, circle and rods need not exceed a dollar or so. Good achromatic lenses with tubes will cost about 3 or 4 dollars each. The slit is an important piece of mechanism and may be purchased or made. The grating, of course, will be the main item of expense, but a good one of about 1¼-inches diameter can be had from Mr. Brashear for about 15 to 20 dollars, which is not expensive, when it is considered that

in resolving power it equals many prisms and will afford the student an interesting and fascinating field of study in the new astronomy.

ALTA Iowa, Sept. 7th 1895.

THE SOLAR EPHEMERIS.*

J. MORRISON, M. A., M. B., PH. D.

FOR POPULAR ASTRONOMY.

For the computation of approximate values of the solar coördinates, etc., for places other than those on the meridian of Greenwich and Washington a column of "hourly variations" is added. The hourly variation of any quantity *at any time*, is the variation or change which would take place in an hour, if the rate of change *at that time*, were to continue uniform throughout the hour. This must not be confounded with the *average variation* which is defined as that hourly variation which if continued during an hour, would produce the change in the value of the quantity that actually does take place during this interval; thus if the difference between the values if a quantity at the times T and $T + t$ be divided by the interval t, the quotient is the average variation, but the hourly variation given in the ephemeris if continued uniform during the hour would *not* necessarily produce the change which the quantity actually undergoes in the same interval.

Let $a_{\prime\prime}$, $a_{\prime}$, etc., be the values of any quantity such as the R A. or Decl. of the Sun or a planet for the dates T_{-2}, T_{-1}, etc., which are consecutive mean noons and let the 1st, 2d, etc., differences be formed as indicated in the following table:

Date.	Function.	1st Diff.	2d Diff.	3rd Diff	4th Diff.	5th Diff
T_{-2}	$a_{\prime\prime}$					
		$b_{\prime\prime}$				
T_{-1}	$a_{\prime}$		$c_{\prime\prime}$			
		$b_{\prime}$		$d_{\prime\prime}$		
T	a		$c_{\prime}$		$e_{\prime\prime}$	
		b		$d_{\prime}$		$f_{\prime\prime}$
T_1	a'		c		$e_{\prime}$	
		b'		d		$f_{\prime}$
T_2	a''		c'		e	
		b''		d'		
T_3	a'''		c''			
		b'''				
T_4	a''''					

* Continued from page 32.

Put $b + b' = 2b_0$, $c' = c_0$, $d'' + d' = 2d_0$, $e'' = e_0$, etc., then we shall readily find

$$b = b_0 + \frac{1}{2}c_0,\ d' = d_0 + \frac{1}{2}e_0,\ d = d' + e' = d_0 + \frac{3}{2}e_0$$

and $$c = c' + d' = c_0 + d_0 + \frac{1}{2}e_0;$$

the fourth differences may be assumed to be equal.

Now if $a^{(t)}$ denote the value of the function at any other date we have by the well known formulæ:

$$a^{(t)} = a + tb + \frac{t(t-1)}{1\ .\ 2}c + \frac{t(t-1)\ (t-2)}{1\ .\ 2\ .\ 3}\ d + \text{etc.},$$

and substituting the values of b, c, etc. as above. We easily find after reduction

$$a^{(t)} = a + t(b_0 - \frac{1}{6}d_0) + \frac{t^2}{2}(c_0 - \frac{1}{12}e_0) + \frac{t^3}{6}d_0 + \frac{t^4}{24}e_0 + \text{etc.},$$

and differentiating we have

$$\frac{da^{(t)}}{dt} = b_0 - \frac{1}{6}d_0 + t(c_0 - \frac{1}{12}e_0) + \frac{t^2}{2}d_0 + \frac{t^3}{6}e_0 + \text{etc.,} \qquad (14)$$

making $t = 0, \pm 1, \pm 2$ etc., we have the variations at the dates $T\ T_{-1}\ T_1$ as follows:

$$\begin{array}{l|l}
\text{At } T_{-2} & b_0 - 2c_0 + \frac{11}{6}d_0 - \frac{7}{6}e_0 \\
T_{-1} & b_0 - c_0 + \frac{1}{3}d_0 - \frac{1}{12}e_0 \\
T & b_0 - \frac{1}{6}d_0 \\
T_1 & b_0 + c_0 + \frac{1}{3}d_0 + \frac{1}{12}e_0 \\
T_2 & b_0 + 2c + \frac{11}{6}d_0 + \frac{7}{6}e_0
\end{array} \qquad (15)$$

Of course each of these must be divided by the number of hours between the dates T, T_1 etc., as in the following example:

Date.	☿'s Decl.	1st Diff.	2d Diff.	3rd Diff.	4thDiff
	° ′ ″	′ ″	″	″	″
1895, May 1	+ 13 6 45.0				
		+ 50 37.5			
2	13 57 22.5		— 29.6		
		+ 50 7.9		— 11.0	
3	14 47 30.4		— 40.6		— 1.0
		+ 49 27.3		— 12.0	
4	15 36 57.7		— 52.6		
		+ 48 34.7			
5	+ 16 25 32.4				

Suppose we require the *hourly* variation in May $3^d\ 0^h$, we shall then have

$$b_0 = \frac{1}{2}(50'\ 7''.9 + 49'\ 27''.3)$$

$$= 49'\ 47''.6,\ c_0 = -\ 40''.6,\ d_0 = -\ 11''.5,\ e_0 = -\ 1''.0$$

and hourly variation

$$= \frac{1}{24}\left(b_0 - \frac{1}{6}\,d_0\right) = +\ 124''.56$$

which is recorded in its appropriate column opposite May 3. For May 3, 1^h we find the hourly variation to be

$$\frac{1}{24}(+\ 49'\ 47''.6 - 40''.6 - 3''.83 - 0''.08) = +\ 122''.63,$$

and so on.

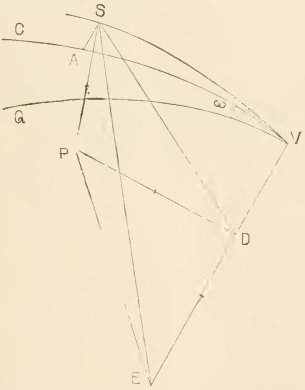
The Washington Ephemeris.

This is computed from the Greenwich ephemeris just described by simply interpolating the various quantities for $5^h\ 8^m\ 12^s.04$—the west longitude of Washington. The R. A. Decl. and equation of time for *apparent* noon, are derived from those for Washington mean noon in precisely the same manner as has already been described, and therefore no further explanation is necessary.

The sidereal time at mean noon is obtained from that of Greenwich mean noon by adding the increase in $5^h\ 8^m\ 12^s.04$ which is $+\ 9^s.8565 \times 5.1367 = 50^s.62988$.

The Sun's Equatorial Rectangular Co-ordinates.

The formulæ are given in the appendix and we here give the demonstration.



Let VQ represent the equator, VC the ecliptic, S the Sun, SA the Sun's latitude $= \beta$, VA the Sun's longitude $= \lambda$, CVQ the obliquity $= \omega$

$ED = X$, $DP = Y$ and

$$SP = Z,\ SE = R.$$

From the right triangle AVS we have

$$\sin\lambda = \tan\beta\cot AVS$$

or
$$\tan AVS = \frac{\tan\beta}{\sin\lambda}$$

or
$$AVS = \frac{\beta}{\sin\ \lambda} = \gamma \text{ suppose.}$$

Also $\cos VS = \cos\beta\cos\lambda = \cos\lambda$, since β is always less than $1''$ and $\cos\beta$ may be taken $= 1$ and therefore VES may be taken equal to λ.

$$X = R\cos\lambda \tag{16}$$

$$\begin{aligned} Y &= DP = DS\cos SDP \\ &= R\sin\lambda\cos(\omega+\gamma) \\ &= R\sin\lambda\cos\omega\cos\gamma - R\sin\lambda\sin\omega\sin\gamma \\ &= R\sin\lambda\cos\omega - R\beta\sin\omega \\ &= R\sin\lambda\cos\omega - R\beta''\sin 1''\sin\omega \\ &= R\sin\lambda\cos\omega - R\beta''\times 19.3 \text{ (in units of the seventh} \end{aligned}$$

decimal place) (17)

$$\begin{aligned} Z &= R\sin\lambda\sin SDP \\ &= R\sin\lambda\sin(\omega+\gamma) \\ &= R\sin\lambda\sin\omega + R\beta\sin 1''\cos\omega \\ &= R\sin\lambda\sin\omega + R\beta''\times 44.5 \text{ (in units of the seventh} \end{aligned}$$

decimal place) (18)

Differentiating (16) we have

$dX = -R\sin\lambda d\lambda$, but $R\sin\lambda = Y\sec\omega$ from (17) neglecting the term 19.3 $R\beta''$, and $d\lambda$ is negative being the reduction of the true longitude for precession and nutation, therefore we get

$$dX = Y\sec\omega d\lambda$$

which must be made homogeneous, dx being expressed in linear measure and $d\lambda$ in angular measure, hence multiplying the latter by $\sin 1''$ and writing $\Delta X'$ and $\Delta\lambda$ for dX and $d\lambda$ respectively since they are not now to be regarded as infinitesimals we have

$$\Delta X' = +Y\sec\omega\,\Delta\lambda\sin 1'' \tag{19}$$

Differentiating (17) we get by the aid of (16) and (18) after rendering both members of the equation homogeneous

$$\Delta Y' = -X\cos\omega\Delta\lambda\sin 1'' + Z\Delta\omega\sin 1'' \tag{20}$$

and in a similar manner from (18) we find

$$\Delta Z' = -X\sin\omega\Delta\lambda\sin 1'' - Y\Delta\omega\sin 1'' \tag{21}$$

In the application of these last three formulæ the signs of $\Delta\lambda$ and $\Delta\omega$ must be regarded as positive; they are, however, *both negative*, being the reduction of the *true* longitude and *true* or *apparent obliquity* of date to the *mean longitude* and *obliquity* respectively. Thus for April 10, 1895, $\Delta\lambda = -15''.73$ and $\Delta\omega = -6''.33$, (see page 278 *Am. Ephem.*) but the negative sign is already taken into account in the formulæ in order to conform to the ephemeris, but the ephemeris is in error in regard to $\Delta\omega$ which is not "the reduction of the mean to the apparent obliquity."

It now remains to explain the terms $-9.4\tau R\sin(\lambda+187°)$ and $+21.7\tau R\sin(\lambda+187°)$ which enter into the values of $\varDelta Y'$ and $\varDelta Z'$ respectively. They appeared for the first time in the *American Ephemeris* for 1869 and were evidently taken, without acknowledgement, from the Connaissance des Temps and up to the year 1886 the formulæ for $\varDelta Y'$ and $\varDelta Z'$ contained two symbols for the Sun's longitude. These terms take into account the motion of the ecliptic itself due to the attraction of the planets. The great circle with which the ecliptic coincided at the beginning of the year 1750 was taken by Laplace as the fundamental plane to which all subsequent and preceding positions of the ecliptic are to be referred. If Π denote the longitude of the ascending node of the actual ecliptic at any subsequent date t, upon the fixed ecliptic reckoned from the equinox of 1750.0 and π, the mean annual motion of the obliquity of the actual ecliptic to the fixed plane, we have according to the researches of physical astronomy

$$\Pi = 171°\ 36'.2 - 5''.21t$$

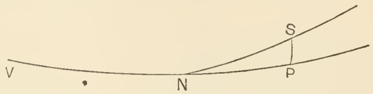
and

$$\pi = 0''.4889t - 0''.00000307t^2$$

and for 1850 we shall find

$$\begin{aligned}\Pi &= 171°\ 36'.2 + 50''.2\times 100 - 5''.21\times 100\\ &= 172°\ 51'.2 \text{ or } 173° \text{ approximately}\end{aligned}$$

and from the second of the above equations we see that the annual change in π is $0''.4889$—some authorities give $0''.48$ and others only $0''.46$.



Now referring the Sun's place to the ecliptic and equinox of the beginning of the year and denoting the changes in Y and Z by $\varDelta Y_2$ and $\varDelta Z_2$ due to this cause and we shall evidently have

$$Y_2 = R\cos\omega \text{ and } Z_2 = R\sin\omega$$

and

$$\begin{aligned}\varDelta Y_2 &= -R\sin\omega\, d\omega\\ \varDelta Z_2 &= +R\cos\omega\, d\omega\end{aligned}\qquad(22)$$

where $d\omega$ may be regarded as the Sun's latitude referred to ecliptic of beginning of the year.

Let NV be the ecliptic of any fixed epoch and SN the ecliptic of any subsequent epoch t: Now we know that NS has a uniform motion around the node N and that $VN = 173°$.

Draw SP perpendicular to NP, then we have

$$SP = d\omega,\ NS = \lambda - 173^\circ, \text{ and } PNS = 0''.4889t$$

t being expressed in years or fractions of a year; then

$$\sin PS = \sin PNS \sin NS$$

that is
$$d\omega = 0''.4889t \sin(\lambda - 173^\circ)$$
$$= 0''.4889t \sin(\lambda + 187^\circ)$$

substituting in (22) we find

$$\varDelta Y_2 = - Rt \sin\omega \sin 1'' \sin(\lambda + 187^\circ) \times 0''.4889$$
$$\varDelta Z_2 = + Rt \cos\omega \sin 1'' \sin(\lambda + 187^\circ) \times 0''.4889$$

Computing the numerical value of the coefficients

$$\sin\omega \sin 1'' \times 0''.4889 \text{ and } \cos\omega \sin 1'' \times 0''.4889$$

we have

$\sin\omega$	= 9.5998756	$\cos\omega$	= 9.9625532
$\sin 1''$	= 4.6855749	$\sin 1''$	= 4.6855749
$0''.4889$	= 9 6892200		9.6892200
.0000009,43	= 3.9746705	.0000021,74	= 4.3373481

Therefore we have for the corrections due to the motion of the ecliptic and referred to mean equinox of the beginning of the year,

$$\varDelta Y_2 = - \ \ 9.4tR \sin(\lambda + 187^\circ)$$
$$\varDelta Z_2 = + 21.7tR \sin(\lambda + 187^\circ)$$

$\varDelta X'$ is evidently not affected by this motion.

We will close this paper with the following example:

Find the Sun's equatorial rectangular coördinates on 1895, Aug. 8.0 G. M. T., having given $\lambda = 135^\circ 39' 4''$; $\beta = + 0''.24$ $\varDelta\lambda = 36''.11$ (see page 278); $\varDelta\omega = 8''.57$, $\log R = 0.0058962$ and $\tau = .616$.

$\log R$	= 0 0058962		0.0058962		0 0058952
$\cos\lambda$	= 9 8543647n	$\sin\lambda$	= 9.8444932		9.8444932
		$\cos\omega$	= 9.9625451	$\sin\omega$	= 9.5999189
$\log X$	= 9.8602609n		9 8129345		9.4503083
X	= −.7248713		+ 0.6500316,3		+ 0 2820384,4
		log 19.3	= 1 2855573	log 44.5	= 1.6483600
		R	= 0 0058962		0 0053962
		β	= 9.3802112		9.3802112
			0.6716647		1.0344674
			4.695		10.826
		Y	= + 0.6500312	Z	= + .2820395
$\log Y$	= 9 8129341	$\log X$	= 9.8602609	$\log - X$	= 9.8602609
$\sec\omega$	= 0.0374549	$\cos\omega$	= 9.9625451	$\sin\omega$	= 9 5999189
$\varDelta\lambda \sin 1''$	= 6.2432024	$\varDelta\lambda \sin 1''$	= 6.2432024	$\varDelta\lambda \sin 1''$	= 6 2432024
$\log \varDelta X'$	= 6.0935914		6.0660084		5.7033822
$\varDelta X'$	= + 0.0001240		+ 0.0001164,15		+ 0.00005051

$$\begin{array}{rlrl}
\log Z & = 9.4503099 & \log - Y & = 9.8129341n \\
\Delta\omega \sin 1'' & = 5.6185557 & \Delta\omega \sin 1'' & = 5.6185557 \\
\hline
 & 5.0688656 & & 5.4314898n \\
 & + 0.0000117{,}18 & & - 0.0000270 \\
\log - 9.4 & = 0.97313n & \log 21.7 & = 1.33646 \\
\tau R & = 9.78602 & \tau R & = 9.78602 \\
\sin(\lambda + 187^\circ) & = 9.78296n & \sin(\lambda + 187^\circ) & = 9.78296n \\
\hline
 & 0.54211n & & 0.90544n \\
 & + 3.48 & & - 8.04 \\
\Delta Y' & = + 0.0001285 & \Delta Z' & = + 0.0000227
\end{array}$$

The use of the Sun's equatorial rectangular coördinates will be shown in a subsequent paper on the planetary ephemerides.

[TO BE CONTINUED].

PROBLEMS.

15. When the altitude of a star is equal to the latitude φ of the place of observation show that

Cos P =

$$\cos = P \tan \varphi \tan \left(45^\circ - \frac{\delta}{2}\right)$$

and
$$\sin \frac{1}{2} A = \sec \varphi \sin \left(45^\circ - \frac{\delta}{2}\right)$$

where P is the hour angle, A the azimuth and δ the declination.

16. When does the altitude of a celestial body vary most rapidly?

17. In latitude $+ 38^\circ$ at the time of the equinox find the time occupied by the Sun in rising or setting, supposing his semi-diameter $16'$.

18. Given the latitude of the place and the Sun's semi-diameter, find the declination when the difference between the times of rising or setting of the upper and lower limbs is a minimum.

19. Is it strictly true that the azimuth of a celestial body is not affected by parallax? And if not, why not?

20. At a given place, show that the velocity in azimuth at rising and setting is constant for all stars.

21. If E denote the *maximum* value of the equation of time due to the *obliquity* alone, prove the following relations

$\cot \lambda = \cos \delta$, $\cot^2 \lambda = \cos \omega$, and
$\sin E = - \cos 2\lambda$, λ being the Sun's long. and δ the Decl.

(This problem proposed by Mr. O. E. Harmon, Chehalis, Wash).

ERRATA.

In Prob. 1, for $\cot \frac{\omega}{2}$ write $\cot^2 \frac{\omega}{2}$.

In Prob. 13, for $\Delta\alpha = - \tan \delta \Delta\omega$ write $\Delta\alpha = - \tan \delta \cos \alpha \Delta\omega$.

M.

2-3-36

is, when $H-\lambda$ is large, this assumption may necessitate a second computation before the error of the computed time is reduced to less than one minute. Secondly, on the score of saving time. This will be evident to anyone that has tried both methods. And it will be still more evident, when we have prepared the materials for the graphic construction of all the occultations visible at our place of observation, by the rapid method to be detailed in a later article of this series.

GEORGETOWN COLLEGE OBSERVATORY,
Washington, D. C.

NEW PHOTOGRAPHIC DISCOVERY.—THE SOLAR CORONA PHOTOGRAPHED IN DAYLIGHT. CHIEF CHARACTERISTIC OF THE CORONA.

D. E. PACKER.

From innumerable experiments made during the last six months it has been found that metallic plates, foils and films are relatively transparent to solar radiance of high refrangibility, and that photographic plates screened by such media during exposure to direct sunlight are affected in proportion to the thinness and celestial conductivity of the interposed screen.

This important discovery has been successfully employed in photographing the solar corona. The results obtained are so remarkable and the recorded changes so great and rapid that great caution had to be exercised till a sufficient mass of confirmatory evidence could be obtained to justify this announcement. The photographs secured range from 1895, July 3 to Dec. 15, on which latter date Comet Perrine is also shown very close to its calculated place.

The earlier photographs were principally taken with a camera of 4-inch aperture, the metallic screens employed being tin and lead foil and sheet copper. Prominent equatorial extensions over the regions of active sunspot groups are the chief features of these pictures.

An immense advance was made by the introduction of a small clear aperture (pin hole) in place of the camera lens. As expected a far greater mass of detail, more sharply definite and exhibiting a considerably greater extension of corona was obtained by this method. Generally three or four exposures by both methods, and through different media were obtained on the

same date, and the more prominent details invariably found to agree; a proof of the objective reality of the phenomena.

A preliminary discussion of the photographs seemed to disclose the following characteristics:

1. A very close and intimate connection with contemporary sunspots and sunspot groups—active sunspots, especially when near the Sun's limb indicated by enormous radiations over the particular region of activity. It may be regarded as an axiom that "every sunspot has its coronal ray," as every prominent radiation may be easily assigned to its particular spot, to which it invariably points.

2. That the well known typic[illegible] maximum and spot-minimum coronal phases alterna[illegible] rapidly, apparently synchronizing with observed phases of short period spot activity and quiescence.

3. That many of the most prominent radiations exhibit a decided helical structure, two or three convolutions, in some instances being distinctly traceable—a surprising and unexpected feature.

4. The great photographic strength of the coronal rays as compared with the feeble image of the solar disc in the photograph.

5. That the corona is an electrical phenomenon. The remarkable association between sunspots and coronal radiations is, perhaps, the most important feature of the research. If, as appears, we are able to associate particular sunspots with their coronal rays, and study the variation of both at the same time, an immense advantage will have been gained. The research is one that appeals to every student of solar physics and as it can be pursued by simple and inexpensive means, we may safely predict a rapid increase in our knowledge of the Sun's immediate surroundings in the near future.

South Birmingham, England.

THE LUNAR EPHEMERIS.*

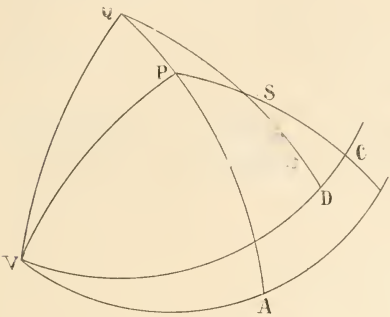
J. MORRISON, M. A., M. B., Ph. D.

For Popular Astronomy.

On pages 272-275 are tabulated the Moon's true longitude and latitude for every mean noon and midnight. The other coördinate—the true distance—is not given but instead of it the equator-

* Continued from page 92, No. 22.

ial horizontal parallax is tabulated on page IV of each month. These coördinates are derived directly from Hansen's Lunar Tables, are fundamental and constitute the basis for all subsequent calculations on the Moon. They are transformed into R. A. and Decl. by the usual formulæ of which the following is a demonstration:



Let VA represent the equator, P its pole, VC the ecliptic and Q its pole, S the position of the Moon or any other celestial body, and let λ and β denote its longitude and latitude respectively. The longitude of the pole of the equator is 90° and the right ascension of the pole of the ecliptic is 270°, therefore in the diagram we have $PQS = \lambda - 90°$, $QS = 90 - \beta$, $PQ = \omega$, the obliquity, $SPQ = 270° - \alpha$ and $PS = 90° - \delta$, of which λ, β and ω are given to find α and δ, that is we have two sides and the included angle of a spherical triangle given to find the other side and one of the other angles. The parallactic angle PSQ is not required. Applying the fundamental formulæ of spherical trigonometry to this triangle we find

$$\sin\delta = \cos\omega \sin\beta + \sin\omega \cos\beta \sin\lambda$$
$$\cos\delta \sin\alpha = - \sin\omega \sin\beta + \cos\omega \cos\beta \sin\lambda$$
$$\cos\delta \cos\alpha = \cos\beta \cos\lambda$$

Dividing the second and first of these, by the third we have respectively,

$$\tan\alpha = - \frac{\sin\omega \tan\beta}{\cos\lambda} + \cos\omega \tan\lambda$$

and $$\tan\delta = \cos\alpha\left(\frac{\cos\omega \tan\beta}{\cos\lambda} + \sin\omega \tan\lambda\right) \qquad (23)$$

These formulæ can be computed in this form by addition and subtraction logarithms and this is the way they have been hitherto computed for every mean noon and midnight in this special case, and then interpolated for intermediate hours. To adapt them to ordinary logarithmic computation, put

$$\sin\lambda = m \cos\varphi$$
and $$\tan\beta = m \sin\varphi$$

that is

$$\tan \varphi = \frac{\tan \beta}{\sin \lambda}$$

and we readily find

$$\tan \alpha = \frac{\cos (\omega + \varphi)}{\cos \varphi} \tan \lambda$$

and $$\tan \delta = \sin \alpha \tan (\omega + \varphi) \tag{24}$$

This is the most convenient form, α and δ being determined by their tangents.

The difference of R. A. and Decl. for one minute is computed by (15) and the semi-diameter by the well known relation between the parallax π and the semi-diameter (d), thus

$$\frac{d}{\pi} = 0.272274 \tag{25}$$

to which $2''.5$ are added for irradiation.

Greenwich Transit.

On page IV of each month there are given mean time of transit at Greenwich and the Moon's age at mean noon.

At the time of transit the Moon's R. A. is equal to the sidereal time. Let Θ = the sidereal time at preceding Greenwich mean noon; α = the Moon's R. A. at the same instant and m = the hourly variation of the Moon's R. A. as found on pages V-XII of each month then $.99727m$ = the variation of R. A. in one *sidereal* hour, and if t_s denote the number of sidereal hours from preceding mean noon till the transit, we must have

$$\begin{aligned} .99727m\, t_s + \alpha &= \text{Moon's R. A. at transit} \\ &= \Theta + t_s \end{aligned}$$

whence $$t_s = \frac{\alpha - \Theta}{1 - .99727m}$$

or in mean solar hours

$$t_m = \frac{\alpha - \Theta}{1 - .99727m} \times .99727$$

$$= \frac{\alpha - \Theta}{3609.856 - m} \tag{26}$$

where $\alpha - \Theta$ and m are expressed in seconds. This will give only an approximate time of transit since m does not remain constant. A small correction to t_m is therefore necessary and can be found as follows:

Compute from the Ephemeris the values of α and Θ for this approximate time or for the nearest hour and let $\Delta\alpha$ be the increase per *minute* in the Moon's R. A. Now in one minute of mean time the sidereal time increases by $60^s.164$ and if τ denote the correction to t_m we shall have

$$(60.164 - \Delta\alpha)\,\tau = \alpha - \Theta$$

whence

$$\tau = \frac{\alpha - \Theta}{60.164 - \Delta\alpha} \tag{27}$$

EXAMPLE.—Find the time of Moon's transit at Greenwich 1895, July 4.

To find the approximate time by (26) we have

$$\begin{array}{lrrrrl}
\text{July 4, mean noon} & \alpha = & 16^h & 44^m & 45^s.83 & \text{page } 114 \\
\text{" " " "} & \Theta = & 6 & 48 & 58\ .75 & \text{" } 111 \\
\hline
& \alpha - \Theta = & 9 & 55 & 47\ .08 & \\
& = & \multicolumn{3}{l}{35747.08 \text{ sec.}} &
\end{array}$$

$m = 2.3863 \times 60 = 143.178$ sec. $\qquad 3609.856 - m = 3466.678$

hence

$$t_m = \frac{35747.08}{3466.678} = 10^h.3116$$

$$= 10^h\ 18^m.696.$$

The Ephemeris gives $10^h\ 18^m.8$. On page 114 we see that m varies quite rapidly, being 2.3955×60 at 10 hours. To find the correct time we have

$$\begin{array}{lrrrr}
\text{July 4, } 10^h & \alpha = & 17^h & 8^m & 40^s.91 \\
\text{" "} & \Theta = & 16 & 50 & 37\ .31 \\
\hline
& \alpha - \Theta = & & 18 & 3\ .60 = 1083.60 \text{ sec.}
\end{array}$$

and $\qquad 60.164 - \Delta\alpha = 60.164 - 2.3955 = 57.7685$ sec.

hence

$$\tau = \frac{1083.60}{57.7685} = 18.7575 \text{ min.}$$

which added to 10 hours gives mean time of transit July 4, $10^h\ 18^m.7575$ or $10^h\ 18^m.8$ as in Ephemeris. When four or five consecutive transits have been computed, the date of the next can be anticipated with sufficient accuracy to supersede the necessity of making a second computation.

The column "Diff. for 1 hour" is computed by (15). The Moon's age at noon is found by subtracting the date of the preceding new moon from the given date.

Moon Culminations at Washington.

By means of the Greenwich transit and the hourly variation we can find an approximate date of the Washington transit and then compute a more accurate date by (27) and after having found the correct date of the transit, the R. A. Decl. equatorial horizontal parallax and semi-diameter are interpolated for this date.

An example will best illustrate the process. Find the time of transit at Washington, 1895, April 4.

From the Greenwich transit we find the approximate time of the Washington transit to be $8^h\ 4^m + 2^m.34 \times 5.136 = 8^h\ 16^m.0$ or $13^h\ 24^m.2$ Greenwich time, say $8^h\ 15^m.8$ Washington time or $13^h\ 24^m$ Greenwich time for convenience of interpolation.

Compute α and Θ for this date, thus

1895, April $4^d\ 13^h\ 24^m$

$$\begin{aligned} \alpha &= 9^h\ 8^m\ 20^s.636 \\ \Theta &= 9\ \ \ 8\ \ \ 12\ .097 \\ \hline \alpha - \Theta &= \qquad +\ 8\ .539 \end{aligned}$$

We also have

$$\varDelta\alpha = 2.3888 \text{ and } 60.164 - \varDelta\alpha = 57.7752$$

therefore $\tau = +\ .14779$ minutes

hence Washington time of transit $= 8^h\ 15^m.8 + 0^m.14779$
$= 8^h\ 15^m.95$

and $\tau\varDelta\alpha = 0.352$ sec. $\quad 60.164\tau = 8.892$ sec.

therefore $\alpha = \Theta = 9^h\ 8^m\ 20^s.989$

which proves the accuracy of the result. For this date interpolate the values of the Decl. and parallax and compute the semi-diameter from (25) which are respectively $+ 19°\ 51'\ 6''.9$, $59'\ 50''$ and $16'\ 20''$.

Sidereal Time of the Semi-Diameter Passing the Meridian.

This is computed in the same way as in the case of the Sun, thus let $d =$ semi-diam. and $\varDelta\alpha =$ the increase of the Moon's R. A. in one minute of mean time, then in one minute of sidereal time, the increase is $\frac{\varDelta\alpha}{\mu}$ where $\mu = 1.00273791$ (page 31) and in the same time the meridian moves 60 seconds. And if t_s denote the number of sidereal seconds in which the semi-diameter passes the meridian we shall have

$$t_s = \frac{60d}{15\left(60 - \frac{\varDelta\alpha}{\mu}\right)} = \frac{4d\mu}{60\mu - \varDelta\alpha}$$

$$t_s = \frac{4d\mu}{60.164 - \varDelta\alpha},$$

when the Moon is on the equator, and for any declination δ, we have

$$t_s = \frac{4d\mu \sec \delta}{60.164 - \varDelta\alpha} \qquad (28)$$

If we put

$$\frac{4\mu}{60.164 - \varDelta\alpha} = A,$$

we can form a table of values of A with $\varDelta\alpha$ as an argument and (28) may be written

$$t_s = Ad \sec \delta \qquad (29)$$

The values of A will be found in Table I.* In our example we have, $d = 980''$, $\delta = 19° 51' 6''.9$ and $\varDelta\alpha = 2^s.3888$, and Table I gives $\log A = 8.84150$ from which we find $t_s = 72.336$ seconds.

Variation of Mean Time of Transit and the Moon's Motion in R. A. and Decl. in One Hour of Longitude.

Let $\varDelta\alpha$ and $\varDelta\delta$ denote the Moon's motion in R. A. and Decl. in one minute of mean time (see pages V-XII of each month), then in the same time the meridian approaches the Moon by $60^s.164 - \varDelta\alpha$ and if $\varkappa$ denote the interval in minutes in which the Moon passes over one hour of longitude we have
$(60^s.164 - \varDelta\alpha)\ \varkappa = 1$ hour $= 60^2$ seconds,

therefore $$\varkappa = \frac{60^2}{60.164 - \varDelta\alpha} \qquad (30)$$

hence the retardation in 1 hr. of long. $= \dfrac{60^2}{60.164 - \varDelta\alpha} - 60$ (31)

and therefore we have

Moon's motion in R. A. in 1 hr. of Long. $= \dfrac{60^2\ \varDelta\alpha}{60.164 - \varDelta\alpha}$ (32)

and Moon's motion in Decl. " $= \dfrac{60^2\ \varDelta\delta}{60.164 - \varDelta\alpha}$ 33)

The values of (30), (31) and (32) are tabulated in Table I with $\varDelta\alpha$ as an argument.

In our example we have $\varDelta\alpha = 2^s.3888$, entering Table I with this argument we get the retardation in 1 hour of longitude $2^s.30968$ or $2^s.310$ which is tabulated in the 3rd column opposite April 4; for the motion in R. A. $148^s.85$ and

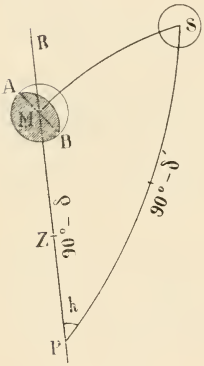
* To appear in next number.

$$\log \frac{60^2}{160.164 - \Delta\alpha} = 1.79456$$

and from page 60, $\Delta\delta = 13''.008$, therefore motion in Decl. is $810''.54$.

The variations of the time of transit, and of the motion in R. A. and Decl. in one hour of longitude are only approximate. They are carried to a degree of exactness not warranted by the original data which are given only to the nearest unit of the first decimal place. The only use they can serve is to give the observer the approximate time of transit and to enable him to set his instrument, but if an accurate comparison of the tabulated and observed place of the Moon be required for any place—such as at the Goodsell Observatory, a computation of all the circumstances of the transit would have to be made de novo.

Illuminated Limbs.



From new Moon to full Moon the first (I) limb is illumined and the second (II) from full Moon to new Moon.

The north and south limbs are determined as follows:

Let M and S be the positions of the Moon and Sun at the time of the Moon's transit, P, the north pole and Z the zenith of the observer. Then in the spherical triangle PSM we have

PS $= 90 - \delta'$, the Sun's north polar distance.

PM $= 90 - \delta$, the Moon's north polar distance.

SPM $= h$, the Sun's hour angle.
$= $ the apparent time of transit.
$= \alpha - \alpha'$,

all of which are given in the Ephemeris. If ACB be the illuminated portion seen by the observer on the meridian PZM, AB will be the line of cusps and the plane of the great circle passing through A, B and M, is perpendicular to the plane of the great circle through M and S. Put the angle ZMB $=$ AMR $= \theta$, then we shall have

$$PMS = \theta + 90^\circ \text{ or } - \theta + 90^\circ$$

and

$$\cot PMS = \frac{\cos\delta \tan\delta' - \sin\delta \cos h}{\sin h}$$

or $$\pm \tan\theta = \frac{\tan\delta' - \tan\delta\cos h}{\sin h \sec\delta} \qquad (34)$$

Now since $\sin h \sec\delta$ is positive when $h < 180°$, and negative when $h > 180°$, the sign of $\tan\theta$ will be the same as that of the numerator of the second member of the equation, that is

$\tan\theta$ is $-$ when $\tan\delta' < \tan\delta\cos h$
" " $+$ " $\tan\delta' > \tan\delta\cos h$
" " 0 " $\tan\delta' = \tan\delta\cos h$

Put $\tan\delta\cos h = \tan\varDelta$ then we shall evidently have the following criteria:

If $\delta' > \varDelta$ or $\varDelta - \delta' =$ a negative quantity the north limb is illuminated, and if $\delta' < \varDelta$ or $\varDelta - \delta' =$ a positive quantity the south limb is illuminated.

When $h = 0°$ or $180°$, $\theta = \pm 90°$ and the line of cusps is at right angles to the meridian, when $h = 90°$ or $270°$, $\varDelta = 0$, and

$$\tan\theta = \frac{\tan\delta'}{\sec\delta}$$

hence if δ' is negative the *south* limb is illuminated and if positive the north limb.

When $\tan\delta' = \cos h \tan\delta$, $\theta = 0$ and the line of cusps coincides with meridian and $\tan\delta > \tan\delta'$ or $\delta > \delta'$ numerically.

The values of $\varDelta$ are tabulated in Table II—a table of double entry—with δ and h as arguments, the former being at the top and bottom and the latter at the left and right sides of the page.

The Sun's approximate declination for every 5th day, is also added for convenience but when a very accurate computation has to be made as in the case where doubt exists as to which limb is actually illuminated, all the quantities must be accurately interpolated for the apparent time of transit.

In our example we have

Mean time of transit........................	April $4^d\ 8^h\ 15^m.95$
Equation of time (page 379)...........	$-\quad 2\ .9$
Apparent time of transit..................	$8^h\ 13^m$
Moon's declination (δ)......................	$+\ 19°\ 51'\ 6''.9$
Sun's declination (δ') approximate..	$+\quad 5\ \ 54\ \ 5\ .0$

Entering Table II we find

$$\varDelta = -11°.4$$

then $$\varDelta - \delta' = -11°.4 - 5°.9 = -17°.3$$

a negative quantity, therefore the north limb is the bright one, and is marked *N*. When both limbs are nearly equally illuminated as on April 8, the one which is slightly deficient is printed in smaller type. At such times a very accurate examination must be made.

(TO BE CONTINUED.)

THE PLANETS AND THE CONSTELLATIONS FOR MARCH.

Mercury will reach greatest western elongation, 27° 20′ from the Sun, March 5, and will therefore be visible as a morning planet during the first half of the month. The observer should look toward the southeast horizon a little way east from Venus in order to see Mercury. The phase of Mercury is at first quarter on March 1 and is increasingly gibbous during the month.

THE CONSTELLATIONS AT 9^h P. M. MARCH 1, 1896.

Venus is to be seen in the morning toward the southeast. Her phase increases from 0.83 to 0.90 while her light diminishes from 66 to 57 during the month. On the morning of the 14th Venus will be close to the fifth magnitude star μ Capri-

dwelling the same as here. Six nebulous objects only are visible to the naked eye, and are all that were known to the ancients until the invention of the telescope. Since then a harvest by many reapers have increased the number to nearly 10,000. Sir John Herschel's general catalogue of clusters and nebulæ contained 5079 objects, about 1100 being clusters, the remainder, nebulæ. Dreyer's New G. C. contains, including the Index Catalogue, 9416, comprising all known up to December, 1887.

The most conspicuous naked eye cluster is the Pleiades. and is familiar to every one as containing six stars, though some claim to see thirteen. The number visible through the telescope is 671, but a photographic camera, by several hours exposure, reveals on the negative plate 2326.

The grandest cluster in the northern heavens (nebulous through common telescopes) is 13 Messier, about one-third the distance from Eta to Zeta Herculis. It is one of the 6 visible to the unassisted eye. It was also discovered by Halley in 1714. Sir William Herschel has stated that it contains 14000 stars. It has never seemed to me that there are one-half as many and I am inclined to think it is a misprint for 4000. The place of the cluster for 1860 is R. A. $16^h\ 36^m\ 40^s$, Decl. $+ 36° 44'$. The place for the Omega Centauri cluster is (same epoch) $13^h\ 18^m\ 24^s$, Decl. $- 46° 35'$, and is No. 3531 of the G. C , and 5139 of the N. G. C.

Near some of Sir John Herschel's southern nebulæ I have discovered several overlooked by him, notably, one between G. C. 4863 and 4865. in field with the latter. It is very faint, very small exceedingly elongated to a ray five times as long as broad. I am surprised at his failure to see it. Such exceedingly long thin nebulæ are undoubtedly of the shape of a double convex lens of long focus, with its longer diameter in the direction of the line of sight.

Echo Mountain, Cal., March 1896.

PRACTICAL SUGGESTIONS

THE LUNAR EPHEMERIS.*

J. MORRISON, M. A., M. B., PH. D.

For Popular Astronomy.

Lunar Distances.

On pages XIII to XVIII inclusive, of each month, are given for every three hours the angular distances of the Moon's centre from the Sun, the planets Venus, Mars, Jupiter and Saturn and certain fixed stars, as seen from the centre of the Earth. These distances are easily computed when we know the R. A. and Decl. of the bodies; thus, if α and δ denote the R. A. and Decl. of the

* Continued from page 369, No. 27.

Moon and α' and δ' those of the Sun, planet or star, we shall have in the spherical triangle formed by the pole and the bodies, the two sides $90 - \delta$ and $90 - \delta'$ and the included angle $\alpha - \alpha'$, to compute the other side which we will denote by *D*, thus

$$\cos D = \sin \delta \sin \delta' + \cos \delta \cos \delta' \cos (\alpha - \alpha') \qquad (35)$$

which may be computed just as it is by addition and subtraction logarithms, or it may be adapted for computation with ordinary logarithms. "The proportional logarithm" of the difference is simply the log of the time in seconds in which the distance changes by 1″; thus on 1895, Jan. 1, midnight, we see that the change in the angular distance of the Moon from the Sun is in three hours 1° 21′ 32″, then to find the time in whicn it changes 1″ we have $1° 21' 32'' : 1'' :: 3^h : \varkappa$

or
$$\varkappa = \frac{10800}{4892} = 2^s.207$$

the log of which is 0.3439, the characteristic and decimal point being omitted. By the aid of this we can easily find the time in which the distance changes by any amount, supposing the change to be uniform which, however, is not the case. The most accurate method of finding the time at which the true is equal to the observed distance, is by the interpolation formula taking into account the second differences:—thus let α^t denote any observed distance and α_0 the next least in the computed lunar distances, then b and c being the first and second differences respectively we have

$$\alpha^t = \alpha_0 + tb + \frac{t\,(t-1)}{2}\,c + \ldots\ldots$$
$$= \alpha_0 + tb - \frac{tc}{2} + \frac{t^2}{2} + \ldots\ldots$$

whence
$$t = \frac{\alpha^t - \alpha_0}{b - \frac{1}{2}c}, \qquad \left(\text{omitting } \frac{t^2}{2}\right) \qquad (36)$$

which will give the time with all the accuracy required.

Example.—On Jan 2, 1895 the Moon's apparent angular distance from the Sun as measured with a sextant was found to be 77° W. find the Greenwich mean time. On page 15, American Ephemeris, we see that the time lies between 15 hours and 18 hours and we have

$$\alpha^t - \alpha_0 = 30'\ 27'', \quad b = +4989'', \quad c = +13'',$$
$$t = \frac{1827}{4989 - 6.5} = .3666\ldots\ldots \text{ or } 1^h\ 6^m,$$

therefore the time is $16^h\ 6^m$.

The method of proportional logarithms gives $16^h\ 5^m\ 55^s$.

TABLE I.

Moon's Retardation and Motion in 1 Hour of Longitude.

Arg. $\Delta\alpha$ ☽'s hr. motion in R.A.	log. A.		Retardation in 1 hour of Longitude.		Moon's Motion in 1 hour of Longitude.		$\log \dfrac{60^2}{60.164 - \Delta\alpha}$	
1.74	8.83666	7	1.618	10	107.22	.63	1.78971	7
1.75	8.83673	8	1.628	11	107.85	.63	1.78978	8
1.76	8.83681	7	1.639	11	108.48	.64	1.78986	7
1.77	8.83688	8	1.650	11	109.12	.64	1.78993	8
1.78	8.83696	7	1.661	10	109.76	.63	1.79001	7
1.79	8.83703	8	1.671	10	110.39	.64	1.79008	7
1.80	8.83711	7	1.681	10	111.03	.63	1.79015	8
1.81	8.83718	7	1.691	10	111.66	.64	1.79023	7
1.82	8.83725	8	1.701	11	112.30	.64	1.79030	8
1.83	8.83733	7	1.712	11	112.94	.63	1.79038	7
1.84	8.83740	8	1.723	11	113.57	.64	1.79045	8
1.85	8.83748	7	1.734	10	114.21	.64	1.79053	7
1.86	8.83755	8	1.744	11	114.85	.63	1.79060	8
1.87	8.83763	7	1.755	11	115.48	.64	1.79068	7
1.88	8.83770	8	1.766	10	116.12	.64	1.79075	7
1.89	8.83778	7	1.776	11	116.76	.63	1.79082	8
1.90	8.83785	8	1.787	10	117.39	.64	1.79090	7
1.91	8.83793	7	1.797	11	118.03	.64	1.79097	8
1.92	8.83800	7	1.808	11	118.68	.63	1.79105	7
1.93	8.83807	8	1.819	11	119.31	.64	1.79112	8
1.94	8.83815	7	1.830	10	1 9.95	.64	1.79120	7
1.95	8.83922	8	1.840	11	120.59	.64	1.79127	8
1.96	8.83830	7	1.851	10	121.23	.64	1.79135	7
1.97	8.83837	7	1.861	11	121.87	.64	1.79142	8
1.98	8.83844	8	1.872	11	122.51	.64	1.79150	7

Arg. $\Delta\alpha$ ☽'s hr. motion in R A	log. A.		Retardation in 1 hour of Longitude.		Moon's Motion in 1 hour of Longitude.		log $\frac{60^2}{60.164 - \Delta\alpha}$	
1.99	8.83852		1.883		123.15		1.79157	
		7		11		.64		8
2.00	8.83859		1.894		123.79		1.79165	
		8		10		.64		7
2.01	8.83867		1.904		124.43		1.79172	
		7		10		.64		7
2.02	8.83874		1.915		125.07		1.79179	
		8		11		.64		8
2.03	8.83882		1.926		125.71		1.79187	
		7		10		.64		7
2.04	8.83889		1.936		126.35		1.79194	
		8		11		.64		8
2.05	8.83897		1.947		126.99		1.79202	
		7		10		.64		7
2.06	8.83904		1.957		127.63		1.79209	
		8		11		.64		8
2.07	8.83912		1.968		128.27		1.79217	
		7		11		.65		7
2.08	8.83919		1.979		128.92		1.79224	
		8		11		.64		8
2.09	8.83927		1.990		129.56		1.79232	
		7		10		.64		7
2.10	8.83984		2.000		130.20		1.79239	
		8		11		.64		8
2.11	8 83942		2.011		130.84		1.79247	
		7		10		.64		7
2.12	8.83949		2.021		131.48		1.79254	
		8		11		.65		8
2.13	8.83957		2.032		132.13		1.79262	
		7		11		.64		7
2.14	8.83964		2.043		132.77		1.79269	
		8		11		.65		8
2.15	8.83972		2.054		133.42		1.79277	
		7		11		.64		7
2.16	8.83979		2.065		134.06		1.79284	
		8		11		.64		8
2.17	8.83987		2.076		134.70		1.79292	
		7		10		.65		7
2.18	2.83994		2.086		135.35		1.79299	
		8		11		.64		8
2.19	8.84002		2.097		135.99		1.79307	
		7		10		.64		7
2.20	8.84009		2.107		136.63		1.79314	
		8		11		.65		8
2.21	8.84017		2.118		137.28		1.79322	
		7		11		.64		7
2.22	8.84024		2.129		137.92		1.79329	
		8		11		.65		8
2.23	8.84032		2.140		138.57		1.79337	
		7		10		.65		7
2.24	8.84039		2.150		139.22		1.79344	
		8		11		.64		8
2.25	0.84047		2.161		139.86		1.79352	
		7		11		.65		7
2.26	8.84054		2.172		140.51		1.79359	
		8		11		.65		8
2.27	8.84062		2.183		141.16		1.79367	
		7		10		.64		7
2.28	8.84069		2.193		141.80		1.79374	
		8		11		.65		8

Arg. $\Delta\alpha$ ☽'s hr. motion in R.A.	log A.		Retardation in 1 hr. of Longitude.		Moon's Motion in 1 hour of Longitude.		log $\frac{60^2}{60.164 - \Delta\alpha}$	
2.29	8.84077		2.204		142.45		1.79382	
		7		11		.64		7
2.30	8.84084		2.215		143.09		1.79389	
		8		11		.65		8
2.31	8.84092		2.226		143.74		1.79397	
		7		11		.65		7
2.32	8.84099		2.237		144.39		1.79404	
		8		10		.65		8
2.33	8.84107		2.247		145.04		1.79412	
		7		10		.64		7
2.34	8.84114		2.257		145.68		1.79419	
		8		11		.65		8
2.35	8.48122		2.268		146.33		1.79427	
		7		11		.65		7
2.36	8.84129		2.279		146.98		1.79434	
		8		11		.65		8
2.37	8.84137		2.290		147.63		1.79442	
		7		10		.65		7
2.38	8.84144		2.300		148.28		1.79449	
		8		11		.65		8
2.39	8.84152		2.311		148.93		1.79457	
		7		11		.64		7
2.40	8.84159		2.322		149.57		1.79464	
		8		11		.65		8
2.41	8.84167		2.333		150.22		1.79472	
		7		10		.65		7
2.42	8.84174		2.343		150.87		1.79479	
		8		11		.65		8
2.43	8.84182		2.354		151.52		1.79487	
		7		11		.65		7
2.44	8.84189		2.365		152.17		1.79494	
		8		11		.65		8
2.45	8.84197		2.376		152.82		1.79502	
		7		11		.65		7
2.46	8.84204		2.387		153.47		1.79509	
		8		11		.65		8
2.47	8.84212		2.398		154.12		1.79517	
		7		11		.65		7
2.48	8.84219		2.409		154.77		1.79524	
		8		11		.65		8
2.49	8.84227		2.420		155.42		1.79532	
		7		11		.66		8
2.50	8.84234		2.431		156.08		1.79540	
		8		11		.65		7
2.51	8.84242		2.442		156.73		1.79547	
		8		11		.65		8
2.52	8.84250		2.453		157.38		2.79555	
		8		10		.65		7
2.53	8.84258		2.463		158.03		1.79562	
		7		11		.65		8
2.54	8.84265		2.474		158.68		1.79570	
		8		10		.65		7
2.55	8.84273		2.484		159.33		1.79577	
		7		11		.66		8
2.56	8.84280		2.495		159.99		1.79585	
		8		11		.65		7
2.57	8.34288		2.506		160.64		1.69592	
		7		11		.65		8
2.58	8.84295		2.517		161.29		1.79600	
		8		10		.66		7

Arg. $\Delta\alpha$ ☽'s hr. motion in R.A.	log. A.		Retardation in 1 hour of Longitude.		Moon's Motion in 1 hour of Longitude.		$\log \dfrac{60^2}{60.164 - \Delta\alpha}$	
2.59	8.84303		2.527		161.95		1.79607	
		7		11		.65		8
2.60	8.84310		2.538		162.60		1.79615	
		8		11		.65		7
2.61	8.84318		2.549		163.25		1.79622	
		7		11		.66		8
2.62	8.84325		2.560		163.91		1.79630	
		8		11		.66		8
2.63	8.84333		2.571		164.57		1.69638	
		7		11		.65		7
2.64	8.84340		2.582		165.22		1.79645	
		8		11		.65		8
2.65	8.84348		2.593		165.87		1.79653	
		7		11		.66		7
2.66	8.84355		2.604		166.53		1.79660	
		8		11		.65		8
2.67	8.84363		2.615		167.18		1.79668	
		7		11		.66		7
2.68	8.84370		2.626		167.84		1.79675	
		8		11		.65		8
2.69	8.84378		2.637		168.49		1.79683	
		7		10		.66		7
2.70	8.84385		2.647		169.15		1.79690	
		8		11		.65		8
2.71	8.84393		2.658		169.80		1.79698	
		7		11		.66		8
2.72	8.84400		2.669		170.46		1.79706	
		8		11		.66		7
2.73	8.84408		2.680		171.12		1.79713	
		8		11		.66		8
2.74	8.84416		2.691		171.78		1.79721	
		8		10		.65		7
2.75	8.84424		2.701		172.43		1.79728	
		7		11		.66		8
2.76	8.84431		2.712		173.09		1.79736	
		8		11		.65		7
2.77	8.84439		2.723		173.74		1.79743	
		7		11		.66		8
2.78	8.84446		2.734		174.40		1.79751	
		8		12		.66		8
2.79	8.84454		2.746		175.06		1.79759	
		7		11		.66		7
2.80	8.84461		2.757		175.72		1.79766	
		8		11		.66		7
2.81	8.84469		2.768		176.38		1.89774	
		8		11		.66		7
2.82	8.84477		2.779		177.04		1.79781	
		8		11		.65		8
2.83	8.84485		2.790		177.69		1.79789	
		7		10		.66		7
2.84	8.84492		2.800		178.35		1.89796	
		8		11		.66		8
2.85	8.84500		2.811		179.01		1.79804	
		7		11		.66		7
2.86	8.84507		2.822		179.67		1.79811	
		8		11		.66		7
2.87	8.84505		2.833		180.33		1.79818	
		7		12		.66		7
2.88	8.84522		2.845		180.99		1.79825	

Table II.—For Finding the Illuminated Limb of the Moon.

Argument 1.—The Moon's Declination at the top and bottom of the page.
" 2.—The Apparent Time of Moon's Transit at the left and right hand columns.

Apparent Time of Transit.		∓ 30°	∓ 25°	∓ 20°	∓ 15°	∓ 10°	∓ 5°	∓ 0	Apparent Time of Transit.		☉'s Decl. δ'	
h m	h m	°	°	°	°	°	°		h m	h m		°
24 0	0 0	∓ 30.0	∓ 25.0	∓ 20.0	∓ 15.0	∓ 10.0	∓ 5.0	0	12 0	12 0	Jan. 0	− 23.1
23 55	5	30.0	25.0	20.0	15.0	10.0	5.0	0	11 55	5	5	22.6
50	10	30.0	25.0	20.0	15.0	10.0	5.0	0	50	10	10	21.9
45	15	30.0	24.9	19.9	14.9	9.9	5.0	0	45	15	15	21.0
40	20	29.9	24.9	19.9	14.9	9.9	5.0	0	40	20	20	20.0
35	25	29.9	24.9	19.8	14.9	9.9	5.0	0	35	25	25	18.9
30	30	29.8	24.8	19.8	14.8	9.8	5.0	0	30	30	30	17.6
25	35	29.7	24.8	19.7	14.8	9.8	4.9	0	25	35	Feb. 4	16.1
20	40	29.6	24.7	19.7	14.7	9.8	4.9	0	20	40	9	14.5
15	45	29.5	24.6	19.7	14.7	9.8	4.9	0	15	45	14	12.9
10	50	29.4	24.5	19.6	14.6	9.7	4.9	0	10	50	19	11.1
5	55	29.2	24.4	19.5	14.6	9.7	4.8	0	5	55	24	9.3
23 0	1 0	29.1	24.3	19.4	14.5	9.7	4.8	0	11 0	13 0	Mar. 1	7.4
22 55	5	28.9	24.1	19.3	14.4	9.6	4.8	0	10 55	5	6	5.5
50	10	28.8	24.0	19.2	14.3	9.5	4.8	0	50	10	11	3.6
45	15	28.6	23.8	19.1	14.2	9.5	4.7	0	45	15	16	− 1.6
40	20	28.5	23.7	18.9	14.1	9.4	4.7	0	40	20	21	+ 0.4
35	25	28.3	23.5	18.7	14.0	9.4	4.7	0	35	25	26	2.4
30	30	28.0	23.3	18.5	13.9	9.3	4.6	0	30	30	31	4.3
25	35	27.8	23.1	18.4	13.8	9.2	4.6	0	25	35	Apr. 5	6.2
20	40	27.6	22.9	18.2	13.6	9.1	4.5	0	20	40	10	8.1
15	45	27.3	22.7	18.0	13.5	9.0	4.5	0	15	45	15	9.9
10	50	27.1	22.4	17.8	13.4	8.9	4.4	0	10	50	20	11.6
5	55	26.8	22.2	17.7	13.2	8.8	4.3	0	5	55	25	13.3
22 0	2 0	26.6	22.0	17.5	13.1	8.7	4.3	0	10 0	14 0	30	14.9
21 55	5	26.3	21.8	17.3	12.9	8.6	4.2	0	9 55	5	May 5	16.4
50	10	25.9	21.5	17.0	12.7	8.5	4.2	0	50	10	10	17.7
45	15	25.6	21.2	16.8	12.6	8.4	4.1	0	45	15	15	19.0
40	20	25.3	20.9	16.6	12.4	8.2	4.1	0	40	20	20	20.1
35	25	25.0	20.6	16.3	12.2	8.1	4.1	0	35	25	25	21.0
30	30	24.6	20.3	16.1	12.0	8.0	4.0	0	30	30	30	21.8
25	35	24.2	20.0	15.8	11.8	7.9	3.9	0	25	35	June 4	22.5
20	40	23.8	19.7	15.6	11.6	7.7	3.8	0	20	40	9	23.0
15	45	23.4	19.3	15.3	11.4	7.5	3.8	0	15	45	14	23.3
10	50	23.0	19.0	15.0	11.2	7.4	3.7	0	10	50	19	23.4
5	55	22.6	18.6	14.7	11.0	7.2	3.6	0	5	55	24	23.4
21 0	3 0	22.2	18.3	14.4	10.7	7.1	3.5	0	9 0	15 0	29	23.2
20 55	5	21.7	17.9	14.1	10.5	6.9	3.5	0	8 55	5	July 4	22.9
50	10	21.2	17.5	13.8	10.3	6.8	3.4	0	50	10	9	22.3
45	15	20.8	17.1	13.5	10.1	6.6	3.3	0	45	15	14	21.6
40	20	20.3	16.7	13.2	9.8	6.5	3.2	0	40	20	19	20.8
35	25	19.8	16.3	12.8	9.5	6.3	3.2	0	35	25	24	19.8
30	30	19.3	15.9	12.5	9.2	6.2	3.1	0	30	30	29	18.7
25	35	18.8	15.5	12.1	9.0	6.0	3.0	0	25	35	Aug. 3	17.4
20	40	18.3	15.0	11.8	8.7	5.8	2.9	0	20	40	8	16.1
15	45	17.7	14.5	11.5	8.4	5.6	2.8	0	15	45	13	14.6
10	50	17.2	14.0	11.1	8.1	5.4	2.7	0	10	50	18	13.0
5	55	16.6	13.6	10.7	7.9	5.2	2.6	0	5	55	23	11.3
20 0	4 0	16.1	13.1	10.3	7.6	5.0	2.5	0	8 0	16 0	28	9.6
19 55	5	15.5	12.6	9.9	7.3	4.8	2.4	0	7 55	5	Sept. 2	7.8
50	10	14.9	12.1	9.5	7.0	4.6	2.3	0	50	10	7	5.9
45	15	14.3	11.7	9.1	6.8	4.5	2.2	0	45	15	12	4.1
40	20	13.7	11.2	8.7	6.5	4.3	2.1	0	40	20	17	2.1
35	25	13.1	10.7	8.3	6.2	4.1	2.0	0	35	25	22	0.2
30	30	12.5	10.2	7.9	5.9	3.8	1.9	0	30	30	27	− 1.8
25	35	11.9	9.7	7.5	5.6	3.6	1.8	0	25	35	Oct. 2	3.7
20	40	11.2	9.1	7.1	5.2	3.4	1.7	0	20	40	7	5.6
15	45	10.5	8.5	6.7	4.9	3.2	1.6	0	15	45	12	7.5
10	50	9.8	8.0	6.3	4.6	3.0	1.5	0	10	50	17	9.4
5	55	9.2	7.4	5.9	4.3	2.8	1.4	0	5	55	22	11.2
19 0	5 0	8.5	6.9	5.4	4.0	2.6	1.3	0	7 0	17 0	27	12.9
18 55	5	7.8	6.4	5.0	3.7	2.4	1.2	0	6 55	5	Nov. 1	14.6
50	10	7.1	7.1	4.5	3.4	2.2	1.1	0	50	10	6	16.1
45	15	6.4	5.8	4.1	3.1	2.0	1.0	0	45	15	11	17.5
40	20	5.7	4.6	3.6	2.7	1.8	0.9	0	40	20	16	18.8
35	25	5.0	4.1	3.2	2.3	1.6	0.8	0	35	25	21	20.0
30	30	4.3	3.5	2.7	2.0	1.4	0.7	0	30	30	26	21.0
25	35	3.6	2.9	2.3	1.6	1.2	0.6	0	25	35	Dec. 1	21.9
20	40	2.9	2.3	1.8	1.3	0.9	0.4	0	20	40	6	22.6
15	45	2.2	1.7	1.3	1.0	0.7	0.3	0	15	45	11	23.0
10	50	1.4	1.1	0.6	0.6	0.4	0.2	0	10	50	16	23.3
5	55	0.7	0.6	0.4	0.3	0.2	0.1	0	5	55	21	23.5
18 0	6 0	∓ 0.0	∓ 0.0	∓ 0.0	∓ 0.0	∓ 0.0	∓ 0.0	0	6 0	18 0	26	23.4
											31	− 23.1
Apparent Time of Transit.		± 30°	± 25°	± 20°	± 15°	± 10°	± 5°	± 0°	Apparent Time of Transit.			

Critera. { This table gives the value of $\varDelta$, $\tan \varDelta = \cos h \tan \delta$
If $\varDelta - \delta'$ { = a positive quantity, the south limb is illuminated.
= a negative " " north " " "

In using this table, the upper signs are to be taken together and the lower signs together; thus: suppose the arguments are $21^h\ 20^m$ and $-\ 15°$. Entering the table with these we find $\varDelta = -\ 11°.6$, again if the arguments are 15^h and $+\ 15°$, we have $\varDelta = -\ 10°.7$, and so on.

130. What localities are best adapted to difficult astronomical work? O. L.

Answer:—Our querist is respectfully referred to an article in this number by Professor William H. Pickering who kindly consented to give some of his valuable experience to aid in answering this question and some others kindred to it.

131. What is the best kind of lantern for the projection of ordinary slides for the illustration of celestial objects? M. E.

Answer: We presume our querist is already aware that nothing in the way of a light can take the place of the electric arc or the oxy-hydrogen burner. Walmsley & Fuller, 136 Wabash Ave. Chicago, make an acetylene burner which is probably as good as any made at present. Their "generator No. 3"—the 2 lb. size—is listed at $20; will run four burners or equivalent for 5 hours. We cannot speak of the Welsbach burner, never having tried it. The best light we know anything about is the direct-current arc—the one made by Colt, of New York and Chicago, is superb—quiet as a mouse and absolutely steady—costs $100. As to lenses, everybody uses Darlot's, half or quarter size according to distance from the screen. We have used Darlot lenses for years—they leave little to be desired. Oxyhydrogen cylinders cost from $30 to $50 according to size.

THE PLANETS AND CONSTELLATIONS FOR MAY.

Mercury will be evening planet during the whole of May, coming to greatest eastern elongation, 22° 9′ east from the Sun, May 16. This will be the best time in the year to see Mercury, for the planet will be farther north than the Sun and will set more than an hour after sunset during the entire month, and about two hours after sunset at the middle of the month. Mercury will be in the constellation Taurus, passing 3° south of the Pleiades May 2, 8° north of Aldebaran May 9 and 3° south of β Tauri May 19.

Venus is morning planet but is getting too nearly in line with the Sun to be conspicuous. The phase is nearly full, increasing from 0.95 to 0.98 during the month. On May 3, Venus will be close to the fourth magnitude star δ Piscium.

Mars is on the meridian at about eight o'clock in the morning, and may be seen toward the southeast from three to four o'clock. He is among the faint stars south of the great square of Pegasus. Mars is still farther from the Earth than from the Sun, his apparent diameter being only 6″. Mars will be in conjunction with the Moon, 3° 37′ south of the latter at 10^h a. m., May 7.

Jupiter may be observed in the early evening, in the constellation Cancer, and is the most prominent object in the western sky. The reader will do well to try early twilight observations of the planet this month. The Moon will be in conjunction with Jupiter, passing about a degree and a half to the north of the planet, May 18, at $1^h\ 18^m$ a. m.

Saturn will be opposite the Sun on May 3, and will be in good position for observation for the next three or four months. The plane of the rings of Saturn is inclined to our line of sight about 21°, so that the details of the ring structure can be plainly seen. Any telescope bearing a magnifying power of 200 ought to show the Cassini division and the inner "crape" ring easily. Among the tables this month we give the times of eastern elongation of the satellites of Saturn, i. e. the times when they attain their greatest distances to the right of the planet, as seen in an inverting telescope. For the slower moving satellites, the times of western elongation and of inferior and superior conjunction, are also given. On

and many fine fields of stars will be discovered. Then look at the star Alpha, in the outer edge of the bowl nearest to the Pole-star, there is a faint star of about the eighth magnitude, near it, in the direction of Beta. It is of a reddish color. The star Alpha is approaching the Earth with an average speed of somewhere about 40 miles a second, and it has been observed "to alternate in color from red to yellow, in a period of 54½ days. The star is under no suspicion of varying in light."—(*System of the Stars*, p. 147, Agnes M. Clerke).

Near the second "pointer," is a spot of faint light, described by Sir John Herschel, as "a most extraordinary object," a large, uniform nebulous disc, quite round, very bright, not sharply defined, but yet very suddenly fading away to darkness. At Parsonstown, in 1848, two stars were perceived in the interior, each surrounded by a dark space encroached upon by nebulous whorls, and the object received the name of the "Owl Nebula," from the appearance of two great eyes thus presented."—(*System of the Stars*, p. 256).

The seven stars in the Great Dipper, are in reality seven splendid suns, probably very much larger than our Sun, and glowing with intense lustre. Iron, sodium, magnesium and other well known elements, exist in the atmospheres of these stars, and their massive globes raging with fiery heat, rush through the depths of space, with inconceivable speed. Five of the stars are receding from us at the rate of seventeen miles per second, the other two are travelling in an opposite direction. It is certain that these two do not belong to the same system as the other five. Thirty-six thousand years hence, the seven stars of the Great Dipper wi l have dissolved partnership, and its appearance will have changed. The handle of the Dipper will be bent and its rim out of place, for the reason that five stars will have drifted in one direction and two in another. During countless ages the stars which seem so steadfast have been rushing onward through space. There are stars travelling in "family parties," as Miss Clerke quaintly expresses it, colonies of stars of a friendly tendency drifting together others less friendly drifting apart. Despite the fact, that each star thus urging its way through space, is an enormous mass of glowing vapor, yet the most perfect order and harmony prevails in the star-depths.

Groombridge 1830, the Runaway Star.

Between the constellation of Ursa Major and Ursa Minor, there is a star of the sixth or seventh magnitude, and therefore invisible to the unaided eye. "It is known as Groombridge 1830 and though small, it is in one respect one of the most remarkable stars in the northern hemisphere. It is at least four or five times as far from the Earth as the star 61 Cygni and travels at the rate of two hundred miles a second In ten minutes it has travelled 120,000 miles. If our Earth moved equally fast the journey around the Sun would be accomplished in a month, instead of the year which is now required. The velocity of Groombridge 1830 is no mere spasmodic effort; with a stately uniformity, worthy of the dignity of a majestic sun, it sweeps along, alike inflexible in the direction of its motion and in the velocity with which its journey is performed."—(*The Story of the Heavens*, p 425, Robert S. Ball). But there are now two other "runaway" stars outrunning even Groombridge. Arcturus moves palpably through the heavens at the rate of 375 miles a second, and the velocity of Mu Cassiopeia is 363 miles a second. "Flying stars" can then no longer be regarded as intruders into stellar society. Whether or not they belong to it for better or for worse, they evidently form at present an important part of its mechanism."—*System of the Stars*, p. 345.

Polaris, the Pole Star.

When Shakespeare wrote the familiar lines: "I am as constant as the Northern Star, of whose true fixed and resting quality there is no fellow in the firmament," he was not aware of the fact that Polaris was anything but fixed. It is urging its way through space at the rate of about forty-six miles a second, and in about nine thousand years Alpha Centauri will take its place as our guide. Polaris has held its post of honor for over a thousand years, and it was preceded in office by Thuban of the Dragon.

Ursa Minor, the Little Bear.

This constellation, though not remarkable in its appearance and containing but few bright stars, is nevertheless justly distinguished on account of its position in the heavens. The stars in this group being situated near the celestial Pole appear to revolve about it very slowly and in circles so small as never to descend below the horizon. Its leading star is Polaris, the North Polar star. It is of the third magnitude and situated a little less than a degree and a quarter from the true Pole of the heavens, on that side of it which is towards Cassiopeia and opposite to Ursa Major. "The pole-star is a famous double but its minute companion can only be seen with a telescope. As so often happens, however, it has another companion for the opera-glass, and this latter is sufficiently close and small to make an interesting test for an inexperienced observer armed with a glass of small power. It must be looked for pretty close to the rays of the large star, with such a glass. It is of the seventh magnitude. With a large field-glass several smaller companions may be seen and a very excellent glass may show an 8th magnitude star almost hidden in the rays of the 7th magnitude companion. The star Beta, which is also called Kochab, has a pair of faint stars nearly north of it, about one degree distant. With a small glass these may appear as a single star, but a stronger glass will show them separately."—(*Astronomy with an Opera-Glass*, p. 27, Garrett P. Serviss.) Ursa Minor contains twenty-four stars including three of the third magnitude and four of the fourth. The seven principal stars form a figure resembling that of the Great Dipper, only that the Dipper is reversed and about one-half as large as the one in that constellation. The first star in the handle is the polar star, around which the rest constantly revolve. The two last in the bowl of the Dipper, corresponding to the Pointers in the Great Bear, are of the third magnitude and situated about 15° from the pole. The brightest of them is called Kochab, and it may be easily known by its being the brightest and middle one of three conspicuous stars forming a row, one of which is about 2° and the other 3° from Kochab. The two brightest of these are situated about 3° apart and are called the *Guards* or *Pointers* of Ursa Minor. Of the four stars which form the bowl of the Dipper, one is so small as hardly to be seen. They lie in a direction towards Gamma in Cepheus.

Legend.

According to Grecian mythology Ursa Major and Ursa Minor are the nymph Calisto and her son Arcas who were transformed into bears by the imperious Juno and afterwards placed among the stars by Jupiter. The unbear-like length of the creatures' tails is explained by the statement that they stretched as Jupiter lifted them to the sky. Calisto was a native of the city of Helice in Achaia, a district near the bay of Corinth, hence the Greater Bear is sometimes called Helice.

Taurus, the Bull.

In the old star-map Taurus, the Bull, is represented as if in an attitude of rage, and about to attack Orion. Only the head and shoulders of the animal are to be

Sid

the air upon the refraction, that of the colour of the stars, influences which are generally neglected, for example by Laplace, seen quite clearly to emerge from the observations made at Madison.

"These same series bring a confirmation to the numbers found in the physical laboratories for the index of refraction and the coefficient of expansion of the atmospheric air. They accord very well with the mean refractions based upon the work at the Observatory of Poulkova, even in indicating as probable a small variation of the constant of refraction having the year as its period. Another conclusion, already rendered probable by different researches, consists in the fact that the meridian instruments appear to give values systematically too great for the right ascensions of faint stars, this effect attaining a possible value of 0.009^s per magnitude of the star. A curious result of these studies is the existence of a personal equation in the measurement of distances, the correction for which varies as the square of the interval measured. The thorough discussion which Mr. Comstock has given leads him to regard this phenomenon as real and gives as definitive for the constant of aberration, the figures $20''.44$, very near to those which were found a half a century ago, by W. Struve. This value is in very satisfactory accord with that furnished by our own investigations. Neither depend in any way upon hypotheses made on the law of the variability of latitude.

"The sagacity and perseverance of which Mr. Comstock has given proof in the course of these difficult researches seem to me to designate his work as peculiarly worthy the attention of the Academy."

The Newtonian Constant of Gravitation.—We have held back the remainder of Professor C. V. Boys's lecture on "The Newtonion Constant of Gravitation" for several months, until we could procure photographs of the instruments used, to illustrate fully the several steps that were taken in this important study. Professor Boys very kindly furnished us twelve fine photographs (the instruments in place) taken by the aid of magnesium light and six of these illustrations are given in this number. We have purposely reserved Plate VIII until next time.

We have also asked Professor Boys to favor our readers by giving a detailed statement of his method of reducing his observations by which his value of G is obtained. This part of his work is not given in the lecture which we are reprinting. There is some reason for this, for prominent American scholars are apparently not agreed as to exact methods of procuring the value of G.—Ed.

PRACTICAL SUGGESTIONS.

THE SIDEREAL EPHEMERIS.

J. MORRISON, M. A., M. B., PH. D.

FOR POPULAR ASTRONOMY.

If the fixed stars were referred to some invariable plane and line of reference, their places would vary only by the amount of their own proper motion in space, and the computation of their coördinates would be comparatively an easy matter, but in order that they may be available for the purposes of astronomy, navigation and geodesy, they must be referred to the movable equator and equinox, that is to say, to the true equator and equinox of the date, for which their positions are required. Their R. A. and Decl. from day to day will therefore be affected by the three astronomical corrections—precession, nutation and aberration, and the computation of their positions will accordingly be laborious, tedious and troublesome. The place of a star referred to the *mean* equator and equinox of *any date*, is its *mean* place for that date; that referred to the *true* equator and equinox, is its *true* place and that in which it is seen, referred to the true equator and equinox, its *apparent* place. The true place is equal to the mean corrected for *nutation*, and the apparent place is equal to the true, corrected for *aberration*. The true and mean places are found from the apparent by applying the same corrections with their signs changed.

The mean R. A. and Decl. of the principal stars for the beginning of each year or rather for the beginning of the fictitious year, that is when the Sun's mean longitude is 280°, are given on pages 293 to 301 of the American Ephemeris, and in the adjacent columns are given the annual variation in R. A. and Decl. by means of which the mean place for any other date may be easily found. The apparent places for the upper transit at Washington are given for every tenth day of the year on the following pages together with the variation at each date. The apparent place, however, must not be confounded with the observed place or the place in which we actually see the stars. The latter is affected by two other corrections, viz: diurnal aberration and refraction. The former depends on the latitude of the place of observation and the declination of the star and is so small that it may generally be neglected except for stars very near the pole when the R.A. at the time of transit will be increased by $0^{sec}.0216 \cos \varphi \sec \delta$, while the latter depends on the altitude or zenith distance of the

star at the place of observation. Of course neither of these can be taken into consideration in the construction of an ephemeris which is always computed for the centre of the Earth, the general station from which all astronomical phenomena are supposed to be viewed.

If α and δ denote the *mean* R. A. and Decl. at the beginning of the fictitious year and α' and δ' the apparent R. A. and Decl. at any subsequent date τ expressed in years or fractions of a year, we shall have,

$$\alpha' = \alpha + \tau \times \text{mean annual precession} + \text{nutation} + \text{aberration} \\ + \tau \times \text{annual proper motion, and similarly for } \delta'.$$

The precession, nutation and aberration in R. A. and Decl. have already been investigated on pages 25-33. The proper motion is always very small and can be found in the star catalogues. It is deduced from observations continued during a long period, the longer the better. Substituting now the expressions for precession, nutation and aberration, we have

$$\begin{aligned}
\alpha' = \alpha &+ \tau\,(m + n \sin\alpha \tan\delta) && \text{(Precession)} \\
&+ \Delta l\,(\cos\omega + \sin\alpha \sin\omega \tan\delta) - \Delta\omega \cos\alpha \tan\delta && \text{(Nutation)} \\
&- h \sec\delta\,(\sin\alpha \sin\odot + \cos\alpha \cos\omega \cos\odot) && \text{(Aberration)} \\
&+ \tau\mu && \text{(Proper Motion)}
\end{aligned}$$

(37)

$$\begin{aligned}
\delta' = \delta &+ \tau n \cos\alpha && \text{(Precession)} \\
&+ \Delta l \cos\alpha \sin\omega + \Delta\omega \sin\alpha && \text{(Nutation)} \\
&- h\,(\cos\odot \sin\omega \cos\delta + \sin\odot \cos\alpha \sin\delta \\
&\qquad\qquad - \cos\odot \cos\omega \sin\alpha \sin\delta) && \text{(Aberration)} \\
&+ \tau\mu' && \text{(Proper Motion)}
\end{aligned}$$

The values of Δl and $\Delta\omega$ vary slightly from year to year and the three most important terms of each are given on page 27. More accurate values for the year 1900 are as follows:

$$\begin{aligned}
\Delta l = &- 17''.2574 \sin \Omega + 0''.2073 \sin 2\Omega - 1''.2695 \sin 2\odot \\
&+ 0''.1474 \sin (82^\circ + \odot) + 0''.0125 \sin (2\odot - \Omega) \\
&- 0''.0058 \sin (3\odot - 281^\circ.2) - 0''.0024 \sin 2(\odot - \Omega) \\
&+ 0''.0053 \sin 2(\odot - \Gamma') - 0''.2041 \sin 2\leftmoon \\
&+ ''.0677 \sin (\leftmoon - \Gamma') + \ldots \\
\Delta\omega = &+ 9''.2240 \cos \Omega - 0''.0895 \cos 2\Omega + 0''.5506 \cos 2\odot \\
&+ 0''.0092 \cos (281^\circ.2 + \odot) - 0''.0067 \cos (2\odot - \Omega) \\
&+ 0''.0027 \cos (3\odot - 281^\circ.2) + 0''.0023 \sin \Gamma' \\
&+ 0''.0885 \cos 2\leftmoon + 0''.0181 \cos (2\leftmoon - \Omega) + \ldots
\end{aligned}$$

(38)

and $h = 20''.4451$.

In these formulæ μ and μ' are the proper motions in R. A. and Decl., $\odot$ the Sun's true longitude, Ω the longitude of the Moon's

ascending node, ω the obliquity of the ecliptic, T′ the longitude of the Moon's perigee and ☽ the Moon's longitude.

Taking $\omega = 23^\circ\ 27'\ 8''$, substituting the values of Δl, $\Delta\omega$ and h, reducing and arranging the coefficients and restoring the values of m and n, we have the following:—

$$
\begin{aligned}
\alpha' = \alpha &+ \tau(46''.0907 + 20''.0410 \sin\alpha\tan\delta) && \text{(Precession)}\\
&- (15''.8318 + 6''.8682 \sin\alpha\tan\delta)\sin\text{☊}\\
&+ (0''.1902 + 0''.0825 \sin\alpha\tan\delta)\sin 2\text{☊}\\
&- (1''.1646 + 0''.5052 \sin\alpha\tan\delta)\sin 2\odot\\
&+ (0''.1352 + 0''.0488 \sin\alpha\tan\delta)\sin(82^\circ + \odot)\\
&+ (0''.0145 + 0''.0050 \sin\alpha\tan\delta)\sin(2\odot - \text{☊})\\
&- (0''.0053 + 0''.0023 \sin\alpha\tan\delta)\sin(3\odot - 281^\circ.2)\\
&- (0''.0022 + 0''.0009 \sin\alpha\tan\delta)\sin 2(\odot - \text{☊})\\
&+ (0''.0049 + 0''.0021 \sin\alpha\tan\delta)\sin 2(\odot - T')\\
&- (0''.1872 + 0''.0812 \sin\alpha\tan\delta)\sin 2\text{☽}\\
&+ (0''.0621 + 0''.0269 \sin\alpha\tan\delta)\sin(\text{☽} - T') && \text{(Nutation)}\\
&- 9''.2240 \cos\text{☊}\cos\alpha\tan\delta\\
&+ 0''.0895 \cos 2\text{☊}\cos\alpha\tan\delta\\
&- 0''.5506 \cos 2\odot\cos\alpha\tan\delta\\
&- 0''.0092 \cos(281^\circ.2 + \odot)\cos\alpha\tan\delta\\
&+ 0''.0067 \cos(2\odot - \text{☊})\cos\alpha\tan\delta\\
&- 0''.0027 \cos(3\odot - 281^\circ.2)\cos\alpha\tan\delta\\
&- 0''.0023 \sin T'\cos\alpha\tan\delta\\
&- 0''.0885 \cos 2\text{☽}\cos\alpha\tan\delta\\
&- 0''.0181 \cos(2\text{☽} - \text{☊})\cos\alpha\tan\delta\\
&- 20''.4451 \cos\alpha\cos\odot\cos\omega\sec\delta && \text{(Aberration)}\\
&- 20''.4451 \sin\alpha\cos\odot\sec\delta\\
&+ \tau\mu && \text{(Proper Motion)}
\end{aligned}
$$

and

$$
\begin{aligned}
\delta' = \delta &+ 20''.041\,\tau\cos\alpha && \text{(Precession)}\\
&- 6''.8682 \cos\alpha\sin\text{☊} \qquad + 9''.2240 \sin\alpha\cos\text{☊}\\
&+ 0''.0825 \cos\alpha\sin 2\text{☊} \qquad - 0''.0895 \sin\alpha\cos 2\text{☊}\\
&- 0''.5052 \cos\alpha\sin 2\odot \qquad + 0''.5506 \sin\alpha\cos 2\odot\\
&+ 0''.0488 \cos\alpha\sin(82^\circ + \odot) + 0''.0092 \sin\alpha\cos(281^\circ.2 + \odot)\\
&+ 0''.0050 \cos\alpha\sin(2\odot - \text{☊}) - 0''.0067 \sin\alpha\cos(2\odot - \text{☊})\\
&- 0''.0023 \cos\alpha\sin(3\odot - 281^\circ.2) && \text{(Nutation)}\\
&\qquad\qquad + 0''.0027 \sin\alpha\cos(3\odot - 281^\circ.2)\\
&- 0''.0009 \cos\alpha\sin 2(\odot - \text{☊}) + 0''.0023 \sin\alpha\sin T'\\
&+ 0''.0021 \cos\alpha\sin 2(\odot - T') + 0''.0885 \sin\alpha\cos 2\text{☽}\\
&- 0''.0803 \cos\alpha\sin 2\text{☽} \qquad + 0''.0181 \sin\alpha\cos(2\text{☽} - \text{☊})\\
&+ 0''.0269 \cos\alpha\sin(\text{☽} - T') \;\ldots\ldots\\
&- 20''.4451 \cos\omega\cos\odot(\tan\omega\cos\delta - \sin\alpha\sin\delta) && \text{(Aberration)}\\
&- 20''.4451 \sin\odot\cos\alpha\sin\delta\\
&+ \tau\mu' && \text{(Proper Motion)}
\end{aligned}
$$

From these formulæ the apparent R. A. and Decl. of any star may be computed, but the computation in this form would be exceedingly laborious and troublesome and therefore we have recourse to the following elegant transformation invented by the

illustrious Bessel. In the formula for α', it is at once seen that the numerical coefficients in the terms involving $\sin \Omega$, $\sin 2\Omega$, etc., are very nearly in the ratio of m to n, that is of $46''.0907$ to $20''.0410$—a circumstance which is not accidental but results from the theory of nutation. Since

	$n = 20''.0410$	and	$m = 46''.0907$
Put............	$nk = 6.8682$	and	$mk + g = 15.8318$
	$nk_1 = 0.0825$		$mk_1 + g_1 = 0.1902$
	$nk_2 = 0.5052$		$mk_2 + g_2 = 1.1646$
	$nk_3 = 0.0488$		$mk_3 + g_3 = 0.1352$
	$nk_4 = 0.0050$		$mk_4 + g_4 = 0.0145$
	$nk_5 = 0.0023$		$mk_5 + g_5 = 0.0053$
	$nk_6 = 0.0009$		$mk_6 + g_6 = 0.0022$
	$nk_7 = 0.0021$		$mk_7 + g_7 = 0.0049$
	$nk_8 = 0.0812$		$mk_8 + g_8 = 0.1872$
	$nk_9 = 0.0269$		$mk_9 + g_9 = 0.0621$

Substituting in the preceding formulæ for α' and δ' and factoring, we have

$$\begin{aligned}
\alpha' = \alpha &+ [\tau - k \sin \Omega + k_1 \sin 2\Omega - k_2 \sin 2\odot + k_3 \sin (82^\circ + \odot) \\
&+ k_4 \sin (2\odot - \Omega) - k_5 \sin (281^\circ.2 + 3\odot) - k_6 \sin 2(\odot - \Omega) \\
&+ k_7 \sin 2(\odot - \mathrm{T}') - k_8 \sin 2☽ + k_9 \sin (☽ - \mathrm{T}') + ..][m + n \sin \alpha \tan \delta] \\
&- [9''.2240 \cos \Omega - 0''.0895 \cos 2\Omega + 0''.5506 \cos 2\odot \\
&+ 0''.0092 \cos (281^\circ.2 + \odot) - 0.0067 \cos (2\odot - \Omega) \\
&+ 0.0027 \cos (3\odot - 281^\circ.2) + 0.0023 \sin \mathrm{T}' - 0.0885 \cos 2☽ \\
&- 0.0181 \cos (2☽ - \Omega)] \cos \alpha \tan \delta \\
&- 20''.4451 \cos \alpha \cos \odot \cos \omega \sec \delta \\
&- 20''.4451 \sin \alpha \sin \odot \sec \delta \\
&- g \sin \Omega + g_1 \sin 2\Omega - g_2 \sin 2\odot + g_3 \sin (82^\circ + \odot) + g_4 \sin 2(\odot - \Omega) \\
&- g_5 \sin (3\odot - 281^\circ.2) - g_6 \sin 2(\odot - \Omega) + g_7 \sin 2(\odot - \mathrm{T}') \;.\;.\;.\;. \\
&+ \tau\mu
\end{aligned} \qquad (39)$$

and

$$\begin{aligned}
\delta' = \delta &+ [\tau - k \sin \Omega + k_1 \sin 2\Omega - k_2 \sin 2\odot + k_3 \sin (82^\circ + \odot) \\
&+ k_4 \sin (2\odot - \Omega) - k_5 \sin (3\odot - 281^\circ.2) - k_6 \sin 2(\odot - \Omega) \\
&+ k_7 \sin 2(\odot - \mathrm{T}') - k_8 \sin 2☽ + k_9 \sin (☽ - \mathrm{T}')]\, n \cos \alpha \\
&+ [9''.2240 \cos \Omega - 0''.0895 \cos 2\Omega + 0''.5506 \cos 2\odot \\
&+ 0''.0092 \cos (281^\circ.2 + \odot) - 0''.0067 \cos (2\odot - \Omega) \\
&+ 0''.0027 \cos (3\odot - 281^\circ.2) + 0''.0023 \sin \tau' + 0''.0885 \cos 2☽ \\
&+ 0''.0181 \cos (2☽ - \Omega)] \sin \alpha \\
&- 20''.4451 \cos \omega \cos \odot (\tan \omega \cos \delta - \sin \alpha \sin \delta) \\
&- 20''.4451 \sin \odot \cos \alpha \sin \delta \\
&+ \tau\mu'
\end{aligned} \qquad (40)$$

The values of k, k_1, etc., g, g_1, etc., obtained by solving the preceding equations, are :

$k = 0.3427$	$g = 0.0360$
$k_1 = 0.0041$	$g_1 = 0.0006$
$k_2 = 0.0252$	$g_2 = 0.0028$
$k_3 = 0.0024$	$g_3 = 0.0230$
$k_4 = 0\ 0002$	$g_4 = 0.0030$
$k_5 = 0.0001$	$g_5 = 0.0000$
$k_6 = 0.00004$	$g_6 = 0.0001$
$k_7 = 0.0001$	$g_7 = 0.0001$
$k_8 = 0.00405$	$g_8 = 0.00045$
$k_9 = 0.00134$	$g_9 = 0.00023$

Substituting these values we finally have

$$\begin{aligned}
\alpha' = \alpha &+ [\tau - 0''.3427 \sin ☊ + 0''.0041 \sin 2 ☊ - 0''.0252 \sin 2☉ \\
&+ 0''.0024 \sin (82° + ☉) + 0.0002 \sin (2☉ - ☊) \\
&- 0.0001 \sin (3☉ - 282°.2) - .00004 \sin 2(☉ - ☊) \\
&+ 0.0001 \sin 2 (☉ - T') - 0.00405 \sin 2 ☽ \\
&+ 0.00134 \sin (☽ - T')] [m + n \sin \alpha \tan \delta] \\
&- [9''.2240 \cos ☊ - 0.0895 \cos 2☊ + 0''.5506 \cos 2☉ \\
&+ 0''.0092 \cos (281°.2 + ☉) - 0''.0067 \cos (2☉ - ☊) \\
&+ 0''.0027 \cos (3☉ - 281°.2) + 0.0023 \sin T' + 0.0885 \cos 2 ☽ \\
&+ 0.0181 \cos (2 ☽ - ☊)] \cos \alpha \tan \delta \\
&- 20''.4451 \cos \alpha \cos ☉ \cos \omega \sec \delta \\
&- 20''.4451 \sin \alpha \sin ☉ \sec \delta \\
&- 0''.0364 \sin ☊ - 0''.0039 \sin 2☊ - 0''.0028 \sin 2 ☉ \\
&\qquad\qquad\qquad\qquad + 0''.0230 \sin (82° + ☉) \\
&+ 0''.0030 \sin (2☉ - ☊) - 0.00001 \sin (3☉ - 281°.2) \\
&\qquad\qquad\qquad\qquad - 0.0001 \sin 2(☉ - ☊) \\
&+ 0.0001 \sin 2(☉ - T') - .00045 \sin 2 ☽ + \ 00023 \sin (☽ - T') + \ldots \\
&+ \tau\mu
\end{aligned}$$

and

$$\begin{aligned}
\delta' = \delta &+ [\tau - 0''.3427 \sin ☊ + 0''\ 0041 \sin 2☊ - 0''.0252 \sin 2☉ \\
&+ 0''.0024 \sin (82° + ☉) + 0''.0002 \sin (2☉ - ☊) \\
&- 0''.0001 \sin (3☉ - 281°.2) - .00004 \sin 2(☉ - ☊) \\
&+ 0.0001 \sin 2(☉ - T') - .00405 \sin 2 ☽ + .00133 \sin (☽ - T')]\ n \cos \alpha \\
&+ [9''.2240 \cos ☊ - 0''.0895 \cos 2☊ + 0''.5506 \cos 2☉ \\
&+ 0''.0092 \cos (282°.2 + ☉) - 0''.0067 \cos (2☉ - ☊) \\
&+ 0.0027 \cos (3☉ - 281°.2) + 0''.0023 \sin T' + 0.0885 \cos 2 ☽ \\
&+ 0.0181 \cos (2 ☽ - ☊)] \sin \alpha \\
&- 20''.4451 \cos \omega \cos ☉ (\tan \omega \cos \delta - \sin \alpha \sin \delta) \\
&- 20''.4451 \sin ☉ \cos \alpha \sin \delta \\
&+ \tau\mu'
\end{aligned}$$

Put

$$\begin{aligned}
A = \tau &- 0''.3427 \sin ☊ + 0''.0041 \sin 2☊ - 0''.0252 \sin 2☉ \\
&+ 0''.0024 \sin (82° + ☉) + 0''.0002 \sin (2☉ - ☊) \\
&- 0.0001 \sin (3☉ - 281°.2) - .00004 \sin 2(☉ - ☊) \\
&+ 0.0001 \sin 2(☉ - T') - .00405 \sin 2 ☽ + .00134 \sin (☽ - T') \\
B = &- 9''.2240 \cos ☊ + 0''.0895 \cos 2☊ - 0''.5506 \cos 2☉ \\
&- 0.0092 \cos (281°.2 + ☉) + 0.0067 \cos (2☉ - ☊) \\
&- 0.0027 \cos (3☉ - 281°.2) - 0.0023 \sin T' - 0.0885 \cos 2 ☽ \\
&- .0181 \cos (2 ☽ - ☊) \\
C = &- 20''.4451 \cos ☉ \cos \omega \\
D = &- 20''.4451 \sin ☉ \\
E = &- 0.0364 \sin ☊ + 0.0039 \sin 2☊ - 0.0028 \sin 2☉ \\
&+ 0''.0230 \sin (82° + ☉) + 0''.0030 \sin (2☉ - ☊) - \ldots
\end{aligned}$$

These quantities depend on the date and are independent of the star's place; they are computed and tabulated for each Washington mean midnight on pages 281-292, under the name of Besselian Star Numbers.

We also put

$$a = m + n \sin\alpha \tan\delta$$
$$= 46''.0907 + 20''.0410 \sin\alpha \tan\delta$$
$$= 3^s.07267 + 1^s.33606 \sin\alpha \tan\delta$$
$$b = \tfrac{1}{15} \cos\alpha \tan\delta$$
$$c = \tfrac{1}{15} \cos\alpha \sec\delta$$
$$d = \tfrac{1}{15} \sin\alpha \sec\delta$$

and

$$a' = n \cos\alpha = 20''.0410 \cos\alpha$$
$$b' = -\sin\alpha$$
$$c' = \tan\omega \cos\delta - \sin\alpha \sin\delta$$
$$d' = \cos\alpha \sin\delta$$

all of which depend on the star's place and are sometimes given in the star catalogues.

Then we have

$$\alpha' = \alpha + \mathrm{A}a + \mathrm{B}b + \mathrm{C}c + \mathrm{D}d + \mathrm{E} + \tau\mu \qquad \text{(in time)}$$
$$\delta' = \delta + \mathrm{A}a' + \mathrm{B}b' + \mathrm{C}c' + \mathrm{D}d' + \tau\mu' \qquad \text{(in arc)}$$

Another transformation which is sometimes more convenient than the above is made as follows:

Put

$$f = 46''.0907\mathrm{A} + \mathrm{E} = 3^s.07267\mathrm{A} + \tfrac{1}{15}\mathrm{E} \qquad \text{(in time)}$$

$g \sin \mathrm{G} = \mathrm{B}$ $\qquad h \sin \mathrm{H} = \mathrm{C}$ $\qquad$ and $\qquad i = \mathrm{C} \tan\omega$

$g \cos \mathrm{G} = 20''.0410\mathrm{A}$ $\qquad h \cos \mathrm{H} = \mathrm{D}$

Then we shall have

$$\alpha' = \alpha + f + \tfrac{1}{15}g \cos \mathrm{G} \sin\alpha \tan\delta + \tfrac{1}{15}g \sin \mathrm{G} \cos\alpha \tan\delta$$
$$+ \tfrac{1}{15}h \sin \mathrm{H} \cos\alpha \sec\delta + \tfrac{1}{15}h \cos \mathrm{H} \sin\alpha \sec\delta + \tau\mu$$
$$= \alpha + f + \tfrac{1}{15}g \sin(\mathrm{G} + \alpha) \tan\delta + \tfrac{1}{15}\sin(\mathrm{H} + \alpha) \sec\delta$$
$$+ \tau\mu \qquad \text{(in time)}$$

(43)

and

$$\delta' = \delta + g \cos \mathrm{G} \cos\alpha - g \sin \mathrm{G} \sin\alpha - h \sin \mathrm{H} \sin\alpha \sin\delta$$
$$+ h \cos \mathrm{H} \cos\alpha \sin\delta + i \cos\delta + \tau\mu'$$
$$= \delta + g \cos(\mathrm{G} + \alpha) + h \cos(\mathrm{H} + \alpha) \sin\delta + i \cos\delta$$
$$+ \tau\mu' \qquad \text{(in arc)}$$

(44)

The quantities f, G, H, g, h, and i are called the Independent Star Numbers and are computed for every Washington mean midnight—see pages 285-292. Since the coefficients of sin ☊, sin 2☉ etc. in the terms of A, B, C and D remain sensibly constant for a long period, the values of the several terms can be tabulated with

Ω, $2\odot$, 2Ω, $82^\circ + \odot$, etc., as arguments and thus the computation of the Besselian Numbers is greatly facilitated.

The preceding formulæ are accurate for all stars except those very near the pole. For the latter all appreciable terms must be included and especially those involving the tangent and secant of the declination. It now remains to determine the value of τ or the date of the beginning of the fictitious year or when the Sun's *mean* longitude is 280°. This date does not correspond to the beginning of the tropical year on the meridian of Greenwich. It is evident that the meridian on which this fictitious year begins, must vary from year to year and since the Sun's mean longitude is always equal to his mean R. A., the sidereal time at this meridian when the fictitious year begins must be $18^h\ 40^m = 280^\circ$. Now according to Bessel, the Sun's *mean* longitude at mean noon at Paris on 1800, Jan. 0, was $279^\circ\ 54'\ 1''.36$ which quantity is usually denoted by E. The student must be careful not to confound this number with the quantity E in the preceding formulæ; he must also remember that by Jan. $0^d.0$ is meant Dec. 31, noon, or Dec. $31^d.0$. The Sun's *sidereal* motion in 365.25 days is

$$360^\circ - 22''.617656.$$

This is estimated from a *fixed* point of the ecliptic, but the *mean* longitude is referred to the *moving equinox*, and it therefore follows that the mean motion is equal to the sidereal plus the general precession in the same time. Bessel's general precession is $50''.2235$, hence the mean motion in 365.25 days (neglecting the secular variation)

$$\begin{aligned} &= 360^\circ - 22''.617656 + 50''.2235 \\ &= 360^\circ + 27''.605844 \end{aligned}$$

Therefore the mean daily motion $= 59'\ 8''.3302$ and the mean motion in 365 days = mean motion in 365.25 days less one fourth of the mean daily motion or $14'\ 47''.083$. Now it is plain that the value of E for any year, say $1800 + t$, is found by adding the motion in 365 days for each common year and the motion in 366 days for each leap year, so that if r denote the remainder after dividing t by 4, we shall have for the date $1800 + t$, Jan. 0, Paris mean time

$$E = 279^\circ\ 54'\ 1''.36 + 27''.6058\ t - (14'\ 47''.083)\ r$$

and if we denote by τ, the mean time interval from the beginning of the fictitious year to Jan. 0 of any year, we must have

$$\tau = \frac{E - 280^\circ}{\text{mean daily motion}} = -0.10107 + 0.00777995\,t - \tfrac{1}{4}r$$

expressed in mean solar days.

Suppose, for example, we require the date of the beginning of the fictitious year for 1890 for Washington, D. C. By the preceding we have for 1890, Jan. 0, Paris M. T.,

$$\begin{aligned} E &= 279^\circ\ 54'\ 1''.36 + 27''.6058 \times 90 - (14'\ 47''.083) \times 2 \\ &= 280^\circ\ 5'\ 51''.72 \end{aligned}$$

which shows that the Sun attained the mean longitude of 280° some time *before* Jan. 0; or the meridian on which the fictitious year began is *east* of Paris. Then we have

$$\begin{aligned} \tau &= -0.10107 + .70020 - .5 = +0.09912d \\ \text{Long. of Paris} &= 5^h\ 17^m\ 33^s \qquad\qquad = \quad 0.22052 \\ \hline \tau &= \quad 0.31964 \text{ or } 0^d.320 \end{aligned}$$

that is, 0.320 days before Jan. 0 or 1889, Dec. $30^d.680 = 1890$, Jan. $0^d.0 - 0^d.320$ Washington mean time.

We will now close this paper with the following example:

Find the Besselian and Independent star numbers for July 1st, 1890, Washington mean midnight—that is for 1890, July $1^d.5$. The interval from 1889, Dec. $30^d.680$ to 1890, July $1^d.5$, is 182.820 days $= 0.5005$ years $= \tau$. On page 278 of the Ephemeris we find for July 1.5, $\Omega = 82^\circ\ 54'\ 15''$; on page 112 we find $\odot = 100^\circ\ 13'\ 28''$ and on page 274, $☽ = 274^\circ\ 49'$. The value of Γ' or the longitude of the Moon's perigee is not given in the Ephemeris and indeed the terms involving it are almost inappreciable, but if it is deemed necessary to include them in the values of A and B, we will here indicate how it may be found with sufficient accuracy for our purpose.

The longitude of the perigee of the Moon or of the perihelion of a planet is measured on the plane of the orbit of the Moon or planet and not on the ecliptic, and is equal to the distance of the Moon or planet from the node plus the longitude of the node. For this reason the perigee or perihelion is not regarded as having a latitude, while the Moon or planet when referred to the ecliptic, has both longitude and latitude. Now when the Moon is in perigee which occurs on July 3^d, $2^h.4$ Greenwich M. T., or July $2^d\ 21^h.264$ Washington M. T., (see page 121) the Moon's longitnde will also be the longitude of perigee *referred to the ecliptic*. From page 274 we then find for this date

☽'s long. $= 295° \ 40' \ 19''$
☽'s lat. $\ \ = -2° \ 38' \ 44''$

approximately. If we now deduct the mean longitude of the node at the same time or $82° \ 50' =$ ☊, we shall have $212° \ 50' \ 19''$ and $-2° \ 38' \ 44''$ for the two sides of a right angled spherical triangle to find the third side or hypotenuse which is the distance from the perigee to the node. Denoting this distance by $\varkappa$, the ☽'s longitude and latitude by λ and β, respectively, we have

$$\cos \varkappa = \cos (\lambda - ☊) \cos \beta$$

from which we find	$\varkappa = 212° \ 56'$
to which add	☊ $= \ \ 82 \ \ \ 50$
and we have	$T' = 295° \ 46'$

The perigee has a mean daily motion of about $+ 6' \ 40''$, and therefore deducting the motion in $1^d \ 9^h.264$ we shall have for

July $1^d \ 12^h$ Washington M. T., $T' = 295° \ 43'.4$

which is of course only an approximate value but quite near enough for our purpose. We will now resume the computation of A, B., etc.

Besselian Star Numbers.

log .3427 = 9.53491	.0041 = 7.61278	.0252 = 8.40140
sin ☊ = 9.99666	sin 2☊ = 9.38946	sin 2⊙ = 9.54330*n*
.340070 = 9.53157	.001005 = 7.00224	− .00880 = 7.94470*n*
.0024 = 7.38021	.0002 = 6.30103	.0001 = 6.00000
sin (82 + ⊙) = 8.58932*n*	sin (2⊙ − ☊) = 9.94775	sin (3⊙ − 281°.2) = 9.52292
− .00093 = 5.96953*n*	.000177 = 6.24878	.000033 = 5.52292
0.0001 = 6.00000	.00405 = 7.60260	.00134 = 7.12710
sin 2(⊙ − τ') = 9.42690	sin 2☽ = 9.22361*n*	sin (☽ − T') = 9.55235*n*
.000027 = 5.42690	− .00067 = 6.82621*n*	− .000478 = 6.67945*n*

A = .50053 − .34007 + .001005 + .00880 − .000093 + .000177 − .000033 + .000027 + .00067 − .000478 = + 0.17055 and log. A = 9.23184.

log 9.2240 = 0.96492	.0898 = 8.95182	.5506 = 9.74084
cos ☊ = 9.09177	cos 2☊ = 9.98654*n*	cos 2⊙ = 9.97173*n*
1.13945 = 0.05669	− .08677 = 8.93836*n*	− .51590 = 9.71257*n*
.0092 = 7.96379	.0067 = 7.82607	.0027 = 7.43136
cos(281°.2+⊙) = 9.96890	cos (2⊙ − ☊) = 9.66505*n*	cos(3⊙ − 281°.2) = 9.97443
.00856 = 7.93269	− .00310 = 7.49112*n*	.00255 = 7.40579
.0023 = 7.36173	.0885 = 8.94694	.0181 = 8.25768
sin T' = 9.95470*n*	cos 2☽ = 9.99383*n*	cos(2☽ − ☊) = 9.45927*n*
− .00207 = 7.31643*n*	− .08720 = 8.94077*n*	− .00521 = 7.71695*n*

$$B = -1.13945 - .08677 + .51590 - .008564 - .003098 - .002545 + .002072 + 0.0872 + 0.00521 = -0.6308, \qquad \log B = 9.79962n.$$

$$\begin{array}{rl@{\qquad}rl}
-20''.4451 = & 1.31059n & -20''.4451 = & 1.31059n \\
\cos \odot = & 9.24921n & \sin \odot = & 9.99305 \\
\cos \omega = & 9.96255 & & \\
\hline
& & \log D = & 1.30364n \\
\log C = & 0.52235 & \text{and} \quad E = & -0''.275
\end{array}$$

These coefficients differ slightly from those in the Ephemeris; this is due partly to the fact that different values of Δl and $\Delta\omega$, as well as of m and n, are here employed and also to the omission of several very small terms.

THE INDEPENDENT STAR NUMBERS.

$$\begin{array}{rl@{\qquad}rl}
46''.0907 = & 1.66362 & 20''.0410 = & 1.30192 \\
A = & 9.23184 & A = & 9.23184 \\
\hline
7''.861 = & 0.89546 & g \cos G = & 0.53376 \\
.15 = & 1.17609 & g \sin G = & 9.79962n \\
\hline
0^s.5241 = & 9.71937 & \tan G = & 9.26586n \\
& & G = & 349^\circ\ 33' \\
1\text{-}15\ E = & -0.014 & \sin G = & 9.25858n \\
\hline
0.510 = & f \text{ (in time)} & \log g = & 0.54104 \\
h \sin H = & 0.52235 & \log C = & 0.52235 \\
h \cos H = & 1.30364n & \tan \omega = & 9.63732 \\
\hline
\tan H = & 9.21871n & \log i = & 0.15967 \\
H = & 170^\circ\ 36'\ 16'' & i = & 1''.45 \\
\sin H = & 9.21309 & & \\
\hline
\log h = & 1.30926 & &
\end{array}$$

These also differ slightly from the values given in the Ephemeris for the reasons already given.

Examples showing the use of these numbers are given in the Explanation of the Ephemeris and therefore need not be repeated here. When the reader shall have gone through this long and tedious investigation, he will doubtless come to the conclusion that the so-called fixed stars are by no means what they are represented to be. The difficulty, however, is not due to any motion in the stars themselves but to the fact that the plane and point of reference are constantly in motion, thus necessitaing the application of three of the astronomical corrections, viz, precession, nutation and aberration.

A number of new problems and acknowledgement of those solved will be given in the next number.

(TO BE CONTINNED.)

THE PLANETS AND CONSTELLATIONS FOR AUGUST, 1896.

Mercury begins the month behind the Sun, having passed superior conjunction July 31, and will not be in favorable position for observation during the month. Mercury will be in conjunction with Jupiter, Aug. 4, at midnight, Central standard time, and with Venus, Aug. 8, at 8^h A. M.

THE CONSTELLATIONS AT 9^h P. M. AUGUST 1, 1896.

Venus is also behind the Sun and will not be in good position during August. Venus and Jupiter will be in conjunction Aug. 2, at 5^h P. M. If we could see Venus at this time her phase would be full and round like that of Jupiter. The two planets will then be less than 1° apart.

the method described below, which promises to determine accurately the relative motion of two or more stars in the line of sight if they are near enough together to be photographed upon the same plate. Let A and B be two such stars, A being at rest and B approaching with such a velocity that a given line in its spectrum is deviated by the amount d, and let a photograph be taken in such a position that the end of shorter wave length of the spectrum of B is turned towards that of A. Then the distance between the images of the given line in the two spectra will be less by the amount d. than it would be if both stars were at rest. Now let another photograph be taken in which, by turning the prism 180°, the spectra are turned by the same amount, so that the end of greater wave length of the spectrum of B is turned towards that of A. The distance between the two lines will then be increased by an equal amount. If two such photographs are superposed and the images of the reference line in the spectra of A are made to coincide, its images in the spectra of B will deviate by $2d$. To apply this method, a photograph of a region a little east of the meridian is taken in the usual way. Then the telescope is reversed and a second photograph of the same region is taken on a plate with the film side away from the star, so that the photograph is taken through the glass. As both photographs are taken near the meridian the lines will be nearly perpendicular to the length of the spectrum, while, at a large hour angle, if the exposure is long, and the spectra narrow, the lines will cross them obliquely, owing to the differential refraction. Reversing the telescope turns the prisms, and with them the spectra exactly 180°. In making the examination the plates are placed film to film so that the spectra are side by side, and one is moved over the other by means of a micrometer screw. The corresponding lines in the two images of each star in turn are made to coincide, and the difference in the readings gives double the displacement of the line. An error in orienting the plates would affect the results when the stars compared are not in the same right ascension. This source of error may probably be made insensible in several ways, such as by marking a reference line on each plate, or by turning the prisms so that their edges are perpendicular to the line connecting the stars and moving the plate slowly by clock-work. Since the ends only of the lines are compared, narrow spectra may be used, and faint stars may therefore be measured. Experiments are now in progress with a cylindrical lens, by which it is expected that the accuracy of setting on lines in very narrow spectra can be still further increased.

Only preliminary tests of this method can be made at Cambridge at present, as our three best prisms are now in Peru. Two photographs of 101 and 102 Herculis were, however, taken on October 9, 1896, with a single prism, giving poor definition, but showed by inspection that the first of these stars was approaching more rapidly than the second. Measures by Mr. King of the lines $H\varepsilon$ and $H\zeta$ indicated the relative velocities 87 and 94 kilometres a second respectively. These results are not corrected for the position of the prism and other sources of instrumental error. The probable error as indicated by the accordance of the individual settings is 5 kilometres in each case. An inspection of two photographs of the Pleiades shows that the relative motions of the seven brightest stars in the group, although perhaps measureable, is not appreciable to the eye, and probably does not exceed 30 kilometres a second.

The advantages of the above method are, first, the directness of the determination of the motion; second, that double the deviation is measured; and third, that as the ends of two similar lines are made to coincide, the accidental errors of measurement are much less than when each in turn is bisected by a spider line. Since each line in the spectrum may be used a large number of independent determinations may be obtained from one pair of plates. On the other hand, as it is only necessary that one line should be in focus, a visual telescope may be employed; that is, one uncorrected for the photographic rays. No corrections need be applied for the motion of the Sun in space or of the Earth in its orbit, since they will affect both stars equally.

HARVARD COLLEGE OBSERVATORY, Circular No. 13.
November 4, 1896.

PRACTICAL SUGGESTIONS.

ECLIPSES.

J. MORRISON, M. A., M. D., PH. D.

FOR POPULAR ASTRONOMY.

From time immemorial eclipses, and especially those of the Sun, have been universally regarded as the most interesting and striking astronomical phenomena. Solar eclipses furnish astronomers with the best opportunity of testing the accuracy of the Solar and Lunar Tables and also for studying the physical

constitution of the Sun. The calculation of all the circumstances connected with eclipses, involves considerable labor and is performed by several methods according to the accuracy which is desired.

In ancient times it was discovered that eclipses both of the Sun and Moon repeat themselves after an interval or cycle of 18 years and 11 or 12 days, known by the name of the Chaldean period or the Saros. This remarkable cycle is due to the fact that 242 *nodal* revolutions of the Moon are performed *in nearly* the same period as 19 *nodal* revolutions of the Sun or in other terms, 223 lunations or synodical revolutions of the Moon require for their completion 6585.321222 days or 18.029627 Julian years or 18.03001 + tropical years, that is, 18 common years and 11 or 12 days according to the number of leap years contained in the period.

Another remarkable relation in connection with this cycle is that 239 anomalistic revolutions of the Moon are performed in 6585.549 days which differs from the above by less than one-fourth of a day. Hence it follows that at the end of a Saros, the Sun, Moon and the Moon's node are found almost in their original relation, nor is this all, the Moon's mean anomaly has the same value to within $\pm$ 3° and the Sun's mean anomaly to within 12°. These circumstances are due to the fact that the times of revolution of Sun, Moon and the Moon's node and perigee are all close multiples of 18 years. The exact changes in the elements during a Saros or 223 lunations are as follows:

In the argument of latitude	— 28′.5
In the Moon's mean anomaly	— 2°.8
In the Sun's mean anomaly	+ 10°.5
In the distance of the Moon's perigee from node	+ 2°.4
In the distance of the Sun's perigee from node	— 11°.0

It is quite evident then that owing to these slight changes, the mean place of the Moon and all its larger inequalities will attain the same or very nearly the same values at the end of the period that they had at the beginning, and not only these, but the parallax and semidiameter of the Moon also return to their former values.

In consequence of the retrograde motion of 28′.5 from the node during each Saros the corresponding eclipses in succeeding cycles will be subject to slow progressive changes in their magnitude and duration. A series of such eclipses will begin with a very small partial eclipse near one pole of the Earth and gradually in-

crease for about eleven or twelve recurrences after which it will become annular or total near the same pole. Some forty or forty-five such eclipses will then follow, the line of annular or total eclipse moving slowly toward the other pole, it will then again become partial and finally cease altogether. The series will contain about 70 eclipses of the Sun, embracing a period of about 1260 years, after which a new series will begin and pass through the same changes.

As already stated, there are several methods of computing eclipses.

1st. There is the graphical method described in Loomis's Practical Astronomy and other similar works. This method does not of course aim at anything like accuracy: it is in fact only a rough approximation at best, and is not to be recommended.

2d. The parallactic method which consists in assuming one or more times for the beginning and end of the eclipse and then computing for the given place, the parallax in R. A. and Decl. which when applied to the true R. A. and Decl. of the Sun and Moon will give their apparent places. The beginning or end of the eclipse will of course take place when the apparent distance between the centres of the Sun and Moon, is equal to the sum of their augmented semi-diameters. This method is fully explained in the work already alluded to and is capable of a high degree of accuracy but it involves considerable labor and lacks the generality of other methods.

3rd. Woolhouse's method which may be regarded to some extent as an extension of the preceding. It is employed in the English Nautical Almanac and gives all the circumstances of an eclipse with all the accuracy desired. The student who wishes to become acquainted with this method will find it fully explained in the appendix of the English Nautical Almanac for the year 1836.

4th. Bessel's method employed in the American Ephemeris and Nautical Almanac, is the one which we propose to explain in the following papers. It furnishes all the circumstances of an eclipse with a degree of accuracy which leaves little or nothing further to be desired.

Whatever method is employed, the first thing to be done is to determine the elements of the eclipse which are as follows:

Greenwich Mean Time of ☌ of Sun and Moon in R. A.

R. A. of the Sun and Moon (at this date).

Decl. of the Sun and Moon.

Sun and Moon's hourly motions in R. A. and Decl.

Sun and Moon's equatorial horizontal parallaxes.

Sun and Moon's semi diameters.

To compute the first, find from the ephemeris, the R. A. of the Sun and Moon for several hours preceding and following conjunction. These quantities are already given for the Moon, but will have to be carefully interpolated for the Sun. Subtract the Moon's R. A. from that of the Sun and let these differences be denoted by α_3, α_2, α_1, α_0, α_{-1}, α_{-2}, α_{-3}, etc., we are now to find the date at which this difference becomes zero. A simple inspection will show the dates between which conjunction will take place. Form the first, second and third differences in the usual way and put $b =$ 1st diff., $c = \frac{1}{2}(c_0 + c_1)$, *i. e.*, half the sum of the second difference which precede and follow b, and $d =$ 3d diff. Then we shall have for any term of the series

$$\alpha^{(t)} = \alpha_0 + tb + \frac{t\,(t-1)}{1,\,2}\,.\,c + \frac{t\,(t-1)\,(t-\frac{1}{2})}{1,\,2,\,3}\,.\,d + \text{etc.}$$

At the time of conjunction $a^{(t)} = 0$, putting then the second member equal to zero and solving for t we have, omitting the square and higher powers

$$t = \frac{\alpha_0}{-b + \frac{1}{2}c - \frac{1}{12}d} \text{ or } = \frac{\alpha_0}{\frac{1}{2}c - b} \qquad (45)$$

when the 3d diff. is neglected.

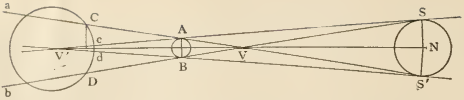
A check on the accuracy of this date will be found by computing the R. A. of the Sun and Moon which must be equal, and if not, the time must be corrected by a simple interpolation until the computed R. A's are exactly equal. The other elements are now interpolated for this date from the data in the ephemeris.

When an eclipse begins or ends at a place, the limbs of the Sun and Moon are in apparent contact and the observer is situated on the surface of the cone which envelopes the two bodies. An inspection of the diagrams shows that there are two such enveloping cones.

First. The cone *aVb* whose vertex *V* lies *between* the Sun and Moon, is called the penumbral cone and

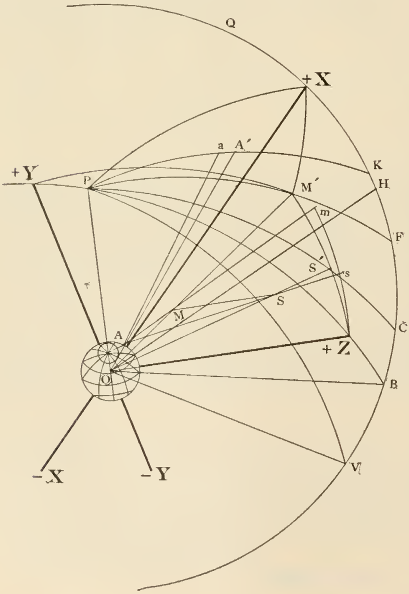
Second, the cone *SV′S′* whose vertex *V′* falls beyond the Moon towards the Earth, is called the umbral cone or cone of total shadow.

An observer at *C* or *D* sees the limbs, *AS′* or *BS* in apparent ex-



terior contact which are the beginning and ending of the entire eclipse, but if the observer be at *c* or *d* he sees the limbs *AS* or *BS′* in apparent *interior contact* which are the beginning and ending of the total or annular eclipse according as the observer is nearer to or farther from the Moon than the vertex *V′* of the umbral cone. The eclipse is partial at points between *Cc* and *Dd*, and total between *c* and *d*. *V′VN* is the axis of the shadow and the condition that an eclipse may occur at any given place, *C*, for instance, is that its distance from the axis must be equal to the radius of the shadow for that point and the analytical expression which fulfills this condition, is the fundamental equation in the theory of eclipses.

POSITION OF THE AXIS OF THE SHADOW.



Let *O* be the centre of the Earth, *A* a place on its surface, *S* the Sun's centre and *M* the Moon's and *S′*, *M′*, *A′*, *s* and *m* the points in which *OS*, *OM*, *OA*, *AS* and *AM* produced, meet the celestial sphere, hence *S′* and *M′* are the *true* places of the Sun and Moon and *s* and *m* their apparent places, and *A′* is the *geocentric* zenith of the place *A*.

Again, let *a* be the *geographical* zenith of the place *A*, *OZ* a straight line *parallel* to *MS* and meeting the celestial sphere in *Z*, *VQ* the equator, *V* the vernal equinox, *P* the

north pole of the equator, and PB, PC, PF and PK declination circles through Z, S', M' and A' and a. The line MS joining the centres of Sun and Moon meets the celestial sphere in the common vanishing point of all lines parallel to it, that is, in the point Z, and therefore the position of the axis of the cone will be determined by the R. A. and Decl. of this point.

Let $r = OM$ the Moon's distance.
$r' = OS$ the Sun's distance.
$G = MS$ the distance between the centres of Sun and Moon.
$\alpha = VF$ the Moon's R. A.
$\delta = FM$ " " Decl.
$\alpha' = VC$, the Sun's R. A.
$\delta' = CS$, " " Decl.
$a = VB$, the R. A. of the point Z.
$d = BZ$, the Decl. " " "

Make VH a quadrant, then OV, OH and OP will be a system of rectangular axes having the centre of the Earth as origin and the axis of X or OV the line through the equinoctial points, the axis of Y or OH the intersection of the planes of the equator and solstitial colure and the axis of Z or OP the axis of the equator. Also let x be positive towards the vernal equinox V; y positive towards that point of the equator whose R. A. is 90° and z positive towards the north, then the coördinates will be

of the Sun	and	of the Moon
$x_0' = r' \cos \delta' \cos \alpha'$		$x_0 = r \cos \delta \cos \alpha$
$y_0' = r' \cos \delta' \sin \alpha'$		$y_0 = r \cos \delta \sin \alpha$
$z_0' = r' \sin \delta'$		$z_0 = r \sin \delta$

If we now transfer the origin of coördinates to the centre of the Moon, the axes remaining parallel, then the R. A. and Decl. of the Sun as seen from this new origin, will be the same as the R. A. and Decl. of the point Z, and therefore the coördinates of the Sun in this new system will be

$$\begin{aligned} x'_m &= G \cos d \cos a \\ y'_m &= G \cos d \sin a \\ z'_m &= G \sin d \end{aligned}$$

But these coördinates must evidently be equal respectively to the difference of the corresponding coördinates of the Sun and Moon in the former system, that is

$$\begin{aligned} x'_m &= x'_0 - x_0 \\ y'_m &= y'_0 - y_0 \\ z'_m &= z'_0 - z_0 \end{aligned}$$

or

$$\begin{aligned} G \cos d \cos a &= r' \cos \delta' \cos \alpha' - r \cos \delta \cos \alpha \\ G \cos d \sin a &= r' \cos \delta' \sin \alpha' - r \cos \delta \sin \alpha \qquad (46) \\ G \sin d &= r' \sin \delta' - r \sin \delta \end{aligned}$$

which determine a, d and G in terms of known quantities, but it is expedient to put these equations in a more convenient form for computation. Multiply the first by cos α' and the second by sin α' and add the products; also multiply the first by sin α' and the second by cos α' and subtract the first product from the second and we find

$$\begin{aligned} G\cos d\cos(a-\alpha') &= r'\cos\delta' - r\cos\delta\cos(\alpha-\alpha') \\ G\cos d\sin(a-\alpha') &= -r\cos\delta\sin(\alpha-\alpha') \\ G\sin d &= r'\sin\delta' - r\sin\delta \end{aligned} \tag{47}$$

Divide each of these by r' and put

$$\frac{G}{r'} = g \text{ and } \frac{r}{r'} = b$$

then they become

$$\begin{aligned} g\cos d\cos(a-\alpha') &= \cos\delta' - b\cos\delta\cos(\alpha-\alpha') \\ g\cos d\sin(a-\alpha') &= -b\cos\delta\sin(\alpha-\alpha') \\ g\sin d &= \sin\delta' - b\sin\delta \end{aligned} \tag{48}$$

Dividing the second of these by the first and the third by the second, we have

$$\tan(a-\alpha') = -\frac{b\cos\delta\sec\delta'\sin(\alpha-\alpha')}{1-b\cos\delta\sec\delta'\cos(\alpha-\alpha')} \tag{49}$$

$$\tan d = -\frac{(\sin\delta' - b\sin\delta)\sin(a-\alpha')}{b\cos\delta\sin(\alpha-\alpha')} \tag{50}$$

and
$$g = \frac{\sin\delta' - b\sin\delta}{\sin d} \tag{51}$$

The quantity b is most easily computed as follows:

Let π = the Moon's equatorial horizontal parallax
π' = the Sun's " " "
π_0 = the Sun's mean horizontal parallax
and r_0 = the Sun's mean distance
then
$$r_0 \sin\pi_0 = r'\sin\pi'$$

or expressing r' in terms of r_0, that is making $r_0 = 1$, we have

$$\sin\pi' = \frac{\sin\pi_0}{r'}$$

and also
$$r\sin\pi = r'\sin\pi'$$

therefore
$$b = \frac{r}{r'} = \frac{\sin\pi'}{\sin\pi}$$

$$= \frac{\sin\pi_0}{r'\sin\pi} \tag{52}$$

in which r' and π are given in the ephemeris and π_0 is constant.

The formulæ for computing a, d and g are perfectly rigorous but as $(\alpha - \alpha')$ at the time of an eclipse can never exceed $1\frac{1}{2}°$, $a - \alpha_1$ is always less than $20''$, and b is about $\frac{1}{400}$, the following approximate formulæ will give the quantities with all the exactness required. At the time of an eclipse δ is nearly equal to δ' and we may write the arc for the sine and tangent of $\alpha - \alpha'$ and $a - \alpha'$ respectively and put $\cos\ (\alpha - \alpha') = 1$, hence (49) becomes

$$(a - \alpha_1)'' = - \frac{b}{1 - b} \cos \delta \sec \delta' \,(\alpha - \alpha')'' \qquad (53)$$

From the 1st and 3d of (48) we have approximately

$$g \cos d = \cos \delta' - b \cos \delta$$
$$g \sin d = \sin \delta' - b \sin \delta$$

Multiplying the first of these by $\sin \delta'$ and the second by $\cos \delta'$ and subtracting the second result from the first and also multiplying the first by $\cos \delta'$ and the second by $\sin \delta'$ and adding we have

$$g \sin (d - \delta') = - b \sin (\delta - \delta')$$

and $$g \cos (d - \delta') = 1 - b \cos (\delta - \delta')$$

whence $$\tan (d - \delta') = - \frac{b \sin (\delta - \delta')}{1 - b \cos (\delta - \delta')}$$

or approximately $$(d - \delta')'' = - \frac{b}{1 - b} (\delta - \delta')'' \qquad (54)$$

and $$g = 1 - b \qquad (55)$$

and then $$G = r' g \qquad (56)$$

To Find the Co-ordinates of the Moon.

In the diagram take BX and ZY each equal to a quadrant, then OX, OY and OZ form a system of rectangular axes, having the origin at O, the centre of the Earth and the axis OZ parallel to MS the line joining the centres of the Moon and Sun and $BZ = PY = d$. The plane of xy is always at right angles to the axis of the shadow and is called the principal to fundamental plane of reference. The plane of yz is the plane of the declination circle through Z and the plane of xz is of course at right angles to the other two.

The axis of Z or OZ parallel to the axes of the shadow will be reckoned *positive* towards the Moon; the axis of y or OY, the intersection of the plane of the declination circle through Z with the principal plane, and will be taken as *positive* towards the north, and the axis of x or OX the intersection of the plane of

the equator with the principal plane, will be taken positive toward that point X whose R. A. is $90° + a$.

Let x, y, z be the coördinates of the Moon, and let XM', YM' and ZM' be joined by arcs of great circles, then we shall have

$$\begin{aligned} x &= r \cos XOM' = r \cos XM' \\ y &= r \cos YOM' = r \cos YM' \\ z &= r \cos ZOM' = r \cos ZM' \end{aligned}$$

From the spherical triangles XPM', YPM', and ZPM' we have by the fundamental formula of spherical trigonometry

$$\begin{aligned} \cos XM' &= \cos PM' \cos PX + \sin PM' \sin PX \cos M'PX \\ &= \sin\delta \cos 90° + \cos\delta \cos (90° + a - \alpha) \\ &= \cos\delta \sin (\alpha - a) \\ \cos YM' &= \cos PM' \cos PY + \sin PM' \sin PY \cos M'PY \\ &= \sin\delta \cos d + \cos\delta \sin d \cos (180° + a - \alpha) \\ &= \sin\delta \cos d - \cos\delta \sin d \cos (\alpha - a) \\ \cos ZM' &= \cos PM' \cos PZ + \sin PM' \sin PZ \cos M'PZ \\ &= \sin\delta \sin d + \cos\delta \cos d \cos (\alpha - a) \end{aligned}$$

Substituting these values in the above we have

$$\begin{aligned} x &= r \cos\delta \sin (\alpha - a) \\ y &= r [\sin\delta \cos d - \cos\delta \sin d \cos (\alpha - a)] \\ z &= r [\sin\delta \sin d + \cos\delta \cos d \cos (\alpha - a)] \end{aligned} \qquad (57)$$

Substituting $1 - 2 \sin^2 \frac{1}{2} (\alpha - a)$ for $\cos (\alpha - a)$ and reducing we obtain

$$\begin{aligned} x &= r \cos\delta \sin (\alpha - a) \\ y &= r \sin (\delta - d) + x \sin d \tan \frac{1}{2} (\alpha - a) \\ z &= r \cos (\delta - d) - x \cos d \tan \frac{1}{2} (\alpha - a) \end{aligned} \qquad (58)$$

which are more easily computed, or we may adapt (57) to logarithmic computation thus,

put
$$\begin{aligned} c \sin C &= r \sin\delta \\ c \cos C &= r \cos\delta \cos (\alpha - a) \end{aligned}$$

then we shall have

$$\begin{aligned} x &= r \cos\delta \sin (\alpha - a) \\ y &= c \sin (C - d) \\ z &= c \cos (C - d) \end{aligned} \qquad (59)$$

To Find the Co-ordinates of the Place of Observation.

Let ξ, η and ζ be the coördinates of A, the place of observation

$\varphi = Ka$ the geographical latitude.
$\varphi' = KA'$ the reduced or geocentric latitude.
$\rho = OA$ the radius of the Earth for latitude φ.
and $\mu = VK$ the sidereal time.

then if we write ρ for r, φ' for δ and $\mu - a$ for $\alpha - a$ in (57) we shall evidently have the co-ordinates required, for the declination of A' is φ' and its longitude VK is μ, thus we have

$$\begin{aligned} \xi &= \rho \cos \varphi' \sin (\mu - a) \\ \eta &= \rho [\sin \varphi' \cos d - \cos \varphi' \sin d \cos (\mu - a)] \\ \zeta &= \rho [\sin \varphi' \sin d + \cos \varphi' \cos d \cos (\mu - a)] \end{aligned} \qquad (60)$$

which may be easily put in the form (58) or (59).

The angle $\mu - a$ or BPA' which is usually denoted by θ is the hour angle of the point Z for the meridian of the place of observation. If μ_1 be its value at Greenwich and λ the longitude of the place then we shall evidently have $\theta = (\mu - a) = \mu_1 - \lambda$.

To Find the Equation of Contact.

The fundamental plane always passes through the centre of the Earth and is perpendicular to the axis of the shadow; the plane through the place of observation and parallel to the fundamental plane is called the *parallel plane*. Now since the coördinates x and y of the Moon are also those of every point of the axis of the shadow, it is evident that with reference to the origin A, the coördinates of the axis of the shadow are $x - \xi$ and $y - \eta$, and if Δ denote the distance of A, the place of observation, from the axis of the shadow, we shall evidently have

$$\Delta^2 = (x - \xi)^2 + (y - \eta)^2 \qquad (61)$$

This equation can be put under another and a more useful form as follows:

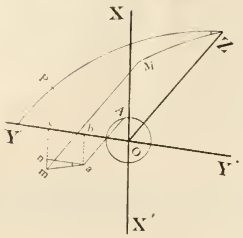
Let M be the Moon and A the place of observation, P the north pole and m and a the projections of M and A respectively on the fundamental plane, then the coördinates of M are

$$x = cm$$
$$y = Oc$$

and of A,

$$\xi = ab$$
$$\eta = Ob$$

Join m and a and draw an parallel to the axis of Y, then $\Delta = am$ which is the projection of the line (AM) joining the place of ob-



servation and the Moon's centre and the plane by which this line is projected contains the axis of the shadow and its intersection with the celestial sphere is a great circle passing through M and Z, therefore its projection am makes the same angle with OY, that MZ makes with the meridian PZ. Put the angle $PZM = nam = Q$ then in the right angled plane triangle amn we have

$$mn = mc - ab = x - \xi$$
$$an = Oc - Ob = y - \eta$$

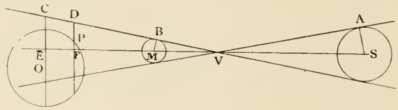
and

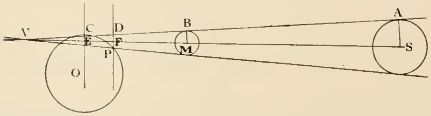
$$\Delta \sin Q = x - \xi$$
$$\Delta \cos Q = y - \eta \qquad (62)$$

The sum of the squares of which is identical with the preceding equation. The significance of the angle Q will be explained in a subsequent paper.

To Find the Radii of the Penumbral and Umbral Cones on the Fundamental and Parallel Planes.

Let CE be the radius of the shadow on the fnndamental and DF the radius on the parallel plane. V the vertex, M and S the centres of the Moon and Sun, and P the position of an observer on the Earth's surface.





Put h = apparent semi-diameter of the Sun at *mean* distance.
k = the ratio of the Moon's radius to the Earth's equatorial radius.
c = distance of the vertex V of the cone from the fundamental plane $= VE$.
ζ = distance between the planes $= EF$.
f = the angle of the cone $= CVE$.
l = radius of shadow on the fundamental plane $= CE$.
and L = radius of the shadow on the parallel plane $= DF$.

Taking the Sun's mean distance as unity, then at *this distance* we have

$$\begin{aligned} &\text{Earth's radius} = \sin \pi_0 \\ &\text{Moon's radius} = k \sin \pi_0 = BM \\ \text{and}\quad &\text{Sun's radius} = \sin h = AS \end{aligned}$$

Now in either figure we have

$$\begin{aligned} & MV \sin f = BM = k \sin \pi_0 \\ & SV \sin f = AS = \sin h \\ \text{then}\quad & SV + MV = G = r'g, \text{ in the first figure} \\ \text{and}\quad & SV - MV = G = r'g, \text{ in the second figure} \end{aligned}$$

therefore we have

$$r'g \sin f = \sin h \pm k \sin \pi_0$$

or

$$\sin f = \frac{\sin h \pm k \sin \pi_0}{r'g} \tag{63}$$

where the plus sign corresponds to the penumbral and the minus sign to the umbral cone. The numerator of the second number of this equation is constant and may be computed once for all. Thus, according to Bessel, $h = 959''.788$ and from recent investigations of the solar parallax, $\pi_0 = 8''.81$* and $k = 0.272274$ therefore we have

$$\log [\sin h + k \sin \pi_0] = 7.6688325 \text{ for penumbral contact.}$$
$$\log [\sin h - k \sin \pi_0] = 7.6666628 \text{ for umbral contact.}$$

We also have $VM = k \operatorname{cosec} f$ and $EM = z$

therefore

$$c = z \pm k \operatorname{cosec} f \tag{64}$$

in which the upper sign applies to the penumbra and the lower to the umbra, then we have

$$l = c \tan f = z \tan f \pm k \sec f$$

and

$$L = (c - \zeta) \tan f = l - \zeta \tan f \tag{65}$$

For the penumbral cone L is always positive since $c > \zeta$, but for the umbral cone $c - \zeta$ is negative when the vertex V falls below the plane of the observer, therefore the criterion of a *total* eclipse is that L or $(c - \zeta) \tan f$ be a negative quantity. If, however,

* In the *American Ephemeris* two values of the Sun's mean equatorial horizontal parallax are used or in other words the solar system is regarded as elastic. This is decidedly wrong and not justified by any facts or theories in Astronomy. Until quite recently Encke's old value, viz, $8''.5776$, a value quite too small, was used in computing eclipses while a value of $8''.848$ was used in other portions. In the volumes of 1897 and 1898 two values are still employed, viz, $8''.800$ for eclipses and $8''.848$ elsewhere—an inconsistency not found in any other similar work but one which might be expected when we consider how the *American Ephemeris and Nautical Almanac* is prepared. A slight change in the Sun's parallax affects the duration and limits of visibility of both a partial and total or annular eclipse, but does not affect the line or path of central eclipse.

the vertex V in the case of the umbral cone falls *above* the plane of the observer, L will be positive which is the criterion of an *annular* eclipse.

For the sake of brevity, it is usual to put

$$\begin{aligned} i &= \tan f \\ l &= ic \\ L &= l - i\zeta \end{aligned} \qquad (66)$$

The criterion of the beginning or ending of an eclipse for a *given place* is

$$\Delta = L$$

or by (61) and (66)

$$(x - \xi)^2 + (y - \eta)^2 = (l - i\zeta)^2 \qquad (67)$$

which, however, is more conveniently expressed by the following two equations in accordance with (62)

$$\begin{aligned} (l - i\zeta) \sin Q &= x - \xi \\ (l - i\zeta) \cos Q &= y - \eta \end{aligned} \qquad (68)$$

Equation (67) or its equivalent (68) is the fundamental equation in the theory of eclipses and its complete discussion furnishes all the circumstances relating to the duration and limits of visibility of these phenomena. The quantities x, y, l, $\tan f$, $\sin d$, $\cos d$ and μ, or $(\alpha - a)$ are independent of the place of observation and can be computed for any dates at the prime meridian. They are computed for several consecutive hours preceding and following conjunction, then interpolated to every ten minutes and tabulated under the head of Besselian Elements. The logarithms of the variations of x and y per minute are also given. These elements furnish the data for the solution of (68) which will be presented in subsequent papers.

(TO BE CONTINUED).

Problems.

22. Find the least possible inclination to the horizon of the line joining the cusps of the Moon.

23. Can a star be found whose real position is unaffected by parallax, refraction and aberration?

24. Given the synodic period of the Moon S, and the length of the sidereal year T_s, find the length of the sidereal period of the Moon.

25. The Moon was observed to rise at the same sidereal time on two successive nights and the inclination of its orbit to the equator was θ, show that sin lat. of the place $= \cos \theta$.

26. To what extent does the diurnal rotation of the Earth affect the duration of a total eclipse of the Sun, at a place on the Earth's equator?

27. If f and f' denote the semi-vertical angles of the umbral and penumbral cones respectively and d the Sun's apparent semi-diameter show that $2 \tan d = \tan f + \tan f'$.

EVENINGS WITH THE STARS.

MARY PROCTOR.

Many delightful evenings, may be spent in observing the starry heavens, if only the observer goes properly to work. Let us suppose that he is provided with a good star-atlas, (I would recommend "Half-Hours with the Stars," published by Putnam's Sons, New York), and an opera glass, and that he is about to commence his astronomical studies. Let us first obtain a general outline of the constellations for each month, and then take three, for instance, for special observation. This month the three constellations will be Ursa Major, Ursa Minor and Taurus.

General Outline.

Rising above the northeast horizon is (*Ursa Major*), the Great Bear; due north is (*Ursa Minor*), the Little Bear, with the stars of (*Draco*), the Dragon winding below the Little Bear towards the west. Above is King Cepheus, and above him (*Cassiopeia*) his queen, their daughter, the Chained Lady, (*Andromeda*) being nearly overhead. Towards the southwest, is (*Aquarius*), the Water Bearer, close by the Winged Horse (*Pegasus*), south of which is (*Pisces*), the Fishes. In the south, is (*Cetus*), the Whale, Phœnix and Eridanus. Almost overhead, (*Aries*) the Ram, whilst towards the east, are (*Gemini*), the Twins, (*Taurus*), the Bull, Orion, (*Lepus*), the Hare, and (*Canis Major*) the Great Dog, and (*Columba*), the Dove, which are rising in the southeast.

Ursa Major.

The first step in studying the stars is to become acquainted with the renowned group of the Great Dipper, as it is called, in Ursa Major. There are many features of interest in this group of stars, and the beginner should first learn to identify the "seven" stars, and then the "two" stars, generally called the "pointers" because they point to the Pole star. First, note the three stars in the handle of the dipper, curving into two of the stars forming the back of the dipper. Now the remaining two stars, forming the front of the dipper, are the "pointers." Continue the line from these stars upward from the dipper, and you come to the Pole star, the first bright star on the way, so you cannot fail to place it correctly. After observing the Dipper with the unaided eye, turn an opera-glass towards it, and ten times as many stars will be seen. If you are the fortunate owner of a telescope, hundreds of stars will be revealed in this small group, and through such an instrument as the one at the Lick Observatory, the number would have to be reckoned in thousands.

Professor Young tells us, in his *Lessons on Astronomy*, that the dimensions of the Dipper furnish a convenient scale of angular measure. From Alpha to

Beta (the "Pointers,") is 5°; from Alpha to Eta, (the star at the extremity of the Dipper Handle,) is 26°. The star Zeta (or Mizar,) at the bend of the handle, is easily recognized by the little star *Alcor* near it. Mizar is a beautiful double star, composed of two stars, shining with a white, and a pale green light. These stars are of the second and fourth magnitude, and they can be easily observed with a small telescope, and even with a very powerful field glass. The small star Alcor can

The Constellations at 9 p. m., Dec. 1, 1896.

be readily seen with the unaided eye, but must not be mistaken for the component of Mizar, for in the magnifying power of a telescope, Alcor is seen removed along way from this star. In *Astronomy with an Opera Glass*, Garrett P. Serviss, recommends the observer, to sweep along the whole length of the Dipper's handle

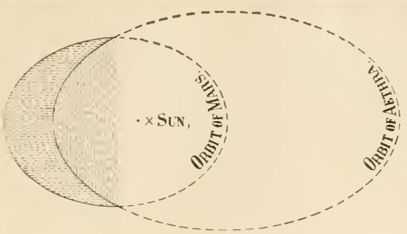


Fig. II.

withstanding the great difference between the angle formed by the intersection of the orbit of the asteroid and the plane of the ecliptic, and that by the intersection of the orbit of Mars and the plane of the ecliptic, there will come a time when the planet and the asteroid will get very near each other. Then the attraction of Mars, at such close range, will be sufficient to change forever the direction of motion, and the plane of the orbit of the asteroid. What will then be the fate of Aethra? Under the conditions named, the asteroid will be transformed into a moon revolving about Mars as its primary, and so constituting one of the family of the planet.

How many asteroids there are, or what the width of the asteroidal belt is, it may be considered probable that there are hundreds and perhaps thousands of them, so small as to be invisible even with the best telescopes, many of which move in orbits quite adjacent to the orbits of both Jupiter and Mars. Not only may some of them become satellites of the two planets, but there may be satellites of the two invisible as yet, that were once asteroids pursuing their way around the Sun influenced by the perturbing influences of either Jupiter or Mars. The asteroidal belt may, for all we know, extend all the way from the orbit of Mars to that of Juriter and the liability that some of them should be captured by their giant neighbors becomes almost, if not altogether, a certainty.

Then if all of the foregoing be true, it may be affirmed with a reasonable degree of probability that the two moons of Mars, Deimos and Phobos, at one time in the far distant past, were members of the asteroidal group. Their size also seems to indicate their origin, one of which, the larger being not more than 16 miles in diameter, and possibly only seven miles. The time of revolution of Phobos the inner moon is $7^h\ 39^m$. That is to say it revolves about Mars a little more than three times every 24 hours, and presents all the different phases of new moon, first quarter, full moon and last quarter, at each revolution. Young says that " this rapidity of revolution raises important questions as to the theory of the development of the solar system, and re-

quires modification of the views which had been held up to the time of their discovery. If the nebular hypothesis is true, a shortening of the satellite's period or a lengthening of the planet's day must have occurred since the satellites came into being." Now a shortening of the satellite's period of revolution, indicates that its orbit has been contracted to smaller dimensions, and this shortening may have been going on for a long time, so that at the time the shortening began the satellite was far enough away to be in the zone of asteriods, in fact, an asteroid itself.

The conclusion, therefore, which was referred to in the beginning of this discussion, is, that owing to the proximity of the asteroidal belt; the intersection of the orbits of Mars and Aethra; the small size of the asteroids and the moons of Mars, the red planet captured its moons from the asteroidal group. And if this be so, then Jupiter some time in the past has done a like thing.

The future, supplied with more powerful telescopes and appliances may reveal to our gaze more moons still revolving about the two planets, of which some were captured, and since that time have been playing the role of satellites.

The University of Kansas.

SOLAR ECLIPSES. (*Continued.*)

J. MORRISON, M. A., M. D., Ph. D.

To Find the Outline of the Penumbra or Umbra on the Surface of the Earth.

The general solution of the fundamental equation (67) or its equivalent (68) determines the position of the penumbra and umbra on the Earth's surface at any time during which an eclipse can occur.

In this discussion we shall use the notation and methods of Bessel as adopted by Chauvenet, but will enter into more details and give some geometrical illustrations which will doubtless aid the student in studying this important and interesting subject.

Resuming equations (60) and (68) writing θ for $(\mu_1 - a)$ in the former, we have

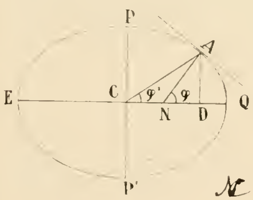
$$\begin{aligned} \xi &= \rho \cos \varphi' \sin \theta \\ \eta &= \rho\, [\sin \varphi' \cos d - \cos \varphi' \sin d \cos \theta) \\ \zeta &= \rho\, [\sin \varphi' \sin d + \cos \varphi' \cos d \cos \theta] \qquad (69) \end{aligned}$$

and

$$\begin{aligned} (l - i\zeta) \sin Q &= x - \xi \\ (l - i\zeta) \cos Q &= y - \eta \end{aligned}$$

These five equations contain six unknown quantities, viz., ξ, η, ζ, φ' θ and Q, the first three being the coördinates of the place at which an *apparent* contact of the limbs of the Sun and Moon may be observed; the fourth, the geocentric latitude of that place; the fifth, the hour-angle of the point Z, and the sixth is the angle PZM or the position-angle of the Moon's centre at this point. The arc ZM' passes through the Sun's centre as well as through the point of contact. At the time of an eclipse Zs never exceeds $5''$, the angle PZM may therefore be taken equal to *Psm*, that is the angle Q is the position angle or the angular distance of the point of contact from the declination circle passing through the apparent centre of the Sun. Any one of these six unknown quantities may be arbitrarily assumed, provided we do not assume such values as would give an impossible result. It will be found most convenient to take Q as the assumed arbitrary quantity and then deduce the other five.

In these equations there is however another unknown quantity, ρ, which depends on φ', and cannot be determined until the latter is found. We might for a first approximation neglect the spheroidal figure of the Earth and put $\zeta = 1$ and after finding an approximate value of φ', compute a more accurate value of ρ and then repeat the computation of φ'. This double computation can be obviated and the ellipticity of the Earth taken into account from the very beginning by the following elegant transformation due to Bessel.



Let $PEP'Q$ be a section of the terrestrial spheroid made by a meridian, PP' the poles and A a point on its surface; draw the normal AN and denote the geographical latitude ANQ by φ. Then if x and y be the coördinates of A, we have the subnormal

$$AD = \frac{b^2}{a^2} x = (1 - e^2)\, x, \text{ when } a = 1,$$

and $$x \tan \varphi' = (1 - e^2)\, x \tan \varphi$$

or $$\tan \varphi' = (1 - e^2) \tan \varphi \qquad (70)$$

the relation between the geocentric and geographical latitudes.

The equation of the ellipse is

$$x^2 + \frac{y^2}{1 - e^2} = 1$$

and $$\frac{y}{x} = \tan \varphi'$$

$$= (1 - e^2) \tan \varphi$$

whence we get

$$x = \frac{\cos\varphi}{\sqrt{(1 - e^2\sin^2\varphi)}} = \rho\cos\varphi'$$

and
$$y = \frac{(1 - e^2)\sin\varphi}{\sqrt{(1 - e^2\sin^2\varphi)}} = \rho\sin\varphi' \qquad (71)$$

If we now put

$$\cos\varphi_1 = \rho\cos\varphi'$$
$$= \frac{\cos\varphi}{\sqrt{(1 - e^2\sin^2\varphi)}}$$

then
$$\sin\varphi_1 = \sqrt{(1 - \cos^2\varphi_1)}$$
$$= \frac{\sqrt{1 - e^2}\,.\,\sin\varphi}{\sqrt{(1 - e^2\sin^2\varphi)}}$$

or
$$\sqrt{(1 - e^2)}\sin\varphi_1 = \frac{(1 - e^2)\sin\varphi}{\sqrt{(1 - e^2\sin^2\varphi)}} = \rho\sin\varphi'$$

whence we get

$$\sqrt{1 - e^2}\tan\varphi_1 = \tan\varphi'$$
$$= (1 - e^2)\tan\varphi \qquad \text{by (70)}$$

and
$$\tan\varphi = \frac{\tan\varphi_1}{\sqrt{1 - e^2}} \qquad (72)$$

which gives the relation between the new variable φ_1 and the geographical latitude φ.

The Clark spheroid of 1866, adopted by the U. S. Coast and Geodetic Survey, gives $\log e = 8.9152515$, and therefore

$$\log\sqrt{(1 - e^2)} = 9.9985251.$$

The first three equations of group (69) now become

$$\begin{aligned} \xi &= \cos\varphi_1\sin\theta \\ \eta &= \sin\varphi_1\cos d\sqrt{1 - e^2} - \cos\varphi_1\sin d\cos\theta \\ \zeta &= \sin\varphi_1\sin d\sqrt{1 - e^2} + \cos\varphi_1\cos d\cos\theta \end{aligned} \qquad (73)$$

which can be simplified by putting

$$\rho_1\sin d_1 = \sin d \qquad \text{and } \rho_2\sin d_2 = \sqrt{1 - e^2}\sin d$$
$$\rho_1\cos d_1 = \sqrt{1 - e^2}\,.\cos d \qquad \rho_2\cos d_2 = \cos d$$

then (73) becomes

$$\begin{aligned} \xi &= \cos\varphi_1\sin\theta \\ \eta &= \rho_1\sin\varphi_1\cos d_1 - \rho_1\cos\varphi_1\sin d_1\cos\theta \\ \zeta &= \rho_2\sin\varphi_1\sin d_2 + \rho_2\cos\varphi_1\cos d_2\cos\theta \end{aligned} \qquad (74)$$

The quantities $\rho_1\,\rho_2$, d_1 and d_2 must be computed and tabulated

for the same dates as the Besselian Elements, and can then be easily interpolated for all intermediate dates.

Again, let us put $\eta_1 = \dfrac{\eta}{\rho_1}$ and assume ζ_1 such that $\xi^2 + \eta_1^2 + \zeta_1^2 = 1$, then the last group becomes

$$
\begin{aligned}
\xi &= \cos\varphi_1 \sin\theta \\
\eta_1 &= \sin\varphi_1 \cos d_1 - \cos\varphi_1 \sin d_1 \cos\theta \\
\zeta_1 &= \sin\varphi_1 \sin d_1 + \cos\varphi_1 \cos d_1 \cos\theta
\end{aligned} \tag{75}
$$

The relation between ζ and ζ_1 will be required and is found as follows: multiply the second of the last group by $\sin d_1$ and the third by $\cos d_1$ and subtract the first product from the second, and again multiply the second by $\cos d_1$ and the third by $\sin d_1$ and add the results, we shall have

$$
\begin{aligned}
\cos\varphi_1 \cos\theta &= -\eta_1 \sin d_1 + \zeta_1 \cos d_1 \\
\sin\varphi_1 &= \eta_1 \cos d_1 + \zeta_1 \sin d_1
\end{aligned} \tag{76}
$$

Substituting these in the third of (74) gives after some simple reductions

$$\zeta = \rho_2 \zeta_1 \cos(d_1 - d_2) - \rho_2 \eta_1 \sin(d_1 - d_2) \tag{77}$$

from which we see that ζ and ζ_1 are very nearly equal, and when very great accuracy is not desired we may use the one for the other. As the result of these transformations group (69) now takes the following form

$$
\begin{aligned}
(l - i\zeta_1)\sin Q &= x - \xi \\
(l - i\zeta_1)\cos Q &= y - \eta = y - \rho_1 \eta_1 \\
\xi^2 + \eta_1^2 + \zeta_1^2 &= 1
\end{aligned} \tag{78}
$$

which determine ξ, η_1 and ζ_1 for each assumed value of Q, and then (75 and 76)

$$
\begin{aligned}
\cos\varphi_1 \sin\theta &= \xi \\
\cos\varphi_1 \cos\theta &= -\eta_1 \sin d_1 + \zeta_1 \cos d_1 \\
\sin\varphi_1 &= \eta_1 \cos d_1 + \zeta_1 \sin d_1
\end{aligned} \tag{79}
$$

determine φ_1 and θ and thence the latitude and longitude by

$$\tan\varphi = \frac{\tan\varphi_1}{\sqrt{1 - e^2}}$$

and $\qquad \lambda = \mu_1 - \theta.$ (See page 317).

In (78) we have put ζ_1 for ζ in the small term $i\zeta$ which can be done here without any appreciable error, but if a more accurate solution is required, we may proceed as follows. In the first two equations of (78) put

$$x - l\sin Q = \sin\beta \sin\gamma$$

and

$$\frac{y}{\rho_1} - \frac{l\cos Q}{\rho_1} = \sin\beta\cos\gamma \tag{80}$$

then they become

$$\xi = \sin\beta \sin\gamma + i\zeta_1 \sin Q$$
$$\eta_1 = \sin\beta\cos\gamma + i\zeta_1 \cos Q \qquad (81)$$

omitting the diviser ρ_1 in the small term $i\zeta_1$ cos Q, because ρ_1 is always nearly equal to unity. Substituting these in the third equation of (78) we find after neglecting terms involving i^2 which are quite inappreciable

$$\zeta_1^2 = \cos^2\beta - 2i\zeta_1 \sin\beta\cos(Q-\gamma)$$

and neglecting the last term of the second member, we have for an approximate value of $\zeta = \cos\beta$, and substituting this in the last term we have

$$\zeta_1^2 = \cos^2\beta - 2i\sin\beta\cos\beta\cos(Q-\gamma)$$

and $\qquad \zeta_1 = \cos\beta - i\sin\beta\cos(Q-\gamma)$, approximately.

It may be observed that the second member ought to have the double sign, but if the eclipse is *visible* ζ_1 must be positive, that is it must lie above the fundamental plane. Substituting these last values of η_1 and ζ_1 in (77) we have, after omitting terms involving i^2 and i sin $(d_1 - d_2)$ as practically insensible,

$$\zeta = \rho_2 \left\{\cos\beta - i\sin\beta\cos(Q-\gamma)\cos(d_1-d_2) - \sin\beta\cos\gamma\sin(d_1-d_2)\right\}$$
$$= \rho_2 \left\{\cos\beta - \sin\beta\Big(i\cos(Q-\gamma)\cos(d_1-d_2) + \cos\gamma\sin(d_1-d_2)\Big)\right\} \qquad (82)$$

Since i and sin $(d_1 - d_2)$ are always very small the last two terms of the second member of (82) are also small and may be computed as follows

put $\qquad i\cos(Q-\gamma) = k\cos K$

and $\qquad \cos\gamma = k\sin K$

then we shall have

$$\zeta = \rho_2\big(\cos\beta - k\sin\beta\cos(K + d_1 - d_2)\big) \qquad (83)$$

When this value of ζ is substituted for ζ_1 in the small term $i\zeta$ in (81), the error committed will be of the order i^3 and the solution will be practically exact.

All the quantities in the second member of (82) are known, Q being of course assumed and β and γ found from (80).

Therefore we have finally from (78) or (81) with all the exactness warranted by the original data

$$\xi = \sin\beta\sin\gamma + i\rho_2\big(\cos\beta - k\sin\beta\cos(K + d_1 - d_2)\big)\sin Q$$
$$\eta_1 = \sin\beta\cos\gamma + \frac{i\rho_2}{\rho_1}\Big(\cos\beta - k\sin\beta\cos(K + d_1 - d^2)\Big)\cos Q \qquad (84)$$
$$\zeta_1 = 1 - \xi^2 - \eta_1^2$$

And if we put $\xi = \sin \beta' \sin \gamma'$
and $\eta_1 = \sin \beta' \cos \gamma'$ (85)
we shall then have $\zeta_1 = \cos \beta'$

Substituting these values of ξ, η_1 and ζ_1 in (79) we have

$$\begin{aligned} \cos \varphi_1 \sin \theta &= \sin \beta' \sin \gamma' \\ \cos \varphi_1 \cos \theta &= -\sin \beta' \cos \gamma' \sin d_1 + \cos \beta' \cos d_1 \\ \sin \varphi_1 &= \sin \beta' \cos \gamma' \cos d_1 + \cos \beta' \sin d_1 \end{aligned}$$

which may be easily adapted to logarithmic computation thus: compute c and C from

$$\begin{aligned} c \sin C &= \sin \beta' \cos \gamma' = \eta_1 \\ c \cos C &= \cos \beta' \qquad\quad = \zeta_1 \end{aligned}$$

then the above becomes

$$\begin{aligned} \cos \varphi_1 \sin \theta &= \xi \\ \cos \varphi_1 \cos \theta &= c \cos (C + d_1) \\ \sin \varphi_1 &= c \sin (C + d_1) \end{aligned} \tag{86}$$

then we have

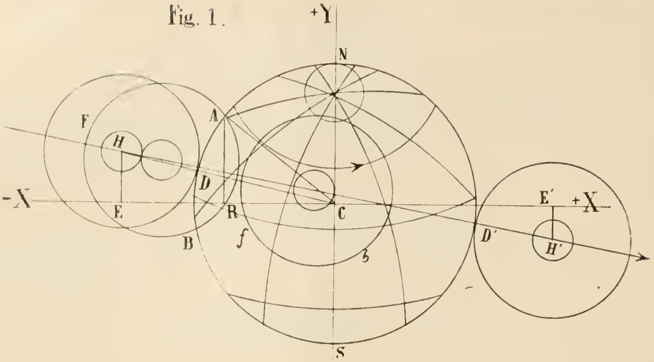
$$\tan \varphi = \frac{\tan \varphi_1}{\sqrt{1 - e^2}} \text{ and } \lambda = \mu_1 - \theta.$$

Before deducing criteria for determining whether the eclipse is beginning or ending at the places found by the preceding formulæ, we shall first determine the times and places at which the eclipse begins and ends on the Earth generally. These dates are the limiting times between which all the circumstances of an eclipse can occur.

If an observer were situated somewhere in the direction of the axis of Z and with his face towards the Earth, he would see the shadow of the Moon traversing the Earth's disk from left to right or from west to east, the Earth also turning in the same direction as indicated by the arrow heads in Figs. 1, 2 and 3.* The Sun will be rising at all points on the left or west side, that is at all points on NDS and setting on the east side or on $ND'S$. Let ABF be a section of the penumbra and the small concentric circle a section of the umbra on the fundamental plane. The first and last

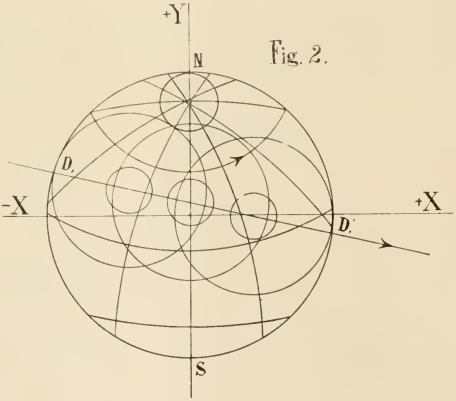
* In Schellen's Spectrum Analysis translated from the German and published in England under the supervision of one of the most distinguished English astronomers and scientists, a page or more is taken up proving that the shadow of the Moon during an eclipse of the Sun moves across the Earth's surface from east to west!! The reasoning employed was somewhat as follows: The Earth makes a revolution on its axis in a day and the Moon makes a revolution around the Earth in about 27 days therefore a spectator on the Earth's surface must move at least 27 times faster than the Moon's shadow and of course would enter the shadow on the west side, pass through it and emerge from it on the east side or regarding for the moment the Earth as fixed, the shadow would appear to move from east to west. The writer first directed the attention of the English reviser to the error which has been corrected or rather removed altogether from subsequent editions. The solution of Problem 1 in this paper shows that the time required for the shadow to cross the Earth's disk can never exceed 7.4 hours.

Fig. 1.



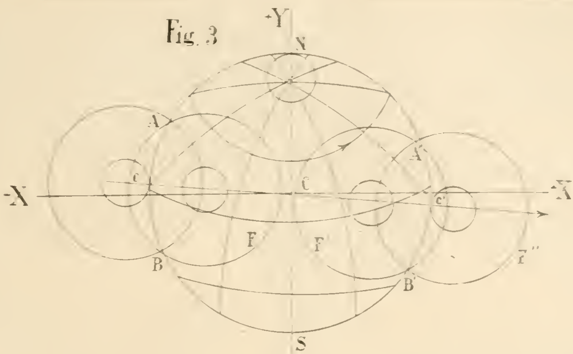
contacts are at the points D and D' respectively, Fig. 1. And we are now to find the times when these contacts take place as well as the latitude and longitude of these points. C being the centre of the Earth and the origin of coördinates CD and CD' are the radii of the Earth for these points; HD, $H'D'$ are the radii of the penumbra and CE, EH and CE', $E'H'$ the coördinates of the centre of the shadow, that is for the first contact $CE = -x$ and $EH = +y$, and for the last $CE' = +x$ and $E'H' = -y$ according

Fig. 2.



to our diagram. When the shadow has advanced a little on the Earth's disk there will be two points A and B in the horizon at which place the eclipse is beginning at sunrise. As the

Fig. 3



shadow advances, the points A and B will separate until their distance apart is equal to the diameter of the shadow after which they will approach each other as in the outline ABF in Fig. 3 and the eclipse is ending at sunrise. Since the first and last contacts and the points A and B where the eclipse is beginning or ending at sunrise or at sunset, are in the horizon and on the fundamental plane, we must have $\zeta_1 = 0$ and therefore (78) becomes

$$\begin{aligned} l \sin Q &= x - \xi \\ l \cos Q &= y - \eta = y - \rho_1 \eta_1 \\ \xi^2 + \eta_1^2 &= 1 \end{aligned} \qquad (87)$$

as the conditions that must be satisfied for these places

Put
$$\begin{aligned} m \sin M &= x \qquad & p \sin \gamma &= \xi \\ m \cos M &= y \qquad & p \cos \gamma &= \eta \end{aligned} \qquad (88)$$

where we see from the diagram that $m = HC$, $\angle M = \angle EHC$, $p = AC$, $\gamma = RAC$, $\xi = CR$ and $\eta = AR$, then (87) becomes

$$\begin{aligned} l \sin Q &= m \sin M - p \sin \gamma \\ l \cos Q &= m \cos M - p \cos \gamma \end{aligned}$$

Squaring and adding we find

$$l^2 = m^2 - 2mp \cos (M - \gamma) + p^2$$

whence
$$\sin^2 \frac{1}{2} (M - \gamma) = \frac{l^2 - (m - p)^2}{4mp}.$$

Let
$$\chi = M - \gamma,$$
then this may be written

$$\sin \frac{1}{2} \chi = \pm \sqrt{\frac{(l + m - p)\ (l - m + p)}{4mp}} \qquad (89)$$

and
$$\gamma = M \pm \chi.$$

Also χ must always be taken less than 180°. The two values of γ are necessary to determine the two points A and B which satisfy the conditions (87). Formula (89) contains one unknown quantity viz., p for m, M and l are accurately known. It is easily seen that p is always very nearly equal to unity, hence if we make $p = 1$ in (89) we shall first get an approximate value of γ, and a more correct value can be found thus:

put $\xi = \sin \gamma'$, then by (87) $\eta_1 = \cos \gamma'$ and $\eta = \rho_1 \eta_1$

we have (88)

$$p \sin \gamma = \sin \gamma'$$
$$p \cos \gamma = \rho_1 \cos \gamma'$$

whence

$$\tan \gamma' = \rho_1 \tan \gamma$$

and

$$p = \frac{\sin \gamma'}{\sin \gamma} = \frac{\rho_1 \cos \gamma'}{\cos \gamma} \tag{90}$$

And with this value of p in (89) a more accurate value of γ is found and we might repeat the computation with this second value of γ, and a still more accurate value of p may be found, but it will never be necessary to proceed beyond the *second* approximation. We remark here however, that in the *second* approximation we must use the *two* values of γ given by (89), that is, we must compute p for *each* of the *two places* (A and B) separately.

When the second value of γ is found the final value of γ' is computed from

$$\tan \gamma' = \rho_1 \tan \gamma \tag{91}$$

and then making $\xi = \sin \gamma'$, $\eta_1 = \cos \gamma'$ and $\zeta_1 = 0$ in (79) we have

$$\begin{aligned} \cos \varphi_1 \sin \theta &= \sin \gamma' \\ \cos \varphi_1 \cos \theta &= - \cos \gamma' \sin d_1 \\ \sin \varphi_1 \quad &= \quad \cos \gamma' \cos d_1 \end{aligned} \tag{92}$$

and

$$\tan \varphi = \frac{\tan \varphi_1}{\sqrt{1 - e^2}} \text{ and } \lambda = \mu - \theta$$

which determine the latitude and longitude of the two points A and B when the eclipse begins or ends at sunrise or sunset.

When the eclipse begins or ends on the Earth generally the cone of the shadow has but a single point in common with the Earth's surface as shown at D and D' Fig. 1. The two solutions of (89) reduce to a single one, and this can happen only when $\chi = 0$ and therefore we must have

$$l + m - p = 0 \qquad \text{and } l - m + p = 0$$

or

$$m = p + l \qquad \text{and} \qquad m = p - l$$

There may be four cases of contact, viz., two exterior as shown

at D and D' Fig. 1 and two interior as shown at D_1 and D'_1 in Fig. 2. In the case of the exterior contacts the axis of the shadow is *without* the Earth and we have $m = \sqrt{(x^2+y^2)} = p + l$. The first interior contact occurs at the last point on the Earth's surface where the eclipse ends at sunrise D_1, and the second at the first point D'_1 where it begins at sunset.

It is evident however that the *interior* contacts can only occur when the *whole* shadow on the fundamental plane, falls *within* the Earth's disk and then we must have $m = p - l$. Therefore for the beginning and ending on the Earth generally we always have

$$(p + l) \sin M = x$$
$$(p + l) \cos M = y$$

We must now find a time T, when these conditions are satisfied

Put $$T = T_0 + \tau$$

where T_0 is the epoch of the eclipse tables which may be the time of conjunction or a more convenient one will be the nearest whole hour to conjunction, and for which the values of x and y are x_0 and y_0.

Now if x' and y' be the hourly* variations of x and y for the term T, we will have

$$x = x_0 + \tau x' \qquad \text{and} \qquad y = y_0 + \tau y'$$

and
$$(p + l) \sin M = x_0 + \tau x'$$
$$(p + l) \cos M = y_0 + \tau y'$$

Put
$$m_0 \sin M_0 = x_0 \qquad \text{and} \qquad n \sin N = x'$$
$$m_0 \cos M_0 = y_0 \qquad\qquad n \cos N = y'$$

then we shall have

$$(p + l) \sin M = m_0 \sin M_0 + \tau \,.\, n \sin N$$
$$(p + l) \cos M = m_0 \cos M_0 + \tau \,.\, n \cos N$$

Multiply the first by cos N and the second by sin N and subtract the second result from the first, and again multiply the first sin N and the second by cos N and add the results, and we get

$$(p + l) \sin (M - N) = m_0 \sin (M_0 - N)$$
$$(p + l) \cos (M - N) = m_0 \cos (M_0 - N) + n\tau$$

and if we now put $M - N = \psi$ we have

$$\sin \psi = \frac{m_0 \sin (M_0 - N)}{p + l}$$

$$\tau = \frac{p + l}{n} \cos \psi - \frac{m_0}{n} \cos (M_0 - N) \qquad (93)$$

$$T = T_0 + \tau$$

* It would be more accurate to let x' and y' denote the changes in x and y during 10 minutes instead of an hour.

The first of these gives two values for ψ, and $\cos\psi$ must be taken first with the *negative* and second with the *positive* sign; the first will evidently give an approximate time for the beginning and the second for the end of the eclipse generally.

For the interior contacts, *when they occur*, we have

$$\sin\psi = \frac{m_0 \sin (M_0 - N)}{p - l}$$

$$\tau = \frac{p - l}{n} \cos\psi - \frac{m_0}{n} \cos (M_0 - N) \tag{94}$$

$$T = T_0 + \tau$$

When $p - l$ is less than $m_0 \sin (M_0 - N)$, $\sin\psi$ has an impossible value and the interior contacts cannot occur.

In the computation of (93) and (94) we first assume $p = 1$ and find an approximate value of ψ, then since in this case $\chi = 0$ and $\gamma = M$ by (89) we have

$$\gamma - N = \psi \text{ or } \gamma = N + \psi \tag{95}$$

with which we find p by (90) and the second computation of (93) and (94) will give the exact times of beginning and ending on the Earth generally. In (95) we must use the two values of ψ found by taking $\cos\psi$ with both the positive and negative signs and therefore we shall find different values of p for the beginning and ending. The second approximation will require separate computations for the two cases.

When the first approximate value of τ is found we can then take the values of x, y, x' y' and l from the eclipse tables for the times $T = T_0 \pm \tau$, for the second approximation. Finally when the correct value of ψ has been found and also the true value of $\gamma = N + \psi$ for each point and also the true values of γ' from (91) the latitude and longitude of the first and last points of contact can be computed from (92).

We will close this paper by computing the beginning and ending on the Earth generally of the eclipse of July 29, 1897 using the elements of the eclipse as given an page 415 of the *American Ephemeris*. We take $T_0 = 4$ hours which is near the middle of the eclipse, and for this date we have

$$x_0 = +\,0.00174 \qquad y_0 = -\,0.06959 \qquad l = 0.55338$$
$$x' = +\,0.08315 \qquad y' = -\,0.03066$$

the variations of x and y in 10 minutes. For brevity we will only put down the results which may be verified by the student. With the above data we easily find

$M_0 = 178° \ 34' \ 4''$ $\quad N = 110' \ 24' \ 26''$
log $m_0 = 8.04269$ $\quad$ log $n = 8.94754$ $\quad M_0 - N = 68° \ 19' \ 38''$

For the first approximation we take $p = 1$, then

$$p + l = 1.55338 \text{ and } \psi = 177° \ 36' \ 47'' \text{ or } 2° \ 23' \ 13''$$

$$\frac{1 + l}{n} \cos \psi = \mp 17.512$$

the unit of time being 10 minutes.

$$-\frac{m_0}{n} \cos (M_0 - N) = - \ 0.290$$

therefore $\quad \tau = - 17.802 = - 178^m.02 = - 2^h \ 58^m.02$
and $\quad\quad\;\; = + 17.222 = + 172 \ .22 = + 2 \ \ 52 \ \ .22$

$$T = T_0 \pm \tau$$
$$= 1^h \ 1^m.98 \text{ beginning and } 6^h \ 52^m.22 \text{ end}$$

The Ephemeris gives

$1^h \ 2^m.0$ " " $6^h \ 52^m.1$ "

The first approximation is therefore accurate enough for all practical purposes.

We will now proceed to the second approximation and then determine the latitude and longitude.

		Beginning.	Ending.
T_0 = approximate time =		$1^h \ 0^m$	$6^h \ 50^m$
$\gamma = N + \psi$	(95)	287° 51′ 13″	112° 37′ 39″
log ρ_1	(73)	9.99868	9.99867
tan γ		0.49201n	0.38005n
tan γ'	(91)	0 49069n	0.37872n
γ'		287° 54′ 17″	112° 41′ 23″
sin γ'		9.97844n	9.96501
sin γ		9.97856n	9 96522
log p		9 99988	9.99979
p		0.99973	.99953

We also have (page 415 *American Ephemeris*) for the above dates

x_0	− 1.49510	+ 1.41511
y_0	+ 0.48130	− 0.59172
l	+ 0.55362	+ 0 55297
x'	+ 0.08315	+ 0.08311
y'	− 0.03055	− 0.03076
$p + l$	1.55335	1.55250
M_0	287° 50′ 40″	112° 41′ 30″
log m_0	0.19608	0 18578
N	110° 10′ 26″	110° 18′ 37″
log n	8.94735	8.94753
$M_0 - N$	177° 40′ 14″	2° 22′ 53″
ψ	177 38 44	2 21 10
$\frac{p+l}{n} \cos \psi$	− 17.525	+ 17.504
$-\frac{m_0}{n} \cos (M_0 - N)$	+ 17.726	− 17.293
τ	+ 0.201	+ 0.211
	$+ 2^m.01$	$+ 2^m.11$
T_0	$1^h \ 0$	$6^h \ 50^m$
T	$1^h \ 2^m.01$	$6^h \ 52^m.11$

which agree exactly with the Ephemeris. We now proceed to find the exact value of γ and thence the latitude and longitude by (92). We first find ρ_1 and sin d_1 and cos d_1 using the values of sin d and cos d for $1^h\ 2^m$ and $6^h\ 52^m.1$

	Beginning.	Ending.
$\log \rho_1$ (73)	9.99868	9.99868
$\sin d_1$	9.50580	9.50454
$\cos d_1$	9 97647	9.97661
$\gamma = N + \psi$, (95)	287° 49′ 9″	112° 39′ 47″
γ'	287 52 12	112 43 30
$\cos \varphi_1 \sin \theta' = \sin \gamma'$	9.97852n	9.96490
$-\cos \gamma'$	9.48694n	9.58694
$\sin d_1$	9 50580	9.50454
$\cos \varphi_1 \cos \theta$	8.99274n	9.09148
$\tan \theta$	0.98578	0.87342
θ	264° 6′ 02″	82° 27′ 37″
$\cos \theta$	9.01192n	9.12273
$\cos \varphi_1$	9.98082	9 96875
φ_1	16° 53′ 48″	21° 28′ 45″
$\cos d_1$	9.97647	9.97661
$\sin \varphi_1$	9.46341	9.56355n
φ_1	16° 53′ 50″	− 21° 28′ 35″
$\tan \varphi_1$	9.48254	9.59488n
$\sqrt{1-e^2}$	9.99853	9.99853
$\tan \varphi$	9.48401	9.59635n
φ	16° 57′.0	− 21° 32′ 5
μ_1	13° 55′.6	101 29 .0
θ	264 6 .0	82 22 .6
λ	109° 49′.6w	19° 6′.4w

Therefore the eclipse begins in latitude 16° 57′ N and long. 109° 49′.6 W and ends in latitude 21° 32′.5 S and longitude 19° 6′.4 W of Greenwich.

The agreement of the two values of φ_1 is a check on the computation; both values are necessary as the sign of φ_1 cannot be determined from its cosine.

TO BE CONTINUED.

Problems.

28. If π and π' denote the equatorial horizontal parallaxes of the Moon and Sun, δ and δ' their semi-diameters and h and h' their hourly motions, show that the duration T of a solar eclipse is

$$T = \frac{2(\pi - \pi' + \delta + \delta')}{h - h'}$$

29. If the angular semi-diameter of the Earth's shadow and of the Moon is seen from the Earth's centre be d and δ respectively and the apparent angular hourly motion of the Moon about the

Earth be h, find expressions for the entire duration of a lunar eclipse and the duration of totality, the Moon being in the node of her orbit when in opposition.

30. If the angle subtended at the Earth by the Sun and the stationary point of a planet's orbit be θ and E the greatest elongation of the planet show that

$$2 \cot \theta = \sec \frac{1}{2} E + \operatorname{cosec} \frac{1}{2} E.$$

31. The R. A. and Decl. of a star being α and δ respectively and $\delta' =$ the Decl. of the Sun when they rise or set together find φ the latitude of the place.

EVENINGS WITH THE STARS.

MARY PROCTOR.

FOR POPULAR ASTRONOMY.

During the month of March, the Great Bear, is in the northeast, the Pointers indicating the Pole Star, in the constellation of the Little Bear. Coiling between the two bears east of the northern horizon, is (*Draco*) the Dragon, with its two gleaming eyes, Beta and Gamma. With an opera glass notice the rich orange color of Gamma and the white of Beta. West of north, is *Cepheus*, beside his wife Cassiopeia and near by is *Andromeda*, their daughter, and her rescuer, *Perseus*. Midway between *Cassiopeia*, and the Great Bear (*Ursa Major*), is Camelopard, an insignificant constellation. The Triangles (*Triangula*) and (*Aries*) the Ram, are approaching the western horizon, and (*Cetus*) the Whale, has almost disappeared. Towards the southwest, are the Twins (*Gemini*), and in the mid-heaven, due west, is (*Taurus*) the Bull, with its blazing eye *Aldebaran*, and the beautiful cluster known as the *Pleiades*. Above Taurus, is the Charioteer (*Auriga*), and below it is *Orion*, already slanting towards "his grave, low down in the west." He treads on (*Lepus*) the Hare, and *Eridanus* flows towards the southwest. The three stars in the Giant's belt, point to *Sirius*, in the constellation of (*Canis Major*) the Great Dog, and higher up, is the Little Dog (*Canis Minor*). In the southern horizon is a part of the great ship *Argo*, near which, low down, is the Dove *Columba*. Almost overhead, is (*Cancer*) the Crab, with the pretty cluster *Praesepe*, or the Beehive, and coiling south of it, and towards the southeastern horizon, is (*Hydra*) the Sea Serpent. Above it, are the constellations (*Corvus*), the Crow, and (*Crater*) the Cup, which are represented as resting on the serpent's back, in the old fashioned star maps. *Virgo*, the Virgin has risen due east, and midway between *Virgo* and Cancer, is the zodiacal sign (*Leo*), the Lion, with the bright, first magnitude star *Regulus*. North of *Virgo*, is (*Coma Berenices*) the Hair of Queen Berenice, and nearly due east, and between this group of stars and the Great Bear (Ursa Major) is the constellation of the Hunting Dogs (*Canes Venatici*). Northeast, the Herdsman (Boötes), is rising, distinguished by the brilliant star Arcturus.

GEMINI, THE TWINS.

This constellation is represented on celestial maps, as the twin brothers Castor, and Pollux. According to Grecian mythology they embarked with Jason in

quest of the golden fleece, at Colchis, on which occasion they distinguished themselves by their bravery. During a battle, Castor was killed, and Pollux was so tenderly attached to his brother, that he was unwilling to survive him. He therefore entreated Jupiter to restore Castor to life, or to be deprived himself of immortality; whereupon, Jupiter permitted Castor, who had been slain, to share the immortality of Pollux. Consequently, as long as the one was upon Earth, so long was the other detained in the underworld, and they alternately lived and died every day. Jupiter further rewarded their love for each other by changing them both into the constellation, known to us by the name of *Gemini*, the Twins.

Castor and Pollux.

Castor is the finest double star visible in the northern heavens, the components being composed of a second and third magnitude star and rather more than 5″ apart. Both are white, according to some authorities, but the smaller is sometimes said to be slightly greenish. Castor is an example of a Sirian star, such stars being dazzling white, often inclining towards a steely blue. The components of Castor can only be seen with a telescope, for the most powerful field-glass fails to separate them. When Castor is observed with an opera-glass, clusters and groups of stars, are seen in the near neighborhood, arranged in a most symmetrical way, as if with a definite plan, recalling the curving streams and wreaths of stars, in the regions of the Milky Way.

As a binary star, Castor excels; the two stars circling around one another during a period of about a thousand years, whilst it seems to have taken captive a tenth magnitude star which follows in its train at a distance of 74″, thus completing this unique system. Castor is receding from the Earth at the rate of 25 miles a second.

Pollux is one of the ten brightest stars north of the equator. Seen with a fine telescope, it presents the appearance of a triple star, whilst in larger instruments it is multiple. The components are orange, grey, and lilac, and if there are planets circling around this system, how strange must be the effect, when the colored suns shine upon these little worlds. Pollux is approaching the Earth with an average speed of somewhere about 40 miles a second.

Other Fine Objects in Gemini.

There is a remarkable cluster of stars in Gemini (N.G.C. 2331) resembling a half opened fan, and composed of six or seven stars drawn closely together, as though by some irresistible power of attraction.

Zeta Geminorum, is a variable star referred to in S. C. Chandler's catalogue of 225 variables (*Astronomical Journal*, September, 1888) and its period is 10 days, 3 hours, 41 minutes and 30 seconds, its range of variation being from the 3.7 to the 4.5 magnitudes.

Eta Geminorum is another variable star referred to in this catalogue, its phases running through a period of 229 days, and varying between 3.2 and 4.2 magnitudes. In 1881, whilst Professor Burnham was observing this star through the great telescope at the Lick Observatory, he noticed that it was a variable double star, "a splendid unequal pair likely to prove an interesting system. Its revolutions deserve the more attention, as no star showing a banded spectrum has yet given signs of orbital movement." The spectrum of this star shows that it belongs to the third type, the orange stars, of which Alpha Orionis, and the variable star *Mira Ceti* are types. Variable stars of irregular period are included in this class. Their spectrum is characterized by dark bands, very dark towards the blue end shading off gradually towards the red end.

Afterward, the writer, disposed to observe the full reflective power of the speculum, when reinforced with a shining coat of metal, silvered it, by simply pouring carefully upon its capacious concave bosom the mixture, into which it was not convenient to suspend it, after the ordinary fashion.

The speculum end of the tube rested firmly upon the ground, and the sky end was elevated by an upright board, which was slowly pushed backward when an object had traversed the field; the azimuth motion being accomplished by gently moving the tube on its earthy bearing. A bored block, fastened to the side of the tube, held the ocular; and a step ladder was necessary to reach it. This clumsy but inexpensive device was quite imposing in appearance, for the tube was eleven feet in length, and made quite an impression upon passers by.

It remains to be said that the definition of the speculum, considering all the circumstances, was remarkable. Comparatively high powers were ventured upon, with surprising satisfaction, and the writer has jocularly asserted, that Galileo would have leaped sky-high had he been permitted to range the celestial vault with this powerful light gatherer.

These are busy days for all of us, and there is grave danger of crowding out such healthful recreation as amateur astronomy affords. In consequence it may not be wise to recommend others to follow in Mr. McCurdy's footsteps, especially since a reasonable sum will secure finished work from such a master hand as that of Brashear of Allegheny. Yet, if some Methuselah among amateurs be ambitious enough to make the attempt, let him be assured that while the outlay of effort and the concentration of mind will necessarily be great, the reward will be commensurate; and that such hours snatched from conventionalities will be found sweeter than the honey of Hymettus.

SOLAR ECLIPSES. (*Continued*).

J. MORRISON, M. A., M. D., PH. D.

FOR POPULAR ASTRONOMY

TO ESTABLISH CRITERIA FOR DETERMINING WHETHER THE ECLIPSE IS BEGINNING OR ENDING AT ANY PLACE PREVIOUSLY FOUND.

When a place is on the surface of the cone of the shadow the eclipse is either beginning or ending there. If T denote the time

of such an occurrence then at the next instant $T + dT$, the place will be either *within* or *without* the shadow according as the eclipse is beginning or ending at the time T, that is to say, the eclipse is beginning or ending according as the distance of the place from the axis of the shadow is, at the time $T + dT$, less or greater than the radius of the shadow at that place, therefore if

$\triangle < l - i\zeta$, the eclipse is beginning.
$\triangle > l - i\zeta$, the eclipse is ending.

In total eclipses $l - i\zeta$ is negative, but by comparing $\triangle^2$ with $(l - i\zeta)^2$ the criterion will still hold.

Since $(l - i\zeta)^2 = (x - \xi)^2 + (y - \eta)^2$, the criterion of beginning or ending for both partial or total eclipses will be the negative or positive value of the differential coefficient of the quantity $(x - \xi)^2 + (y - \eta)^2 - (l - i\zeta)^2$ taken with regard to the time.

Differentiating this expression, T being the independent variable we have

$$(x - \xi)\left(\frac{dx}{dT} - \frac{d\xi}{dT}\right) + (y - \eta)\left(\frac{dy}{dT} - \frac{d\eta}{dT}\right) - (l - i\zeta)\left(\frac{dl}{dT} - i\frac{d\zeta}{dT}\right)$$

in which we regard i as constant, its variation during an eclipse being practically insensible. For the sake of brevity, it is expedient to write, x', y', ξ', etc., for

$$\frac{dx}{dT}, \frac{dy}{dT}, \frac{d\xi}{dT} \text{ etc.}$$

and to represent the quantity by P, after first substituting the values of $x - \xi = (l - i\zeta) \sin Q$, $y - \eta = (l - i\zeta) \cos Q$ and $l - i\zeta = L$, we get

$$P = L\,[(x' - \xi') \sin Q + (y' - \eta') \cos Q - (l' - i\zeta')] \tag{96}$$

and if we represent by P' the quantity within the brackets we shall have

$$P = P'L \tag{97}$$

from which we see that P will be positive or negative according as P' and L have like or unlike signs.

Now in total eclipses L is negative, therefore from the preceding we shall have when

P' is *positive*, eclipse *beginning*.
P' is negative, eclipse ending.

But in external contacts and internal contacts in the case of annular eclipses L is positive, therefore we have when

P' is negative, eclipse beginning.
P' is positive, eclipse ending.

We must now put the expression for P', viz:

$$P' = (x' - \xi') \sin Q + (y' - \eta') \cos Q - (l' - i\zeta') \quad (98)$$

in a convenient form for computation. The quantities x', y', l', ξ' η' and ζ' may be taken for the hourly variations of x, y, l, ξ, η and ζ respectively, but greater accuracy will be secured by taking for them the variations in 10 minutes. The first three can be easily deduced from the eclipse tables or Besselian Elements for the given date, but the last three can only be obtained by differentiating (60), the latitude and longitude of the place being regarded constant, and since

$$\theta = \mu - a = \mu_1 - \lambda, \ \frac{d\theta}{dT} = \frac{d\mu_1}{dT},$$

but θ is expressed in angular measure and in order that all the quantities may be homogeneous, we must express $\frac{d\mu_1}{dT}$ in circular measure by multiplying it by sin 1″, therefore we have $\mu' = \frac{d\mu_1}{dT} \sin 1''$ and for a similar reason we have $d' = \frac{dd}{dT} \sin 1''$ where μ' and d' are the hourly variations or variations for 10 minutes of μ or θ and d respectively.

Differentiating (60) we have

$$\begin{aligned}
\xi' &= \mu' \rho \cos \varphi' \cos \theta \\
&= \mu' (-\eta \sin d + \zeta \cos d) && \text{by the aid of (60)} \\
&= \mu' (-y \sin d + \zeta \cos d + l - i\zeta) \sin d \cos Q) && \text{by (68)} \\
\eta' &= -\rho d' \sin \varphi' \sin d - \rho d' \cos \varphi' \cos d \cos \theta + \rho\mu' \cos \varphi' \sin d \sin \theta \\
&= \mu'\xi \sin d - d'\zeta \\
&= \mu' [x \sin d - (l - i\zeta) \sin d \sin Q] - d'\zeta && \text{by (78)} \\
\zeta' &= \rho d' \sin \varphi' \cos d - \rho d' \cos \varphi' \cos \theta \sin d - \rho\mu' \cos \varphi' \cos d \sin \theta \\
&= -\mu'\xi \cos d + d'\eta \\
&= \mu' [-x \cos d + (l - i\zeta) \cos d \sin Q] + d' [y - (l - i\zeta) \cos Q]
\end{aligned}$$

Substituting these in (98) reducing and omitting terms involving i^2 and id' as practically inappreciable we have

$$\begin{aligned}
P' = &(-l' - i\mu' x \cos d) - (-y + \mu' x \sin d) \cos Q \\
&+ (x + \mu' y \sin d + i\mu' l \cos d) \sin Q \\
&- \zeta (\mu' \cos d \sin Q - d' \cos Q)
\end{aligned}$$

For brevity put

$$\begin{aligned}
a' &= -l' - i\mu' x \cos d \\
b' &= -y + \mu' x \sin d \\
c' &= +x + \mu' y \sin d + i\mu' l \cos d
\end{aligned} \qquad (99)$$

then the above becomes

$$P' = a' - b' \cos Q + c' \sin Q - \zeta\,(\mu' \cos d \sin Q - d' \cos Q) \qquad (100)$$

The quantities a', b', c' can be computed for the same dates as the other quantities in the eclipse tables and interpolated for intermediate dates. They must also be computed for both penumbral and umbral contacts; the values of b' will be the same for both but those of a' and c' will differ by reason of the different values of l' and l.

For convenience of logarithmic computation we put

$$\begin{aligned} e \sin E &= b' \qquad & f \sin F &= d' \\ e \cos E &= c' \qquad & f \cos F &= \mu' \cos d \end{aligned} \qquad (101)$$

then we get

$$P' = a' + e \sin (Q - E) - \zeta f \sin (Q - F) \qquad (102)$$

The quantities a' and F are always very small and may generally be neglected when the sign of P' is alone required, we may also use ζ_1 for ζ the former being derived from

$$\zeta_1 = \cos \beta - i \sin \beta \cos (Q - \gamma)$$

(81 *et seq*) and the latter from (83). Therefore we have with all the accuracy necessary in such cases, the following simple criteria for partial or annular eclipses, viz:

$$\begin{aligned} &e \sin (Q - E) < \zeta f \sin Q, \text{ eclipse beginning} \\ &e \sin (Q - E) > \zeta f \sin Q, \text{ eclipse ending} \end{aligned} \qquad (103)$$

and for total eclipses

$$\begin{aligned} &e \sin (Q - E) > \zeta f \sin Q, \text{ eclipse beginning} \\ &e \sin (Q - E) < \zeta f \sin Q, \text{ eclipse ending} \end{aligned} \qquad (104)$$

Before we can use these criteria we first determine the latitude and longitude of the place by (86) for any assumed time between the beginning and ending of the eclipse on the Earth generally and also for such an assumed value of Q as will not give an impossible result. When the penumbra falls completely within the Earth's disk Q can of course have any value from 0° to 360° as in the outline *bf* Fig. 1.

We then compute the values of e, f and E from (101), b' and c' being interpolated for the given date from the tables of values previously computed from (99).

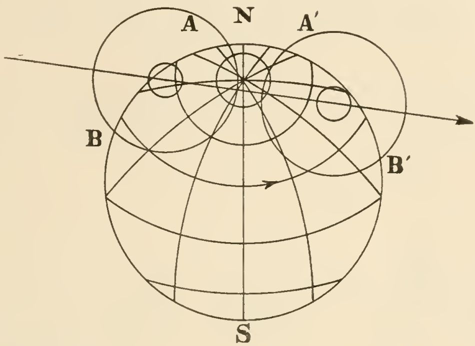
Rising and Setting Limits.

These limits are the curves passing through all the points on the Earth's surface where the eclipse begins or ends at sunrise or

sunset. In this case we have the point Z in the horizon and $\zeta_1 = 0$ as the conditions which must be satisfied.

The latitude and longitude are first determined by (92). The criterion for determining whether the Sun is rising or setting at the place thus found is easily established by the value of θ, the hour angle of the point Z. If this angle lies between 180° and

FIGURE 4.



360°, which includes all points on the horizon, $NABS$, Fig. 4, the Sun is rising, and if between 0° and 180° which embraces all points on the horizon, $NA'B'S$, the Sun is setting. The sign of P' (102) will then determine whether the eclipse is beginning or ending. As no accurate observations can be made in the horizon, it will be sufficient to put both a' and $\zeta = 0$ and then we shall have

$$\sin (Q - E), \text{ negative, eclipse beginning}$$
$$\sin (Q - E), \text{ positive, eclipse ending}$$

But $$l \sin (Q - E) = l \sin Q \cos E - l \sin E \cos Q$$

and from (87) and (88) we find

$$l \sin Q = m \sin M - p \sin \gamma$$
$$l \cos Q = m \cos M - p \cos \gamma$$

Substituting these in the above we have

$$\begin{aligned} l \sin (Q - E) = & \quad m \sin M \sin E - p \sin \gamma \cos E \\ & - m \cos M \cos E + p \cos \gamma \sin E \\ = & \quad m \sin (M - E) - p \sin (\gamma - E) \end{aligned}$$

Therefore for all points in the rising or setting limits we have the following criteria:

$$\begin{aligned} &m \sin (M - E) < p \sin (\gamma - E), \text{ eclipse beginning} \\ &m \sin (M - E) > p \sin (\gamma - E), \text{ eclipse ending} \end{aligned} \qquad (105)$$

where m, M, p and γ are already found by (88).

If the interior contacts exist (see Fig. 2*) the rising and setting limits form on the Farth's surface, two separate enclosed oval curves. See chart of total eclipse of June 16, 1890 as given in the *American Ephemeris.*

If T_1 and T_2 denote the times of beginning and ending on the Earth generally as determined by (93) and T'_1 and T'_2 the times of interior contact determined by (94), a series of points on the rising limit, that is, when the eclipse begins or ends at *sunrise*, will be found for a series of times assumed between T_1 and T'_1, and the points of the setting limit, that is when the eclipse begins or ends at *sunset*, for a series of times assumed between T'_2 and T_2. But when the interior contacts (94) do not exist, as in Fig. 4, the curve on which the eclipse begins at sunrise is continuous with that on which it begins at sunset and the curve of ending at sunrise is continuous with that of ending at sunset; the two curves meeting and forming a single closed curve extending throughout the whole eclipse and resembling a figure 8 much distorted. See chart of eclipse of January 21, 1898 as delineated in the *American Ephemeris.* A series of points on these curves will be found by assuming a series of dates between T_1 and T_2 equidistant say for every 10, 15, 20 or 30 minutes, according to the degree of accuracy desired.

To Find the Curve of Maximum Eclipse in the Horizon.

In this case we must have $\zeta = 0$ since the Sun is in the horizon, and if $\triangle$ denote the distance of the place from the axis of the shadow we have

$$\begin{aligned} \triangle \sin Q &= x - \xi \\ \triangle \cos Q &= y - \eta \end{aligned} \qquad (106)$$

The amount of obscuration will of course depend upon the value of $l - \triangle$, and for the maximum eclipse we have the condition

$$\frac{dl}{dT} - \frac{d\triangle}{dT} = 0$$

Differentiating (106) we have

$$\frac{d\triangle}{dT} \sin Q + \triangle \cos Q \frac{dQ}{dT} = x' - \xi'$$

$$\frac{d\triangle}{dT} \cos Q - \triangle \sin Q \frac{dQ}{dT} = y' - \eta'$$

* This cut will be given later.

whence we get

$$(x' - \xi') \sin Q + (y' - \eta') \cos Q - l' = 0 \qquad (107)$$

since

$$\frac{d\triangle}{dT} = \frac{dl}{dT} = l'.$$

Equation (107) is identical with (98) when $\zeta = 0$ or when the place is in the horizon, therefore the condition which characterizes the maximum eclipse in the horizon is $P' = 0$, and this condition will evidently be satisfied when in (102) we put

$$\sin (Q - E) = 0$$

because $\zeta = 0$ and a' being always very small may be neglected. This condition gives us $Q = E$, or $Q = 180 + E$, therefore (106) becomes

$$\pm \triangle \sin E = x - \xi$$
$$\pm \triangle \cos E = y - \eta$$

and we also have

$$\xi^2 + \eta_1^2 = 1$$

The angle E is known at any time from its tabulated values computed by (101); x and y are also known for the same time, but $\triangle$, ξ and η are unknown.

Put
$$m \sin M = x \qquad p \sin \gamma = \xi$$
$$m \cos M = y \qquad p \cos \gamma = \eta$$

then we have

$$\pm \triangle \sin E = m \sin M - p \sin \gamma$$
$$\pm \triangle \cos E = m \cos M - p \cos \gamma$$

whence we easily find

$$0 = m \sin (M - E) - p \sin (\gamma - E)$$
$$\pm \triangle = m \cos (M - E) - p \cos (\gamma - E)$$

Put
$$\psi = \gamma - E$$

and
$$\sin \psi = \frac{m \sin (M - E)}{p}$$

$$\pm \triangle = m \cos (M - E) - p \cos \psi \qquad (108)$$

The first of these determines ψ when p is known. As p always lies between ρ_1 and unity we may take for p the geometric mean between ρ_1 and unity, that is $\log p = \frac{1}{2} \log \rho_1$ which will be sufficient for all ordinary purposes, but if a more accurate value is

required we put $\gamma = \psi + E$ and then find p by (90) after which (108) must be re-computed, and the true value of ψ having been thus found and also of γ we have γ' by (91) and the latitude and longitude of each point on the curve by (92).

The first of (108) will give two values of ψ and therefore also two values of $\triangle$, but in this particular case $\triangle$ can never be greater than l and therefore that value of ψ which would make $\triangle > l$ must be rejected, so that in general we have only one solution.

The limiting times between which the curve can exist, are known from the computation of the rising and setting limits, or they are the times when

$$m \sin (M - E) = p \sin (\gamma - E)$$

that is, the curve will be computed for those dates for which $m \sin (M - E)$ is less than l.

The degree of obscuration is the fraction of the Sun's apparent diameter covered by the Moon. When the place is on the edge of the umbra, the obscuration is total and the distance of the place from the *edge* of the penumbra is equal to the absolute difference of the radii of the penumbra and umbra or the algebraic sum of l and l_1, the latter being negative; but in any other case the distance of the place within the penumbra is $l - \triangle$, therefore if D denote the degree of obscuration as above defined we shall have when the place is in the horizon

$$D = \frac{l - \triangle}{l + l_1} \tag{109}$$

In annular eclipses l_1 (the radius of the umbral penumbra) is essentially positive. When the place *is not* in the horizon we shall of course still have

$$D = \frac{L - \triangle}{L + L_1} \tag{110}$$

where L and L_1 are the radii of the penumbra and umbra respectively on the *parallel* plane at the distance ζ from the fundamental plane.

Problems.

32. If π and π' denote the equatorial horizontal parallaxes of the Moon and Sun, f and f' the semi-angles of the penumbral and umbral cones and δ' the Sun's semi-diameter as seen from centre of the Earth, show that

$$f + f' = 2\delta' . \frac{\sin \pi}{\sin \pi - \sin \pi'}$$

33. Find the time the Sun's semi-diameter will take to cross the meridian, the declination being 23° 2′ and the semi-diameter 16′ 17″.

34. Given the Sun's declination (δ) and the interval between the times at which he is west and sets (h) to find the latitude of the place, refraction and the Sun's motion in declination being neglected.

35. Find the Sun's declination when he is at the same instant on the horizons of two places whose latitudes are 53° 23′ N., and 3° 12′ S., and longitudes 6° 20′ W., and 35° 10′ W., respectively.

36. If v and v' be the angular velocities of the Moon about the Earth and of the Earth about the Sun, both orbits supposed circular, and if α be the Moon's greatest elongation from the Earth as seen from the Sun, show that the times between the successive greatest elongations are alternately

$$\frac{\pi - 2\alpha}{v - v'} \text{ and } \frac{\pi + 2\alpha}{v - v'}.$$

EVENINGS WITH THE STARS.

MARY PROCTOR.

FOR POPULAR ASTRONOMY.

The Great Bear (*Ursa Major*) is now at its highest and nearly overhead, the Pointers indicating the Pole star (*a* of the Little Bear, *Ursa Minor*). Below the Little Bear, is Cepheus low down to the east of north, and Cassiopeia low down to the west of north. Perseus, the rescurer, is setting in the northwest, and above is the Camelopard. The Charioteer (*Auriga*) is nearing the northwest horizon, and is distinguished by the bright Capella. The Twins (*Gemini*) are also nearing the western horizon, and further west and higher we find the Crab (*Cancer*) below which is the Little Dog (*Canis Minor*). There are very few bright stars in the southwestern sky, *Alfard*, the heart of the Sea Serpent, *Hydra*, shining alone in a great blank space. Nearly overhead is *Leo*, the Lion, and *Coma Berenices* is close by the Hunting Dogs (*Canes Venatici*) being almost overhead, between *Coma* and the Great Bear.

Lower down in the south, we find the Crow (*Corvus*), and the Cup (*Crater*), resting on the back of the Serpent. Extending in the mid-heavens from the southeast to the south, between the Lion and the Crow, is *Virgo*, the Virgin. Low down in the south we find the head and body of the Centaur (*Centaurus*), supposed to typify the patriarchal Noah.

The Scorpion's heart has risen in the southeast and the stars of the Scales (*Libra*), glisten between the head of *Scorpio* and the Virgin's robes. Due east,

low down in *Ophiuchus*, the Serpent-Holder, the Serpent (*Serpeus*), he holds in his hand curving upward toward the Crown (*Corona Borealis*) its head being due west. Above it we see the bright Arcturus, in the constellation of the Herdsman (Boötes). In the northeast is *Hercules*, his head close to the head of the Serpent-Holder. Beneath his feet glows the Lyre (*Lyra*) with the brilliant Vega, and the Swan (*Cygnus*) has already half risen above the northeastern horizon. *Draco*, the Dragon, curves between the Great and Little Bear, towards Cepheus, and then curves its head, towards the heel of Herculus.

The Constellation Virgo.

This constellation occupies a considerable space in the heavens, and contains 411 stars. Its mean R. A. is 13^h 16^m, Decl. — 2° 30′. It is situated next east of Leo, and about midway between Coma Berenices on the north, and Corvus on the south. According to the ancient writers this constellation represents the virgin Astræa, the goddess of justice, who lived upon the Earth during the golden age, but being offended at the wickedness of man during the brazen and iron ages of the world, she returned to heaven, and was placed among the constellations of the zodiac, with a pair of scales, (*Libra*) in one hand and a sword in the other.

"Faith flees, and piety in exile mourns;
And *Justice*, here oppressed, to heaven returns."

Several bright stars are scattered about in this constellation, and may be traced out by the map:

"Her lovely tresses glow with starry light;
Stars ornament the bracelet on her hand;
Her vest in ample fold, glitters with stars:
Beneath her snowy feet they shine, her eyes
Lighten, all glorious, with the heavenly rays,
But *first* the star which crowns the golden sheaf."

The Leading Brilliant Spica Virginis.

Spica Virginis, is in the ear of corn, which in some of the olden star maps, is represented as being held in the left hand of the Virgin, in place of the sword. According to Egyptian mythology, Isis was said to have dropped a sheaf of corn as she fled from Typhon, who, as he continued to pursue her, scatterd it over the heavens. The Chinese call the zodiac, the *yellow road*, as resembling a path over which the ripened ears of corn are scattered. *Spica* may be easily recognized as it glows in solitary splendor, there being no visible star near it except one of the 4th magnitude, situated about 1° below it on the left. The position of this star in the heavens, has been determined with great exactness for the benefit of navigators. It is one of the stars from which the Moon's distance is measured for determining longitude at sea. Its situation is highly favorable for this purpose, as it lies within the Moon's path, and little more than 2° below the Earth's orbit. It comes to the meridian at 9 o'clock about the 28th of May, in that point of the heavens where the Sun is at noon, about the 20th of October. (Burritt's *Geography of the Heavens* p. 92-93.)

According to the spectroscopic observations made by Vogel, *Spica* revolves at a minimum rate of fifty-six miles a second, round the common centre of gravity of itself and an obscure companion. It is of a brilliant white hue, and of the first magnitude whilst the companion-star is of the 10th magnitude and of a bluish color. The movement of Spica, in the line of sight, is 14 English miles a

you see right overhead;" "oh yes, that is simple enough;" "well, now look at that star over there, can you make out a triangle in the heavens?" "Well I guess I can Jack, is that all?" "Not quite; you know the Earth moves but never mind that, suppose the arcs of the triangle move and that one from Pole to star comes nearer and nearer to the meridian making an angle less and less and until it vanishes." "Yes, I can follow that." "Well all you have to do is to measure that angle, and your book will show you how to do that." The problem was clear.

"It was on board the good ship Eliza Everett, of Yarmouth, N. S., bound out to Philadelphla, and a little later we lay in the Deleware River gazing at the great comet of 1874."

Now every one is not sufficiently interested in practical astronomy to follow out in detail all the variations of the time problem, but I think we should all know in a general way how these questions are to be worked; we should not regard as mysterious the connection between the clock and the stars. There is, indeed, a lot of work to be gone over if one would learn all the theorems involved in the astronomical triangle but they are all deductions from a few elementary formulæ and they do not carry us into the region of the higher branches of analysis. They do require however, that the student be able to mentally picture the star sphere and the circles drawn upon it. For my own part I remember most vividly the hour when it was made clear to me that a diagram on paper would represent great circles in the heavens and in the same way as we would work out a problem in school from an illustration on the blackboard, we would arrive at important truths in practical astronomy. And I believe that no one will take up the subject without feeling something of the fascination. The more difficult the problems are the more fascinating they become, the longer we look at them the more we admire them and eventually, like some of Dr. M.'s they become things of beauty, and I am sure to poor Jack Evans and his like, they would have been—joys forever!

Toronto, May 17, '97.

SOLAR ECLIPSES. (*Continued*).

J. MORRISON, M A., M. D , Ph. D.

For Popular Astronomy.

To Determine Formulæ for Finding the Northern and Southern Limits of the Eclipse.

The intersection of the penumbral cone with the surface of the Earth assumes the form of a complete or partial oval, (see Fig. 5),

according as the cone falls wholly or partially on the Earth's illuminated disk, that is according as there is *interior* or exterior contacts. See Figs. 2, 3 and 4.*

Suppose now that the outline of the shadow assumes consecutive positions, corresponding to very small intervals of time, the Earth also turning on its axis, we shall then have a series of these ovals, and it is evident that the *extreme* geographical limits of the eclipse will be the curve which envelopes these ovals, also that at each instant the place of limit by reason of the two motions will be proceeding relatively in the direction of a tangent to the oval.

There will evidently be two such limiting curves—a northern and southern—when interior contact takes place or when the oval becomes entire during the eclipse but only one when it overlaps the Earth's disk, or in other words, if the curves of rising and setting do *not* extend throughout the entire eclipse, there will be both a northern and a southern limit, but when they continue throughout the entire eclipse there will be only one of these limits, a southern when the rising and setting curves extend through the whole eclipse in north latitude and vice versâ. The extreme points of these curves will have the Sun in the horizon and therefore $\zeta_1 = 0$, and it is also evident that these points are the first and last points of the curve of maximum eclipse in the horizon (Fig. 5) and consequently the limiting times are the same as for that curve.

For any point on the northern or southern limiting curve, simple contact is the *maximum* of the eclipse for that place and accordingly we must have $P' = 0$ (see page 35) and therefore we also have by (102)

$$a' + e \sin (Q - E) = \zeta f \sin (Q - F) \qquad (111)$$

Hence for any given date T for which the phase is possible, we must find that point on the outline of the shadow for which the value of Q and its corresponding ζ, satisfy this equation which contains two unknown quantities.

It can therefore be solved only by indirect processes or by successive approximations and for this reason the computation of this curve is usually regarded as the most difficult, complex and unmanageable of all the circumstances connected with the general eclipse. Fortunately however, a precise determination of these curves is of very little importance, since no valuable observations can be made on them. For all practical purposes then, an approximate determination is all that is necessary, and indeed

* See March number, pages 508 and 509, and May number, page 33.

this is all that is aimed at in the charts of eclipses in all Nautical Almanacs, for if we calculate the eclipse for any point on the northern or southern limiting curve as laid down in the Almanacs, we shall almost always find it either within or without the curve.

To solve (111) even approximately we must first arrive at an approximate value of Q or at least determine the limits between which Q must be assumed. Neglecting both a' and F in (111) since they are *always* very small we have approximately

$$e \sin (Q - E) = \zeta f \sin Q \tag{112}$$

in which e, E and f are known from the eclipse tables for any date. We still have two unknown quantities, viz, Q and ζ; the extreme values of the latter are 0 and 1.

The first gives $\sin (Q - E) = 0$

or $$Q = E \text{ and } Q = E + 180^\circ \tag{113}$$

and the second gives

$$e \sin (Q - E) = f \sin Q$$

whence we easily get

$$\tan (Q - \tfrac{1}{2} E) = \frac{e + f}{e - f} \tan \tfrac{1}{2} E$$

$$= \tan V, \text{ suppose} \tag{114}$$

then we have

$$Q = \tfrac{1}{2} E + V \text{ or } Q = \tfrac{1}{2} E + V + 180^\circ \tag{115}$$

Hence Q must be assumed between

$$E \text{ and } \tfrac{1}{2} E + V$$

or between

$$E + 180 \text{ and } \tfrac{1}{2} E + V + 180^\circ.$$

It may be observed, however, that great precision in the angle Q is not necessary, for let AB, Fig. 5,* be the limiting curve which is tangent at P to the outline of the shadow $DEFPHK$ whose axis or centre is at C, then if Q be in error by the angle PCP_1 or PCP_2, the point determined will be either P_1 or P_2 instead of P. Now, although this error may be several minutes or even degrees the erroneous point P_1 or P_2 will never be far removed from the true curve. Accordingly (112) may without any appreciable error be written thus:

$$e \sin (Q - E) = \zeta_1 f \sin Q$$

* Engraver's errors throw out the plate this month. It will appear next time.

in which we may put $\zeta_1 = \cos\beta$ (see page 506*) β being found from (80) with the first assumed value of Q, x, y, l and ρ, being all known for any date from the eclipse tables, we shall then have

$$\frac{\sin(Q - E)}{\sin Q} = \frac{f\cos\beta}{e}$$

whence

$$\tan\left(Q - \frac{1}{2}E\right) = \tan(45^\circ + V')\tan\frac{1}{2}E \qquad (116)$$

where

$$\tan V' = \frac{f}{e}\cos\beta.$$

We may, if necessary, use this value of Q in (80) to determine a more accurate value of $\cos\beta$ and then repeat the computation of (116); but this will in general hardly ever be necessary. When a tolerably correct value of Q has thus been found, we can proceed to compute the latitude and longitude of the point by (80) (81) (83) (85) and (86).

In the computation of a series of points D, E, F, H, etc., on the outline of the shadow DPK for any given date we may of course determine the point P by the formulæ 80—86 for this particular date and it will serve as a check or guide in the determination of preceding or subsequent points.

The limiting dates between which the curve APB in Fig. 5 exists, can be determined as follows, assuming that the curves of rising and setting and maximum in the horizon have not already been computed.

Since the curves begin and end with the Sun in the horizon we have $Q = E$ or $Q = E + 180^\circ$ and at the required time we have

$$\begin{aligned} \xi &= x \mp l\sin E \\ \eta &= y \mp l\cos E \qquad (117) \end{aligned}$$

and

$$\xi^2 + \eta^2 = 1$$

If we put $\xi = \sin\gamma$ this last condition gives $\eta = \cos\gamma$ we also have

$$\sin E = \frac{b'}{e} \text{ and } \cos E = \frac{c'}{e}$$

by (101) in which we may regard e as constant.

Let T be the required time and let $T = T_0 + \tau$, where T_0 is the epoch of the eclipse tables, and if b'_0, c'_0 be the values of b' and c' for the date T_0 and b_1 and c_1 their hourly changes. we shall have for the time T

$$b' = b'_0 + b_1\tau \text{ and } c' = c'_0 + c_1\tau$$

and if E_0 be the value of E for the date T_0 we shall also have approximately

* Vol. IV.

$$\sin E = \sin E_0 + \frac{b_1}{e}\tau \text{ and } \cos E = \cos E_0 + \frac{c_1}{e}\tau$$

We also have with the usual notation

$$x = x_0 + x'\tau \text{ and } y = y_0 + y'\tau$$

With these transformations (117) becomes

$$\sin \gamma = x_0 \mp l \sin E_0 + \left(x' \mp \frac{l}{e} b_1\right)\tau$$

$$\cos \gamma = y_0 \mp l \cos E_0 + \left(y' \mp \frac{l}{e} c_1\right)\tau$$

Put

$$m \sin M = x_0 \mp l \sin E_0$$
$$m \cos M = y_0 \mp l \cos E$$

and

$$n \sin N = x' \mp \frac{l}{e} b_1 \qquad (118)$$

$$n \cos N = y' \mp \frac{l}{e} c_1$$

the upper sign for the southern and the lower for the northern limit, then we have

$$\sin \gamma = m \sin M + n \sin N \,.\, \tau$$
$$\cos \gamma = m \cos M + n \cos N \,.\, \tau$$

whence we easily find

$$\sin (\gamma - N) = m \sin (M - N)$$
$$\cos (\gamma - N) = m \cos (M - N) + n\tau$$

Put $\gamma - N = \psi$ and we have

$$\sin \psi = m \sin (M - N)$$

$$\tau = \frac{\cos \psi}{n} - \frac{m \cos (M - N)}{n} \qquad (119)$$

and

$$T = T_0 + \tau$$

In this equation $\cos \psi$ is to be taken with the negative sign for the first point, A and with the positive sign for the last point, B of the curve. For the latitude and longitude of the extreme points A and B, we take $\gamma = N + \psi$, $\tan \gamma' = \rho_1 \tan \gamma$ and employ (92).

In computing a series of points on the northern and southern limits, it will be found convenient to form a table of the extreme values of Q at intervals of an hour between the extreme dates; we may then assume Q equal to the arithmetical mean of these values and proceed to approximate to the true value by (80) and (116). After three or four consecutive points have been determined we may be able to anticipate the next approximate value

of Q and then correct it by (116) if necessary. When a sufficient number of points have been established a curve drawn through them will be the limiting curve required.

Curve of Central Eclipse.

This curve is the path described by the axis of the shadow as it passes across the Earth's disk. The problem is only a particular case of the one just solved, the shadow being reduced to a point and therefore we must have $l - i\zeta = 0$, and the fundamental equation (68) gives

$$x = \xi \text{ and } y = \eta.$$

But by (78)

$$y - \eta = y - \rho_1\eta_1 = 0 \text{ or } \eta_1 = \frac{y}{\rho_1}$$

and we also have

$$\zeta_1 = \cos\beta$$

which can be computed from (80) with sufficient accuracy for our present purpose. Substituting in (79) we have after first putting

$$c \sin C = \frac{y}{\rho_1}$$

$$c \cos C = \cos\beta$$

$$\begin{aligned} \cos\varphi_1 \sin\theta &= x \\ \cos\varphi_1 \cos\theta &= c\cos(C + d_1) \\ \sin\varphi_1 &= c\sin(C + d_1) \end{aligned} \qquad (120)$$

and $$\tan\varphi = \frac{\tan\varphi_1}{\sqrt{1-e^2}} \quad \text{and } \lambda = \mu_1 - \theta$$

which determine for any given time T a point on the curve, x, y, ρ_1, d_1, μ_1 and l being taken from the eclipse tables for the given date.

As we are now dealing with the umbra only, the value of l is the radius of that cone. In the *American Ephemeris* this radius is denoted by l', but we prefer to use l bearing always in mind that in all problems relating to the line of central eclipse the umbra only is considered. The angle Q is already known from (116).

The extreme dates between which the solution of (120) is possible or the times of beginning and ending of the central eclipse on the Earth generally, are found as follows:

When the central eclipse begins or ends the axis of the shadow is tangent to the Earth's surface and the point Z will be in the horizon and also $\zeta_1 = 0$; therefore the last of (78) gives

$$\xi^2 + \eta_1^2 = 1 \text{ or } x^2 + \frac{y^2}{\rho_1^2} = 1 \qquad (121)$$

as the condition which must be fulfilled.

Since $\zeta_1 = 0 = \cos\beta$, therefore $\sin\beta = 1$ and (79) becomes

$$\begin{aligned} \cos\varphi_1 \sin\theta &= x \\ \cos\varphi_1 \cos\theta &= -\frac{y}{\rho_1}\sin d_1 \\ \sin\varphi_1 &= \frac{y}{\rho_1}\cos d_1 \end{aligned} \tag{122}$$

and also $$\tan\varphi = \frac{\tan\varphi_1}{\sqrt{1-e^2}} \quad \text{and } \lambda = \mu_1 - \theta$$

which determine the latitude and longitude of the place where the central eclipse is first and last seen.

In (80) we neglect the terms $l \sin Q$ and $\frac{l}{\rho_1}\cos Q$ since l (the radius of the umbra) is always a small quantity we shall then have, since $\sin\beta = 1$, $\sin\gamma = x$ and $\cos\gamma = \frac{y}{\rho_1}$ which is in accord with (121).

If now we denote the time of beginning or ending by $T = T_0 + \tau$, T_0 being as before the epoch of the eclipse tables which is generally taken to be the nearest whole hour to the time of conjunction and if x_0 and y_0 be the values of x and $\frac{y}{\rho_1}$ for the date T_0 and x' and y' their hourly variations we shall have

$$\begin{aligned} \sin\gamma &= x = x_0 + x'\tau \\ \cos\gamma &= \frac{y}{\rho_1} = y_0 + y'\tau \end{aligned}$$

Put as before

$$\begin{aligned} m\sin M &= x_0 \qquad & n\sin N &= x' \\ m\cos M &= y_0 \qquad & n\cos N &= y' \end{aligned}$$

and we have

$$\begin{aligned} \sin\gamma &= m\sin M + n\sin N \,.\, \tau \\ \cos\gamma &= m\cos M + n\cos N \,.\, \tau \end{aligned}$$

when we easily obtain in the usual manner

$$\begin{aligned} \sin(\gamma - N) &= m\sin(M-N) \\ \cos(\gamma - N) &= m\cos(M-N) + n\tau \end{aligned}$$

and putting

$\psi = \gamma - N$, the solution becomes

$$\sin\psi = m\sin(M-N)$$

$$\tau = \frac{\cos\psi}{n} - \frac{m\cos(M-N)}{n} \tag{123}$$

and $$T = T_0 + \tau$$

where $\cos\psi$ is to be negative for beginning and positive for the end.

Central Eclipse at Noon.

During the course of the umbra across the Earth some place must have the eclipse central at apparent noon, In this case we evidently have $x = 0$, and neglecting the small terms $l \sin Q$ and $\frac{l}{\rho_1} \cos Q$ in (80) squaring and adding and putting $x = 0$, we get

$$\sin \beta = \frac{y}{\rho_1} \qquad (124)$$

by which β may be found from that value of y which corresponds to the date when $x = 0$. We then have from (120) $\tan C = \tan \beta$ or $C = \beta$, $c = 1$ and $\theta = 0$ since $x = 0$

therefore $\tan \varphi_1 = \tan (\beta + d_1)$

or $\varphi_1 = \beta + d_1$

and $\tan \varphi = \frac{\tan \alpha_1}{\sqrt{1 - e_2}}$ and $\lambda = \mu_1$ (125)

where of course d_1 and μ_1 are taken for the time when $x = 0$.

Duration of Total or Annular Eclipse at any Point on the Curve of Central Eclipse.

Let T denote the time of *central* eclipse and $2t$ the duration, then $T \pm t$ is the time of beginning or end, then from the fundamental equation (68) we have with the usual notation

$$(l - i\zeta) \sin Q = x \pm x't - (\xi \pm \xi' t)$$
$$(l - i\zeta) \cos Q = y \pm y't - (\eta \pm \eta' t)$$

We here have $x = \xi$, $y = \eta$ and we may use $\zeta = \zeta_1 = \cos \beta$ as determined by (80) the terms $l \sin Q$ and $\frac{l}{\rho_1} \cos Q$ being neglected, so that we now have

$$(l - i \cos \beta) \sin Q = \pm (x' - \xi')t$$
$$(l - i \cos \beta) \cos Q = \pm (y' - \eta')t \qquad (126)$$

The values of ξ' and η' are found on page 31, and are

$$\xi' = - \mu' y \sin d + \mu' \cos \beta \cos d$$

and $\eta' = \mu x \sin d$

and from (99) we have

$$x' = \quad c' - \mu' y \sin d - \mu' il \cos d\,^{*}$$
$$y' = - b' + \mu' x \sin d$$

*Errata:—In (99) for the value of b' write $-y'$ for y, and for that of c', write x' for x. Also in the preceding equation in the coefficient of $\cos Q$ write $-y'$ for y and in the next term write x' for x.

Therefore we have

$$x' - \xi' = c' - \mu' \cos \beta \cos d$$

and
$$y' - \eta' = - b'$$

omitting the term $- \mu' i l \cos d$ since both i and l (radius of umbra) are very small.

For brevity put $l - i \cos \beta = L$, then (126) becomes

$$L \sin Q = \pm (c' - \mu' \cos \beta \cos d) t$$

and
$$L \cos Q = \pm b' t$$

whence
$$\tan Q = \frac{c' - \mu' \cos \beta \cos d}{b'}$$

and
$$t = \frac{L \cos Q}{b'}, \text{ or } t = \frac{L \sin Q}{c' - \mu' \cos \beta \cos d} \qquad (127)$$

It may be remarked here that the value of c' from (99) is to be computed for the umbra.

In (127) t is expressed in hours. if we multiply by 3600 the number of seconds in an hour we shall have t in seconds of time,

thus
$$t = \frac{3600 L}{b'} \cos Q.$$

In these formulæ $\cos \beta$ is to be taken for the time T in computing the central curve and all the other quantities viz, l (radius of umbra) i, d, b' and c' from the eclipse tables already computed.

Northern and Southern Limits of a Total or Annular Eclipse.

These limits can be determined with great exactness by the formulæ just derived for the limits of the penumbra taking care to put l equal to the radius of the umbra, and since the northern and southern curves of a total or annular eclipse lie near the central curve, we can always take $\zeta_1 = \cos \beta$ for any date T already found in computing the latter curve, and the value of Q can be found at once from (116) where the quantities e, E and f are to be taken from the eclipse tables for the time T.

Sometimes it happens that an eclipse may be annular near the beginning and ending and total during the middle of the eclipse; in such cases we must bear in mind that the northern limit of the annular eclipse becomes the southern in the total eclipse and vice versâ. The curve of total or annular eclipse like the northern and southern limits of the penumbral cone will of course begin and end on the curve of maximum eclipse in the horizon.

Prediction of a Solar Eclipse for a Given Place.

This is performed by successive approximations. If we have a

chart of the eclipse such as is given in the *American Ephemeris*, we can easily obtain from it an approximate time of the beginning and ending at any place within the limits of visibility, and then compute a correction τ (to be explained presently) to these approximate times and repeat the computation as often as necessary. But if we have no such chart we must assume a time T such that $T = T_0 + \tau$ where T_0 is the epoch of the eclipse tables (any arbitrary date near conjunction, usually the nearest whole hour to that time) and τ a correction to this epoch.

Take x_0, y_0, x', y', d, l and i, or log i, from the eclipse tables for the date T_0 then we shall have for the time T,

$$x = x_0 + x'\tau \text{ and } y = y_0 + y'\tau$$

The coördinates ξ_0 and η_0 of the place of observation at the time T_0 are found by (60) or the first three of (69) and their hourly variations ξ', η' and ζ' are found by differentiating (60) which is done on page 31, May '97 and are found to be

$$\xi' = \mu'\rho \cos \varphi' \cos \theta$$
$$\eta' = \mu'\xi \sin d - d'\zeta$$

where μ' and d' are the hourly change of μ and d multiplied by sin 1″ (see page 31) but for a first approximation we may omit $d'\zeta$. Then for the date T we shall have

$$\xi = \xi_0 + \xi'\tau$$
$$\eta = \eta_0 + \eta'\tau$$

And if we put for brevity $L = l - i\zeta$ and neglect its variation in the first approximation, the fundamental equation (68) becomes

$$L \sin Q = x_0 - \xi_0 + (x' - \xi')\,\tau$$
$$L \cos Q = y_0 - \eta_0 + (y' - \eta')\,\tau \qquad (128)$$

Put $\quad m \sin M = x_0 - \xi_0, \qquad n \sin N = x' - \xi'$

$\quad\quad\; m \cos M = y_0 - \eta_0, \qquad n \cos N = y' - \eta'$

then

$$L \sin Q = m \sin M + n \sin N \,.\, \tau$$
$$L \cos Q = m \cos M + n \cos N \,.\, \tau$$

whence we get as before

$$L \sin (Q - N) = m \sin (M - N)$$
$$L \cos (Q - N) = m \cos (M - N) + n\tau$$

and if $\quad \psi = Q - N$

then

$$\sin \psi = \frac{m \sin (M - N)}{L} \qquad (129)$$

and

$$\tau = \frac{L \cos \psi}{n} - \frac{m \cos (M - N)}{n}$$

The first of these gives two values of ψ which must be taken so that cos ψ may have both the negative and positive sign, the former being used for the beginning and the latter for the ending of the eclipse at the given place. The first approximation just explained may begreatly in error—ten, fifteen or twenty minutes or even more. For the 2d approximation, take the computed times just found or two times nearly equal to them and repeat the computation for each separately. The second approximation will generally be correct within a few seconds and sufficient for all purposes—a perfect prediction not being attainable in the present state of the solar and lunar tables.

From (129) we have

$$Q = \psi + N \tag{130}$$

which is the angular distance of the point of contact measured from the north point of the Sun's limb towards the east. (See page 503, March '97).

CONDITION OF VISIBILITY OF INTRA-MERCURIAN PLANETS.—CAN SUCH BODIES BE PHOTOGRAPHED?*

SEVERINUS J. CORRIGAN.

FOR POPULAR ASTRONOMY.

The periodic times corresponding to the mean distances of the celestial bodies aforesaid, are respectively, 36.29, 15.89 and 6.95 days, both distances and periods being thus somewhat greater than those set forth in my paper published in the February number of POPULAR ASTRONOMY (in which, by the way, I note a typographical or clerical error in the statement of the largest mean distance, which should have been 29 77 instead of 20.71) but the values given them were not intended to be *definitive* being only rough approximations. The values of the diameters of the two inner planetary bodies given above are considerably greater than those in the paper aforesaid, but the latter values have been derived from more definite data, and, I think, are more nearly the true ones.

At the distances of these bodies from the Earth during the total phase of the solar eclipse on July 29, 1878, the true, angular diameters were $1''.3$, $1''.2$ and $1''.0$ respectively; but the surfaces of these bodies being heated considerably above the point of luminosity their diameters must be apparently, enlarged by

* Continued from page 96.

"irradiation" and from an examination of the effect of "irradiadiation" upon the incandescent filament of an electric lamp, I have found that the apparent augmentation of the diameter must be fully three times, and therefore, that the three planetary bodies aforesiad, must have presented *spurious* discs having apparent diameters of 3″.9, 3″.6 and 3″.0 respectively.

Now, Professor Swift, as is well known, reported that the peculiar celestial objects seen by him were alike, red, and that they presented discs about equal to that of the planet Uranus whereof the mean angular diameter is approximately 3″.9 or almost the same as the apparent diameters of the first and second of the true hypothetical planetary bodies above mentioned, therefore the first and second bodies aforesaid answer well, the descriptions of the two celestial objects observed by Swift. Furthermore, one of Professor Watson's positive statements in this connection was that the celestial object observed by him, near Theta Cancri appeared to have a disc "*decidedly greater than the spurious disc of a star*"—a description which, I think, coincides with that of the third hypothetical body aforesaid and that it was "*intensely ruddy*" and also somewhat brighter than the object seen by Swift—which facts indicate that the object observed by Watson was nearer to the Sun than those observed by Swift.

The positions of each of these planetary bodies at the time of observation was in that part of its orbit between the point of "superior conjunction," and that of the greatest western "elongation,"—the first body being at nearly one-third, and the other two at about one-half, of the distance between the two points, and all, therefore, well within the comparatively narrow limits between which according to my hypothesis stated on a preceding page—such bodies can be distinctly seen—they must be toward "superior conjunction," and yet at a considerable distance from the centre of the solar globe, before they are in position to distinctly impress themselves upon the organs of vision; they must be in the *superior* part of their orbits and in this connection it is worthy to note that Professor Watson remarked that the celestial object seen by him near Theta Cancri, seemed to be beyond the Sun. Consideration of all the facts stated above leads me directly to the conclusion that the two celestial objects observed by Swift were two intra-mercurial planetoids corresponding to the hypothetical bodies at mean distances 0.2145 and 0.1237 and that Watson's object near Theta Cancri was a like planetoid at mean distance 0.0713.

It should be noted that in this determination, based upon the

itarian end, has been at the basis of our progress in the application of knowledge. If in the last century such men as Galvani and Volta had been moved by any other motive than love of penetrating the secrets of nature they would never have pursued the seemingly useless experiments they did, and the foundation of electrical science would never have been laid. Our present applications of electricity did not become possible until Ohm's mathematical laws of the electric current, which when first made known seemed little more than mathematical curiosities, had become the common property of inventors. Professional pride on the part of our own Henry led him, after making the discoveries which rendered the telegraph possible, to go no further in the application of his discoveries, and to live and die without receiving a dollar of the millions which the country has won through his agency.

"In the spirit of scientific progress thus shown, we have patriotism in its highest form; a sentiment which does not seek to benefit the country at the expense of the world, but to benefit the world by means of one's country. Science has its competition, as keen as that which is the life of commerce. But its rivalries are over the question who shall contribute the most and the best to the sum total of knowledge, who shall give the most, not who shall take the most. Its animating spirit is love of truth. Its pride is to do the greatest good to the greatest number. It embraces not only the whole human race but all nature in its scope.

The public spirit of which this point is the focus has made the desert blossom as the rose, and benefited humanity by the diffusion of the material product of the Earth. Should you ask me how it is in the future to so use the influence thus acquired for the benefit of humanity at large, I would say, look at the work now going on in these precincts, and study its spirit. There are the agencies which will make 'the voice of law the harmony of the world.' Here is the love of country blended with the love of the race. Here the love of knowledge is as unconfined as your commercial enterprise. Do not think that the main object here is to learn the forms of vertebrates and the properties of oxides, but rather to imbibe that catholic spirit which, animating your energies, shall make your power an agent of beneficence to all mankind."

SOLAR ECLIPSES.—(*Concluded.*)

J. MORRISON, M. A., M. D., PH. D.

FOR POPULAR ASTRONOMY.

When all the circumstances of an eclipse are required, such for instance, as those given in Nautical Almanacs, Bessel's method is decidedly the best although it involves a very considerable amount of labor. The computation of the Besselian elements and of the auxiliary quantities ρ_1, ρ_2, d_1, d_2, a', b', c', e, f, E and F should be made with great care and with seven figure logarithms, except in the case of ρ_1 ρ_2 d_1 and d_2 where five figure tables will be sufficient. The Moon's parallax should be interpolated from the ephemeris to at least *two places* of decimals and should difference regularly and if it does not, it must be adjusted until it does do so, for any error here will be a disturbing factor throughout the entire computation. But when only a few of the circumstances of the eclipse are wanted—such as the dates and places of beginning or ending on the Earth generally, the place where the eclipse is central at any given time, where it is central at noon, etc.—the following method will be far more convenient, just as accurate and requires very little computation compared with the Besselian method.

An eclipse of the Sun first begins on the Earth generally when the Moon becomes tangent to the cone which envelopes the Sun and Earth. To the spectator this apparent contact will be in the horizon, the Moon's disk being wholly above and the Sun's below it, the point of contact and the centres of the Sun and Moon will evidently be in the same vertical circle and both bodies will be depressed by the horizontal parallaxes which at that time belong to that place. As the Sun's parallax is very small, it will be found most convenient to retain the Sun in his true place and to give to the Moon the effect of the difference of their parallaxes. This difference is that which influences the relative position of the two bodies and should be reduced to the place of observation, but since this is unknown, the least deviation from the truth will be obtained by reducing the difference of parallax to latitude 45°. If ρ_0 denote the radius of the Earth in latitude 45° and Π, the reduced relative parallax we shall have

$$\Pi = \rho_0 \, (\pi - \pi')$$

The places where the central eclipse (either total or annular) is first seen, is where the axis of the Moon's shadow first becomes

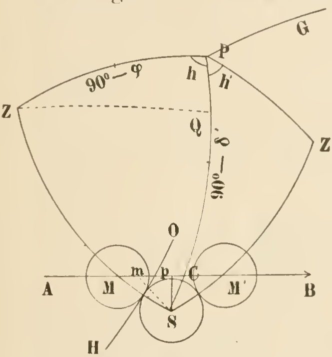
tangent to the Earth; the centres of the Sun and Moon will then be in the horizon.

Similar circumstances will evidently have place when the eclipse finally leaves the Earth. Let s and s' denote the semi-diameters of the Moon and Sun respectively and Δ, the angular distances between their centres as seen from the centre of the Earth, then it is evident we shall have the following limiting positions:

For beginning and ending on Earth generally	$\Delta = \Pi + s + s'$	
" " of total eclipse	$\Delta = \Pi + s - s'$	
" " of annular eclipse	$\Delta = \Pi + s' - s$	
" " of central eclipse	$\Delta = \Pi$	(131)

The eclipse is of course total when $s > s'$ and annular when $s < s'$, provided internal contacts exist.

Referring the Sun and Moon to the surface of a sphere concentric with the Earth, let AB be a portion of the Moon's relative orbit or that which is generated by the relative motion of the Moon; P, the north pole; S the Sun; Sp a perpendicular to the relative orbit or the line of near est approach of the centres; C the point where the Moon's comes into conjunction in R.A., CS being the difference of declination at that time; M and M' the positions of the Moon when the eclipse first begins and finally leaves the Earth, $MS = M'S = \Delta = \Pi + s + s'$, and Z and Z' the geocen tric zeniths of the places where the eclipse is first and last seen and which must be in the continuations of SM and SM' respectively. HO is an arc of the horizon of the place whose zenith is Z and the direction of motion is indicated by the arrowhead.



ZS or $Z'S = 90° + s'$ but for practical purposes may be taken equal to 90° without any appreciable error and the Moon being so near the horizon will not be sensibly affected by augmentation.

Let φ' and φ'_1 be the *geocentric* latitudes of Z and Z' and h and h' the hour angles ZPS, $Z'PS$ and at the time of conjunction let

a = relative motion in R. A. *i. e.*, ☽'s motion in R. A. — ☉'s
d = " Decl., " " Decl. "
D = Difference of decl. at ☌, *i. e.* ☽'s decl. — ☉'s decl.
i = inclination of relative orbit AB to a parallel of declination through C or the angle CSp.
v = angle MSp (this angle will be greater than 90° when D is negative)

Note.—The ′ mark on Z in the cut was omitted by engraver.

The relative motion in R. A. must be reduced to an arc of a great circle by multiplying it by the cosine of the Moon's declination and since the triangles pCS, MSp, etc., are very small, we can regard them as plane triangles whose sides are expressed in seconds of arc of a great circle.

From the diagram we easily deduce the following equations:

$$\tan i = \frac{d}{\alpha \cos \delta}, \qquad Sp = D \cos i$$

$$Cp = D \sin i, \qquad (132)$$

and relative hourly motion in orbit $= \dfrac{d}{\sin i}$.

The time of describing Cp, or the interval between the middle of the general eclipse and the time of conjunction, is found by dividing Cp by the relative hourly motion and denoting this interval by t we have

$$t = \frac{D}{d} \sin^2 i \qquad (133)$$

the sign of t will be determined by those of D and d. If T_m denote the time of the middle of the eclipse we have

$$T_m = \text{time of } \sigma - t \qquad (134)$$

We also have $$\cos v = \frac{Sp}{SM} = \frac{D \cos i}{\Delta} \qquad (135)$$

and $$Mp = D \cos i \tan v$$

and if t_1 denote the semi-duration of the eclipse or the time of describing Mp, we have

$$t_1 = \frac{D}{2d} \sin 2i \tan v \qquad (136)$$

and therefore, $$\begin{aligned} \text{Beginning} &= T_m - t_1 \\ \text{Ending} &= T_m + t_1 \end{aligned} \qquad (137)$$

on the Earth generally.

Let the angle $PSZ = S$ and $PSZ' = S'$, both being estimated from PS toward the east, then we shall have

$$S = (-i) - v \quad \text{and} \quad S' = (-i) + v \qquad (138)$$

and the spherical triangles PSZ, PSZ' give

$$\cos PZ = \cos PSZ \sin PS, \qquad \tan ZPS = -\frac{\tan PSZ}{\cos PS}$$

$$\cos PZ = \cos PSZ' \sin PS, \qquad \tan Z'PS = -\frac{\tan PSZ'}{\cos PS}$$

or $\sin\varphi' = \cos S \cos\delta'$, $\tan h = -\frac{\tan S}{\sin\delta'}$

$$\sin\varphi'_1 = \cos S' \cos\delta', \qquad \tan h' = -\frac{\tan S'}{\sin\delta'} \qquad (139)$$

The latitudes thus found are of course geocentric and are reduced to geographical latitudes by (70); the hour angles are measured from PS towards the east and if PG is the meridian of Greenwich, then SPG is the apparent time of beginning or ending and if λ and λ' be the east longitudes of the places and H and H' the Greenwich apparent times we shall evidently have

$$\lambda = h - H \text{ and } \lambda' = h' - H' \qquad (140)$$

Beginning and Ending of Central Eclipse.

In this case we put $\varDelta = \Pi$ according to (131) and then compute v, t_1, S and S' by (135) and (138) and then the geocentric latitude and the longitude by (139) and (140).

To find when and where the *total* eclipse first and last appears at sunrise or sunset, we must use $\varDelta = \Pi + s - s'$ for external contact and $\varDelta = \Pi - (s - s')$ for internal contact, and for an annular eclipse, $\varDelta = \Pi + s' - s$ and $\varDelta = \Pi - (s' - s)$ respectively.

To Find Where the Eclipse is Central (Total or Annular) at any Time Between the Dates of Beginning and Ending of the Central Eclipse on the Earth Generally.

Let m be the position of the Moon t hours before or after the time T_m or the middle of the eclipse, then

$$mp = \frac{dt}{\sin i}, \quad \tan v = \frac{2dt}{D \sin 2i} \qquad (141).$$

$$\varDelta_0 = \frac{D \cos i}{\cos v} = \frac{dt}{\sin i \sin v} \qquad (142)$$

$v > 90°$ when Sp is negative

and $$S = (-i) \mp v \qquad (143)$$

the upper sign for time before the middle and the lower for time after.

Now in order that an eclipse may be central at any place, the parallax in altitude must be such as will bring the centres of the Moon and Sun into apparent coincidence, and hence by the nature of parallax we have

$$\sin \Pi \sin ZS = \sin \varDelta_0$$

or
$$\sin z = \frac{\sin \Delta_0}{\sin \Pi} \tag{144}$$

where z denotes the zenith distance which is of course less than 90°, the Sun being now above the horizon.

In the spherical triangle PZS, we have $ZS = z$ just determined, $PS = 90° - \delta'$ and the angle S, to find the co-latitude PZ and the hour angle ZPS or h. Draw ZQ perpendicular to PS and put $\theta = QS$, then we shall have $\tan \theta = \tan z \cos S$.

$$\tan h = \frac{\sin \theta}{\cos (\theta + \delta')} \tan S \tag{145}$$

and
$$\tan \varphi' = \tan (\theta + \delta') \cos h$$

θ less than 90° and with the same sign as $\cos S$, h in same quadrant as S and δ' the Sun's declination at conjunction.

A check on this formula is easily derived thus

$$\frac{\sin \theta}{\cos (\theta + \delta')} = \frac{\cos S}{\cos h} \cdot \frac{\sin z}{\cos \varphi'},$$

in which all the quantities are involved and which will be satisfied only when all are correct. Finally if H be the Greenwich apparent time we have

$$\lambda = h - H \tag{146}$$

For a more accurate determination at any time, reduce the difference of parallaxes to the latitude just found, that is correct the value of Π and recompute as before.

Central Eclipse at Noon.

During the course of the general central eclipse some place will have the eclipse central at apparent noon. At this time the Sun and Moon will evidently have true as well as apparent conjunction in R. A. and $S = 0$.

The parallax in altitude will bring the centres into apparent coincidence and we shall have

$$\sin z = \frac{\sin D}{\sin \Pi'}$$

and
$$\varphi' = \delta' + z \tag{147}$$

z to have the same sign as D. Apparent time of ☌ $= \lambda$ (west longitude of place).

We will now make an application of these formulæ (131) to (147) in the case of the total eclipse of January, 1898, using the elements as given in the *American Ephemeris*, which are as follows:

Greenwich M. T. of ☌ in R. A. January 21^d			19^h	37^m	$26^s.5$			
⊙ and ☽'s R.A. 20^h 18^m $32^s.82$, Hourly motions $10^s.52$ and $147^s.72$								
⊙'s Declination	$-19°$	$38'$	$40''.1$	Hourly motion	$+$	$0'$	$34''.5$	
☽'s "	-19	6	$27\ .1$	" "	$+11$	34	$.0$	
⊙'s Equa. Hor. Par.			$8\ .9$	⊙'s semi-diam.	16	15	$.0$	
☽'s " " "	1	0	$11\ .6$	☽'s " "	16	23	$.3$	

From these data we have

$$a = 2058'', \qquad d = +659''.5, \qquad D = +32'\ 13''$$
$$\pi - \pi' = 60'\ 2''.7, \qquad \rho_0\,(\pi - \pi') = 3596''.62, \qquad \Delta = 5554''.9$$

and by (132) we deduce the following results which may be verified by the reader:

$$i = 18°\ 44'\ 2''.1, \qquad Sp = +1830''.59, \qquad Cp = +620''.82$$
$$t = +\ 18^m\ 8^s.42, \qquad T_m = 19^h\ 19^m\ 18^s.1$$
$$v = 70°\ 45'\ 31''.8, \qquad t_1 = 2\ \ 33\ \ 14\ .7$$

Beginning	$=16$	46	$3\ .4$
Ending	21	52	$32\ .8$

These results differ from those in the ephemeris by only $0^m.16$ and $0^m.05$ respectively.

By (138) we have $\quad S = -89°\ 29'\ 33''.9$
and $\quad S' = +52\ \ 1\ \ 29\ .7$

By (139) we have

$\cos S = 7.9470938$	$\cos S' = 9.7890998$
$\cos \delta' = 9.9739572$	$\cos \delta' = 9.9739572$
$\sin \varphi = 7.9210510$	$\sin \varphi' = 9.7630570$
$\varphi' = +0°\ 28'\ 39''.8$	$\varphi' = +35°\ 24'\ 56''.6$
$\tan \varphi' = 7.9210661$	$\tan \varphi'_1 = 9\ 8519160$
$\log(1 - e^2)\ \ 9.9970502$	$\log(1 - e^2)\ \ 9\ 9970502$
$\tan \varphi\ \ 7.9240159$	$\tan \varphi_1\ \ 9.8548658$
$\varphi + 0°\ 28'\ 51''\ 5$	$\varphi_1 + 35°\ 35'\ 59''$

The ephemeris gives $+\,0°\ 28'.9$ and $+\,35°\ 36'.9$

If we now reduce the difference of the horizontal parallaxes to these latitudes and repeat the computation we shall find the agreement practically complete.

For the longitude we have by (139) and (140)

$-\tan S = 2\ 0528892$	$-\tan S'\ \ 0.1075785n$
$\sin \delta' = 9\ 5265748n$	$\sin \delta'\ \ 9.5265748n$
$\tan h = 2.5263144n$	$\tan h'\ \ 0.5810037$
$h = -89°\ 49'\ 46''.08$	$h'\ \ 75°\ 17'\ 45''.6$

The Greenwich apparent time of beginning is January 21, 16^h 34^m 15^s and for ending 21^h 40^m 41^s, therefore $H = 248°\ 33'\ 45''$ and $H' = 325°\ 10'\ 15''$

then	$\lambda = h - H$	and $\lambda' = h' - H'$
	$= - 338° \ 23' \ 31''$	$= - 249° \ 52' \ 29''.4$
or	$\lambda = 21° \ 36' \ 29''$ East	or $\lambda' = 110° \ 7' \ 30''.4$ East
ephemeris gives	21 38.7	and 110 4.0
difference	$2^m.2$	$3^m.5$

If, however, we proceed to the second approximation we shall obtain almost identical results.

For the beginning and end of central eclipse we have by (131) $\Delta = \rho_0 (\pi - \pi')$, and by (135) and (136) we have

$$v = 59° \ 24' \ 15''.7 \qquad \text{and } t_1 = 1^h \ 30^m \ 28^s.4$$

and therefore central eclipse begins $17^h \ 48^m \ 49^s.7$
and ends 20 49 46 .5

By (138) $S = - 78° \ 8' \ 17''.8$, and $S' = + 40° \ 40' \ 13''.6$
and (139) gives

$$\varphi = + 11 \ \ 14 \ \ 11 \qquad h = - 85° \ 57' \ 40''$$
$$\varphi_1 = + 45 \ \ 46 \ \ 57 \qquad h' = + 68 \ \ 37 \ \ 54$$

The Greenwich apparent times of beginning and end of the central eclipse reduced to arc are respectively

$$H = 264° \ 11' \ 9'' \qquad \text{and } H' = 309° \ 28' \ 50''$$

hence $\lambda = 9° \ 47'.2$ east and $\lambda' = 119° \ 9'.1$ east

To Find Where the Eclipse is Central at Say $20^h \ 20^m$ Greenwich Mean Time.

This date is $1^h \ 0^m \ 42^s$ after the middle of the eclipse, therefore $t = + 1.0116 +$ and by (141), (142), (143) and (144) we easily find $v = 48° \ 36' \ 47''.6$, $\log \Delta_0 = 3.44230$ $S = + 29° \ 52' \ 45''.5$ and $z = 50° \ 20' \ 28''$ and then by (145) and (146) we have

$$\theta = 46° \ 17' \ 9''.3 \qquad h = 24° \ 55' \ 11''.6$$
$$H = 302° \ 2' \ 15'' \qquad \varphi = + 24° \ 36' \ 38''.5$$

and $\lambda = 82° \ 52' \ 56''$ east, which point is near the western coast of India.

Central Eclipse at Apparent Noon.

By (147) we have, since $D = + 32' \ 13''$ and $\Pi = 1° \ 0' \ 2''.1$ (the difference of the horizontal parallaxes reduced to latitude 13° instead of 45°)

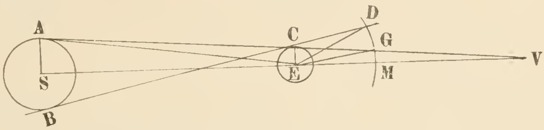
	$z = 32° \ 27' \ 21''$
and	$\varphi' = \delta' + z = + 12° \ 48' \ 41''$
reduction to geograph. latitude	$+ \qquad 5 \ \ 1$
	$\varphi = + 12° \ 53'.7$ See (70).
We also have	$H = \lambda = 291° \ 24' \ 3''$ west

To compute the circumstances of an eclipse for a given place by this method, it would be necessary to compute the parallax in

R. A. and Decl. or the apparent places, then by means of the relative hourly motion in apparent orbit to determine when the apparent distance between the centres, is equal to the sum of the semi-diameters plus the augmentation of the Moon's semi-diameter, all of which involves a considerable amount of labor and is very well explained in detail in Loomis's Practical Astronomy and other similar works to which the reader is referred for further information on this method.

Lunar Eclipses.

The apparent semi-diameter of the Earth's shadow where the Moon crosses it is evidently equal to the sum of the horizontal parallaxes minus the Sun's semi-diameter, for in the diagram we have



Semi-diameter of the umbra $= GEM$

$$\begin{aligned} &= CGE - EVG \\ &= CGE - (AES - EAV) \\ &= \pi - (s' - \pi') \\ &= \pi + \pi' - s' \end{aligned}$$

For the semi-diameter of the penumbra we have

$$\begin{aligned} DEM &= GEM + DEG \\ &= GEM + DCG \text{ or } ACB \\ &= \pi + \pi' - s' + 2s' \\ &= \pi + \pi' + s' \end{aligned}$$

If L denote the distance between the centres of the Moon and shadow we shall evidently have for first and last contacts with the penumbra

$$L = (\pi + \pi' + s') + s \qquad (148)$$

For first and last contacts with the umbra

$$L = (\pi + \pi' - s') + s \qquad (149)$$

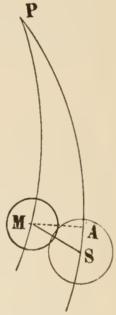
For first and last internal contacts with the penumbra

$$L = (\pi + \pi' + s') - s \qquad (150)$$

and for the first and last internal contacts with the umbra

$$L = (\pi + \pi' - s') - s \qquad (151)$$

It is necessary to increase the radii of the umbra and penumbra by about $\frac{1}{50}$ or $\frac{1}{60}$ of their values by reason of the effect of the Earth's atmosphere. Let S be the geocentric position of the centre of the Earth's shadow which is directly opposite to that of the Sun; M the geocentric position of the Moon and P the north pole.



Put α = R. A. of the Moon.
α' = R. A. of S.
= R. A. of the Sun + 180°.
δ = Declination of the Moon.
δ' = " of the Sun.
Q = the angle PSM.
and $L = MS$.
then $-\delta'$ = Declination of S.
and $\alpha - \alpha'$ = angle MPS.

From the spherical triangle PMS we have

$$\begin{aligned} \sin L \sin Q &= \cos\delta \sin(\alpha - \alpha') \\ \sin L \cos Q &= \cos\delta' \sin\delta + \sin\delta' \cos\delta \cos(\alpha - \alpha') \end{aligned} \qquad (152)$$

The eclipse will begin or end when L has the values given by (148)—(151). As the section of the shadow will differ slightly from a circle by reason of the spheroidal figure of the Earth it will be sufficiently accurate to regard the Earth as a sphere with the radius of latitude 45°, which will be equivalent to reducing the equatorial horizontal parallax of the Moon or π to latitude 45, that is, by writing $\rho_0\pi$ for π where $\log \rho_0 = 9.999266$. Owing to the indefinite outline of both the umbra and penumbra, an accurate computation is unnecessary and therefore we may, instead of, (152) use the following approximate formulæ easily deduced from them:

$$\begin{aligned} L'' \sin Q &= (\alpha - \alpha')'' \cos\delta \\ L'' \cos Q &= (\delta + \delta')'' - \frac{\sin 2\delta \sin^2 \frac{1}{2}(\alpha - \alpha')}{\sin 1''} \end{aligned} \qquad (153)$$

The term

$$\frac{\sin 2\delta \sin^2 \frac{1}{2}(\alpha - \alpha')}{\sin 1''}$$

which we will represent by β, can never exceed 30″ and may generally be omitted; its value however can be readily computed or taken from a table of double entry, in which δ and $(\alpha - \alpha')$ are arguments. Such a Table is given in Loomis's *Practical Astronomy*, Table XVIII, page 385, 1st Ed.

If we take S as the origin of a system of rectangular co-ordinates and draw MA perpendicular to PS, we evidently have

$$\begin{aligned} L \sin Q &= x = (\alpha - \alpha') \cos \delta = AM \\ L \cos Q &= y = (\delta + \delta') - \beta = AS \end{aligned} \tag{154}$$

Now if x and y be computed for several hours preceding and following opposition we shall obtain x' and y' their hourly variations and as in solar eclipses, taking T_0 any assumed epoch near opposition and denoting by x_0 and y_0 the values of x and y for this epoch we shall have for the time of contact $T = T_0 + \tau$, the equations

$$\begin{aligned} L \sin Q &= x_0 + x'\tau \\ L \cos Q &= y_0 + y'\tau \end{aligned} \tag{155}$$

from which τ can easily be obtained by processes already explained

Put $\qquad m \sin M = x_0 \qquad n \sin N = x'$

$\qquad\qquad m \cos M = y_0 \qquad n \cos N = y'$

then we get

$$\sin \psi = \frac{m \sin (M - N)}{L}$$

$$\tau = \frac{L \cos \psi}{n} - \frac{m \cos (M - N)}{n} \tag{156}$$

and $\qquad T = T_0 + \tau$

where $\cos \psi$ is negative for beginning and positive for the end.

The angle $Q = N + \psi$ is very nearly the supplement to PMS and hence the angle of position of the point of contact, reckoned on the Moon's limb from the north toward the east is $N + \psi + 180^\circ$.

The time of middle of eclipse

$$\text{or } T_m = T_0 - \frac{m}{n} \cos (M - N) \tag{157}$$

and for the least distance D between the centres we have

$$D = m \sin (M - N) \tag{158}$$

and if M denote the magnitude of the eclipse we shall evidently have

$$M = \frac{L - D}{2s}, \text{ where } L \text{ is given by } (149).$$

STATIONS FOR OBSERVING THE TOTAL ECLIPSE OF THE SUN IN JANUARY, 1898

The land path of the line of the total eclipse of the Sun commences from a little south of Ratnagiri, on the Bombay coast, and runs in a north-easterly direction to Nepal, passing nearly over Mount Everest, and then disappears in Thibet. The shadow of the Moon will therefore pass through parts of the Bombay Presidency, through Hyderabad, Berars, Central Provinces, and parts of Central India, Bengal, and Northwest Provinces. The length of the path through India is about a thousand miles, and the width of the shadow roughly fifty miles. Hence the area from which observations *could* be taken is enormous. In India, however, facilities for travelling simply do not exist at all over by far the greater part of the country; and as accommodation for European travellers is even more scanty than the means of transport, the number of stations from which observations of the forthcoming eclipse are likely to be made is much smaller than would be expected. As the duration of the total phase of the eclipse on the central line decreases from about two minutes ten seconds on the Bombay coast, to about one minute forty seconds in parts of Bengal and the Northwest Provinces, the natural tendency will be for observers to prefer the western stations. In addition, too, it would appear that the meteorological conditions are more favorable at the western than at the eastern or central stations on the line of totality.

The majority of travellers visiting India for the purpose of observing or seeing the total eclipse will land either at Bombay or Calcutta, probably at the former station. From Bombay several parts of the line of totality can be comfortably visited. The stations on the Bombay coast can be very easily reached by the local steamers of the Bombay Steam Navigation Company. As at present arranged, there is a daily passenger steamer to and from Bombay, calling at such ports as Ratnagiri and Viziadurg, which are close to the central line, and at Jaygad, which is close to the north limit of the line of totality, and at Dewgad, which is just within the southern limit. Combined passenger and cargo steamers also leave Bombay for some of these ports twice or three times weekly. The journey only takes from twelve to eighteen hours each way. The fares are cheap, the first-class fare from Bombay to Ratnagiri being seven rupees; the second, two rupees; and the third, one rupee four annas. It should be remembered that these fares do not include food; also that the

Stars." This title was given to him by the soldiers as soon as they learned he was an astronomer. An amusing incident shows how his soldiers regarded him, as an astronomer as well as a general:

"In camp at Bacon Creek, Ky., on a very dark and stormy winter night, the guard was placed as usual, but along about three o'clock had grown careless—more anxious to find shelter than to note everything that was stirring. Suddenly those of us who were 'off duty' received a startling surprise. The men on watch had permitted somebody to come up to our post unchallenged, and we knew nothing of it until this person was in our midst, seizing the soldiers in no gentle manner by arm and collar, and shaking them or tumbling them out of the guard tent, as he exclaimed, 'Why don't you turn out the guard?' Some of the soldiers were for resisting; but all were submissive enough when the word passed around, 'It's old Mitchel himself.'

We were very soon in our places, and then we listened to a lecture, as we stood in the rain, *not* on the subject of astronomy! When the general was gone, the soldiers grumbled, and wished they had an officer 'who had not studied the stars so long, that he could not sleep at night himself and would not let anybody else sleep!' But we resolved not to be caught in the same way again; and we never were!

We now knew, in our division, that the only way to get along in peace with our commander was to faithfully perform every part of military duty.

We exercised the soldier's prerogative in grumbling, but we loved and trusted him for all that, and would have followed him to Mobile or Savannah without hesitation, assured that he would have carried safely through what he undertook."

Liberal, Barton Co., Missouri.

OCCULTATIONS.

J. MORRISON, M. A., M. D., PH. D.

For Popular Astronomy.

An occultation of a fixed star is a total eclipse of it by the Moon, and is computed in the same manner as an eclipse of the Sun, but owing to the fact that the star has neither parallax nor semi-diameter the formulæ become considerably simplified. The shadow is a cylinder with a constant radius equal to k which is therefore to

be substituted for $L = l - i\zeta$ in Eq. (66); the coördinates of the point Z become the R. A. and Decl. of the star and z and ζ are not in general required.

Let α' and δ' denote the R. A. and Decl. of the star, then by (57) we have, r being equal to $\frac{1}{\sin \pi}$,

$$x = \frac{\cos \delta \sin (\alpha - \alpha')}{\sin \pi}$$
$$y = \frac{\sin \delta \cos \delta' - \cos \delta \sin \delta' \cos (\alpha - \alpha')}{\sin \pi} \tag{159}$$

and at the moment of geocentric conjunction $\alpha = \alpha'$, the above become

$$x = 0$$

and
$$Y = \frac{\sin (\delta - \delta')}{\sin \pi} \tag{160}$$

For the coördinates of the place of observation we shall have from Eq. (60)

$$\xi = \rho \cos \varphi' \sin (\mu - \alpha')$$
$$\eta = \rho \left(\sin \varphi' \cos \delta' - \cos \varphi' \sin \delta' \cos (\mu - \alpha') \right) \tag{161}$$

where μ is the sidereal time and therefore $\mu - \alpha'$ is the star's hour angle. The fundamental equation (67) becomes

$$(x - \xi)^2 + (y - \eta)^2 = k^2 \tag{162}$$

which of course can be expressed in the form

$$k \sin Q = x - \xi$$
$$k \cos Q = y - \eta$$

To obtain the variations in a unit of time, of x, y, ξ and η, at the time of conjunction we have only to differentiate Eqs. (159) and (161) and put $\alpha = \alpha'$, $\sin \pi$ being regarded as constant; thus we have

$$\frac{dx}{dT} \text{ or } x' = \frac{1}{\sin \pi} \left(\cos \delta \frac{d\alpha}{dT} \sin 1'' \right)$$
$$\frac{dy}{dT} \text{ or } y' = \frac{1}{\sin \pi} \left(\cos (\delta - \delta') \frac{d\delta}{dT} \sin 1'' \right) \tag{163}$$

and

$$\frac{d\xi}{dT} \text{ or } \xi' = \rho \cos \varphi' \cos (\mu - \alpha') \frac{d\mu}{dT} \sin 1''$$

$$\frac{d\eta}{dT} \text{ or } \eta' = \rho \cos \varphi' \sin \delta' \sin (\mu - \alpha') \frac{d\mu}{dT} \sin 1'' \tag{164}$$
$$= \xi \sin \delta' \frac{d\mu}{dT} \sin 1''$$

If one hour be taken as the unit of time

$$\frac{d\mu}{dT} \sin 1'' = 54147''.84 \sin 1'' = \mu'$$

and $$\log \mu' = 9.419156$$

and if one minute be the unit, then

$$\log \mu' = 7.641005$$

For any time τ we shall have

$$\begin{aligned} x &= x'\tau \\ y &= Y + y'\tau \end{aligned} \qquad (165)$$

THE ELEMENTS OF OCCULTATIONS.

To facilitate the computation of occultations at any given place certain quantities called elements are given in the American Ephemeris which we now proceed to explain.

In the columns refering to the star, those headed $\varDelta\alpha$ and $\varDelta\delta$ are the corrections in R. A. and Decl., to be applied to the mean R. A. and Decl. at the beginning of the year to obtain the apparent position at the date of the occultation. The computation of these quantities which are the results of precession, nutation and aberration, has been explained in a former paper. The Washington mean time of conjunction is equal to the Greenwich time of conjunction minus the longitude of Washington. On pages V to XII inclusive of each month is given the R. A. of the Moon for every hour of Greenwich time and a mere inspection will show the hours between which geocentric conjunction will take place. For some two or three hours before and after conjunction subtract the Moon's R. A. from the star's and let the results which we denote by f, be arranged as follows, and let $\varDelta_1$, $\varDelta_2$ and $\varDelta_3$ be the 1st, 2d, &c., differences:

DATE.	f	$\varDelta_1$	$\varDelta_2$	$\varDelta_3$
T_{-2}	a_{+2}			
		b		
T_{-1}	a_{+1}		c	
		b_1		d
T_0	a_0		c_1	
		b_2		d_1
T_1	a_{-1}		c_2	
		b_3		
T_2	a_{-2}			

Now we have to find when this variable function becomes zero. If conjunction takes place between the dates T_{-1} and T_0 for instance and $a^{(t)}$ be the value of the function at the date t, we shall have by the ordinary formulæ of interpolation

$$a^{(t)} = a_1 + tb_1 + \frac{t\,(t-1)}{2}\,c_0 + \text{etc.} = 0 \qquad (166)$$

where $c_0 = \frac{1}{2}(c + c_1)$, and solving for t, neglecting the square, we get

$$t = \frac{2a_1}{c_0 - 2b_1}$$

Therefore the date of conjunction is $T_{-1} + t$ and as a check the Moon's R. A. computed for this state must be equal to α', and if not, a simple interpolation will be necessary to make them so. For the Washington mean time of conjunction thus found, we find the sidereal time μ, then the hour angle $H = \mu - \alpha'$, positive when west, negative when east.

The quantities Y, x' and y' are found from equations (160) and (163). The limiting parallels in the last two columns will be explained hereafter.

We are now prepared to explain the computation of an occultation for any place where it is visible.

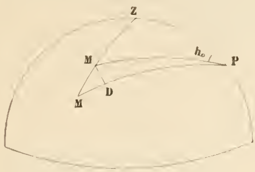
If λ be the west longitude of the place we shall have for the local hour angle

$$h_0 = H - \lambda.$$

The next step is to find the time of *apparent* conjunction, which will be near the middle of the occultation; we can then assume two dates equally distant from the time of apparent conjunction, for the approximate times of immersion and emersion. By reason of the Moon's parallax in R. A. the time of apparent conjunction will be accelerated or retarded according as the hour angle is east or west of the meridian, and the amount of this acceleration or retardation is found by dividing the parallax in R. A. by the Moon's variation (per minute or hour) in R. A. as given on pages V to XII of each month.

In the diagram let P be the north pole, Z the zenith of the place, M the Moon's true place and M' its apparent place when in apparent conjunction with the star; then MPM' is the parallax in R. A. which we denote by Π; ZPM is the Moon's true local hour angle h_0 just found, MM' the parallax in altitude, $ZP = 90 - \varphi'$ and $PM = 90° - \delta$.

Draw the arc MD perpendicular to PM', then we shall have



$$\sin \Pi = \frac{\sin MD}{\sin PM}$$

$$= \frac{\sin MM' \sin ZM'P}{\sin PM}$$

$$= \frac{\sin \pi \sin ZM' \sin ZM'P}{\sin PM}$$

$$= \frac{\sin \pi \sin ZP \sin ZPM'}{\sin PM}$$

$$= \frac{\sin \pi \cos \varphi' \sin (h_0 + \Pi)}{\cos \delta} \quad (167)$$

But since Π is always small and we require only an approximate value, it will be practically sufficient to use

$$\sin \Pi = \frac{\sin \pi \cos \varphi' \sin h_0}{\cos \delta}$$

or expressing Π in seconds of arc

$$\Pi'' = \frac{\sin \pi \cos \varphi' \sin h_0}{\cos \delta \sin 1''} \quad (168)$$

Now if we denote by $\frac{d\alpha}{dT}$ the Moon's variation in R. A. in one minute of mean time as given in the ephemeris and by τ the acceleration or retardation in minutes we shall evidently have

$$\tau = \frac{\Pi}{\frac{d\alpha}{dT}} = \frac{\sin \pi \cos \varphi' \sin h_0}{\cos \delta \frac{d\alpha}{dT} \sin 1''}$$

$$= \frac{\cos \varphi' \sin h_0}{x'} \quad \text{by (163)} \quad (169)$$

The sign of τ will be the same as that of h_0, since $\cos \varphi'$ and x' are always positive.

If T be the time of geocentric conjunction then the date of apparent conjunction will be $T + \tau$ and the apparent hour angle at the place $h_0 + \tau$.

Eq. (169) furnishes the means of computing Downes's Table which gives the values of τ with x' and h_0 as arguments, and for every 6° of latitude from 0° to 72°. It is a Table of double entry, the extreme and mean values of x' at the top, and h_0 at the left hand side of the page.

For intermediate values of the Lat, and of x' and h_0 a troublesome interpolation will be required, but it will be far more convenient to compute τ directly from (169).

Assuming the average duration of an occultation to be one hour, if we subtract and add $0^h.5$ to $T + \tau$ we shall have approximate dates for the computation of immersion and emersion.

Accordingly if we put $\tau_1 = \tau - 0^h.5$
and $\tau_2 = \tau + 0\ .5$

the approximate time of immersion $= T + \tau_1$
" " emersion $= T + \tau_2$

and, the corresponding hour angles $h_0 + \tau_1$ and $h_0 + \tau_2$ which of course must be expressed in arc.

Therefore Eqs. (161), (164) and (165) now become

$$\begin{aligned}
\xi &= \rho \cos \varphi' \sin (h_0 + \tau_0) \\
\eta &= \rho \left(\sin \varphi' \cos \delta' - \cos \varphi' \sin \delta \cos (h_0 + \tau_0)\right) \\
\xi' &= [9.419156]\ \rho \cos \varphi' \cos (h_0 + \tau_0) \\
\eta' &= [9.419156]\ \xi \sin \delta' \\
x &= x'\tau_0 \\
y &= Y + y'\tau_0 \quad \text{where } \tau_0 \text{ stands for } \tau_1 \text{ or } \tau_2
\end{aligned} \tag{170}$$

Having computed $x, y, x', y', \xi, \eta, \xi'$ and η' for the approximate dates $T + \tau_1$ and $T + \tau_2$ put as in the case of eclipses

$$m \sin M = x - \xi \qquad n \sin N = x' - \xi'$$
$$m \cos M = y - \eta \qquad n \cos N = y' - \eta'$$

then we shall have

$$\sin \psi = \frac{m \sin (M - N)}{k}, \qquad k = 0.27227$$

and $$\tau = \frac{k \cos \psi}{n} - \frac{m \cos (M - N)}{n} \tag{171}$$

and $$T_1 = T \pm \tau$$

in which $\cos \psi$ is negative for immersion and positive for emersion.

For a second approximation, recompute for the dates just found which will frequently be considerably in error especially if the duration of the occultation be only a few minutes.

In solar eclipses the position angle was referred to the Sun's centre but in occultations it must be referred to the Moon's, and will therefore be equal to $Q + 180°$, that is

$$P = N - \psi \text{ for immersion}$$
$$P = N + \psi + 180° \text{ for emersion.}$$

If in any case $\sin \psi > 1$, the star can not be occulted at the given place.

Limiting Parallels.

At any time $T_1 = T_0 + \tau_0$, the least distance from a point on the limiting curve to the axis of the cylinder is $k = 0.27227$, the Earth's

radius being unity. Then as in the case of eclipses we shall have for this date the equations

$$\begin{aligned} k \sin Q &= x - \xi + (x' - \xi')\tau_0 \\ k \cos Q &= y - \eta + (y' - \eta')\tau_0 \end{aligned} \qquad (172)$$

and putting

$$\begin{aligned} m \sin M &= x - \xi \qquad & n \sin N &= x' - \xi' \\ m \cos M &= y - \eta \qquad & n \cos N &= y' - \eta' \end{aligned} \qquad (173)$$

the above become after reduction

$$\begin{aligned} k \sin (Q - N) &= m \sin (M - N) \\ k \cos (Q - N) &= m \cos (M - N) + n\tau_0 \end{aligned}$$

the sum of whose squares, is

$$k^2 = m^2 \sin^2 (M - N) + [m \cos (M - N) + n\tau_0]^2$$

which will be a minimum when the last term is zero or when

$$\tau_0 = - \frac{m \cos (M - N)}{n}$$

and therefore $\qquad k = \pm\, m \sin (M - N)$

Expanding this equation and restoring the values of $m \sin M = x - \xi$ and $m \cos M = y - \eta$, and also neglecting the changes of ξ and η for the present purpose, that is, taking $n \sin N = x'$ and $n \cos N = y'$, we obtain

$$(x - \xi) \cos N - (y - \eta) \sin N = \pm k \qquad (174)$$

Now if x_0 and y_0 be the values of x and y for the assumed epoch T_0 which may be the time of geocentric conjunction, we shall have for any other time $T_1 = T_0 + \tau_0$, the following values of x and y, viz:

$$x = x_0 + n \sin N \,.\, \tau_0 \qquad \text{and} \qquad y = y_0 + n \cos N \,.\, \tau_0$$

and the Eq. (174) becomes for this epoch

$$(x_0 - \xi) \cos N - (y_0 - \eta) \sin N = \pm k \qquad (175)$$

Taking the Earth as a sphere, the last of Eq. (79) is

$$\sin \varphi = \eta \cos \delta' + \zeta \sin \delta' \qquad (176)$$

and the last of Eq. (78) is

$$\xi^2 + \eta^2 + \zeta^2 = 1 \qquad (177)$$

The problem is now reduced to finding the maximum and minimum values of φ which fulfill the conditions expressed by these last three equations.

The solution is obtained by the following peculiar and elegant transformation due to Bessel.

Expanding (175) we have after transposing

$$-\xi \cos N + \eta \sin N = - x_0 \cos N + y_0 \sin N \pm k$$

Put $\qquad a = -\xi \cos N + \eta \sin N$

and take $\qquad b = \xi \sin N + \eta \cos N \quad$ so that $\xi^2 + \eta^2 = a^2 + b^2$

and solving for ξ and η we have

$$\begin{aligned} \xi &= -a \cos N + b \sin N \\ \eta &= a \sin N + b \cos N \end{aligned}$$

and (177) becomes $1 = a^2 + b^2 + \zeta^2 \qquad (178)$

where $\qquad a = - x_0 \cos N + y_0 \sin N \pm k$, a *constant* quantity, since we may assume x' and y' to be constant, $\tan N = \dfrac{x'}{y'}$ and N to be taken less than 90° and positive.

The last of (178) can be put in the following form by assuming A and χ so as to satisfy the equations

$$\begin{aligned} \cos A &= a \\ \sin A \cos \chi &= b \\ \sin A \sin \chi &= \zeta \end{aligned} \qquad (179)$$

where $\sin A$ is restricted to positive values.

Substituting in the second of (178) we get

$$\eta = \cos A \sin N + \sin A \cos \chi \cos N$$

and substituting these values of η and ζ in (176) we have

$$\sin \varphi = \cos A \sin N \cos \delta' + \sin A \cos \chi \cos N \cos \delta' + \sin A \sin \chi \sin \delta' \qquad (180)$$

Again assume

$$\begin{aligned} \sin B &= \sin N \cos \delta' \\ \cos B \cos \theta &= \cos N \cos \delta' \\ \cos B \sin \theta &= \sin \delta' \end{aligned} \qquad (181)$$

then $\qquad \sin \varphi = \cos A \sin B + \sin A \cos B \cos (\theta - \chi) \qquad (182)$

in which the only variables are φ and χ, for A, B and θ are known from (179) and (181).

Since $\cos A \sin B$ is positive, $\sin \varphi$ is a maximum

when $\cos (\theta - \chi) = 1 \qquad$ or when $\theta - \chi = 0$

and a minimum when $\cos (\theta - \chi) = -1$ or when $\theta - \chi = 180°$.

Therefore for the limits we have $\quad \sin \varphi = \sin (B \pm A)$
that is, for the northern limit $\quad \varphi = B + A$
southern " $\quad \varphi = B - A$

Since ζ is always positive $\sin \chi$ is positive, and when $\theta = \chi$, $\sin \theta$ must also be positive when $\sin \delta'$ is positive. Therefore the formula

$$\varphi = B + A$$

gives the northern limit of visibility only when the star has north

declination and for similar reasons, $\varphi = B - A$ gives the southern limit only when the star has south declination.

The other limit in each case must evidently be one of the points in which the northern or southern limiting curve intersects the rising and setting limits or when $\zeta = 0$ which according to (179) gives $\sin \chi = 0$ and $\cos \chi = \pm 1$ and these substituted in (180) give

$$\sin \varphi = \sin (N \pm A) \cos \delta'$$

The upper sign gives the northern limit when $\varphi = B - A$ gives the southern and the lower sign gives the southern limit when $\varphi = B + A$ gives the northern.

Now the epoch T_0 for which x_0 and y_0 are taken, is arbitrary and may be the time of geocentric conjunction, when $x_0 = 0$ and $y_0 = Y$, given in the ephemeris, then we shall have

$$\begin{array}{llll} & \cos A_1 = a & A < 180^\circ & \\ & \quad\;\;\, = Y \sin N \pm k & & \\ \text{and} & \sin B = \sin N \cos \delta' & B < 90^\circ & \\ & \varphi_1 = B \pm A_1 & & (183) \\ & \cos A_2 = Y \sin N \mp k & & \\ & \sin \varphi_2 = \sin (N \mp A_2) \cos \delta' & N < 90^\circ & \end{array}$$

Use the upper sign when Decl. of star is north, φ_1 will then be the northern and φ_2 the southern limit.

Use the lower sign when Decl. of star is south, φ_1 will then be the southern and φ_2 the northern limit.

If the shadow extends beyond the Earth, $\cos A_1$ and $\cos A_2$ will have imaginary values; in the former case the occultation is visible beyond the elevated pole and therefore the extreme limits will be $\varphi_1 = +90^\circ$ or -90° according to the sign of δ', and in the latter the value of φ_2 will be the latitude of the point nearest the depressed pole and then $\varphi_2 = \delta' - 90^\circ$ or $\varphi_2 = \delta' + 90^\circ$ according as δ' is positive or negative.

It sometimes happens that the numerical value of $\varphi_1 = B \pm A_1$ in (183) exceeds 90°, in which case $\varphi_1 = 180^\circ - (B \pm A_1)$ or $\varphi_1 = -180^\circ - (B \pm A_1)$, these having the same sign.

Since the limiting curves cannot coincide with the parallels of latitude but intersect the meridians at various angles, an occultation is not visible at all places between the extreme latitudes. In most cases a special computation will be necessary to determine whether or not the occultation will be visible at any given place, and especially is this so when the place is near the limiting curves which are only approximations at best and may be in error a degree or even more.

In the occultations of planets, we proceed in the same manner, using the relative parallax and taking the semi-diameter of the planet into account as in the case of eclipses but the shadow may still be regarded as a cylinder of constant radius.

LIST NO. 3 OF NEBULÆ DISCOVERED AT THE LOWE OBSERVATORY FOR 1900.0

LEWIS SWIFT.

For Popular Astronomy.

List No. 1 of 50 nebulæ discovered here was published in *Gould's Astronomical Journal* of Nov. 13, 1896. List No. 2 of 25 was recently published in *Monthly Notices* and in *Publications of the Astronomical Society of the Pacific*. This one as will be seen consists wholly of southern nebulæ. It is a field rich in nebulæ which that "mighty Nimrod" Sir Wm. Herschel who hunted the sky over could not reach. Several are quite bright and a few are interesting.

I have examined Gale's ring nebula, R. A. $21^h 53^m 10^s$ Decl. — 39° 53′ 42″ and find it an interesting one, increasing the number now known to seven. It bears considerable resemblance to the one between Beta and Gamma Lyræ, but is not as bright, nor will it bear magnifying like that celebrated one, though it is too far south for me to do justice to it. It is remarkable that it escaped detection so long. Numbers 1 and 6 are singular specimens of nebulæ, perhaps deserving of a new classification. I have lately seen three, all looking exactly alike. N. G. C. 1288 is considerably elongated in meridian. It is not round as Sir J. Herschel says. N. G. C. 1340 must be struck out, it is identical with 1344 as has been suspected. I examined the locality thoroughly for 1340 and I am certain that it does not exist. Sometime I intend to take up this matter of doubtful nebulæ. I am glad I have at length found in Barnard's field a nebula his keen eye failed to see. See No. 24.

No.	Date.	Right Ascension h m s	Declination. ° ′ ″	Description.
1	Aug. 10	0 46 45	— 35 0 43	*pB. eeS. E.* with 200 looks like a nebulous *D* Uranus.
2	Sept. 4	0 55 0	— 40 53 51	*vE. vS. R.*
3	" 4	1 9 45	— 33 11 33	*eeF. S. eeE* a ray no * star near.
4	" 4	1 23 35	— 36 17 3	*eeF. pS. R. v dif.*
5	" 4	1 33 10	— 34 29 45	*vF. S. R. eF* * near *nf.*
6	" 6	1 46 45	— 30 26 20	*pB. eS. lE.* like a *D* nebulous no * near with 132 and 200. See No. 1.

ment from the incident wave surface in which the vibration is represented by $\cos \frac{2\pi}{\lambda} \alpha t$. The integration is extended over the whole of the surface of the mirror.

Let us call $\xi\eta$ the coördinates of the point p (referred to the center of the field) and x, y, z the coördinates of the element $dx\,dy$, with reference to the same origin. Then if f denote the principal focal length of the mirror and m the parameter of the parabolic surface, we will evidently have for ρ

$$\rho = \sqrt{(x-\xi)^2 + (y-\eta)^2 + z^2}$$

$$= \sqrt{(x-\xi)^2 + (y-\eta)^2 + \left(f - \frac{x^2+y^2}{m}\right)^2} \qquad (2)$$

and for ε

$$\varepsilon = x\cos\alpha + y\cos\beta + \left(f - \frac{x^2+y^2}{m}\right)\cos\gamma - l \quad (3)$$

where α, β, γ are the direction cosines of the incident wave front (supposed plane), and l the distance of the chosen wave front, (in which the vibration is represented by $\cos \frac{2\pi}{\lambda} \alpha t$), from the central focal point.

When ξ, η, are small and f is large the expression for I^2 is readily reduced to the form

$$I_1^2 = \frac{1}{\lambda^2 f^2}\left[\int\int \sin\left(\frac{2\pi\xi}{\lambda f}x + \frac{2\pi\eta}{\lambda f}y\right)dx\,dy\right]^2$$

$$+ \frac{1}{\lambda^2 f^2}\left[\int\int \cos\left(\frac{2\pi\xi}{\lambda f}x + \frac{2\pi\eta}{\lambda f}y\right)dx\,dy\right]^2$$

which has already been investigated.*

The investigation of the general integral, (1), in the case of a parabolic mirror is not in the opinion of the writer, of much *practical* importance, for the reason that the most useful field of work for reflectors is stellar spectroscopic and allied branches of astrophysical work, in which only the very center of the field is utilized†. Such an investigation however would have great theoretical interest and it is to be hoped that Professor Schæ-

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* See Rayleigh's Wave Theory, Enc. Brit. Vol. 24, §§ 11 and 12, also paper already referred to "General theory of Telescopic Images," *Astrophysical Journal*, Aug. 1897.

† For further remarks on this point see my previous papers in the Astrophysical Journal, Jan. 1895, p 52, March, 1895 p 232, March, 1896, p 182-3, Feb. 1897, p 132 etc.

berle or Professor Poor* who have begun it on the lines of geometrical optics will complete it by the methods of physical optics and thus obtain for the first time complete results which will enable us to judge of the character as well as the mere size, of the images formed by a reflector at some distance from the optical axis.

Owing to the concentration of the attention of opticians during recene years on the problem of the improvement of the refractor the subject of the reflector has not received the general attention that it should from either the theoretical or the practical side.†

If one-tenth the time had been devoted to the problems arising in connection with the figuring, mounting and use of specula as has been spent on the computation and figuring of the lenses of refractors, the reflecting telescope would occupy a much higher place in the estimation of astronomers and spectroscopists, than it does at present. As an example of the really primatic state of our knowledge of the theory of the reflector, it need only be remarked that it is not at all certain that the parabolic form of surface is the best for mirrors, a fact which was long ago pointed out by Lord Rayleigh.‡

YERKES OBSERVATORY,
Dec. 8, 1897.

TRANSITS OF VENUS AND MERCURY.

J. MORRISON, M. A., M. D., PH. D.

FOR POPULAR ASTRONOMY.

A transit of Venus or Mercury may be regarded as an annular eclipse of the Sun, the planet taking the place of the Moon, but with this difference viz: When the Moon is the eclipsing body the umbral penumbra covers only a small portion of the Earth's

* I have just received word from Professor Poor that he proposes to take up this investigation. See his paper, "Aberration of Parabolic Reflectors" which will be published in the Feb. number of the *Astrophysical Journal*.

† In this connection it is only proper to refer to Sir Howard Grubb's loyal and constant advocacy of the reflector for many years past. See particularly his papers "Great Telescopes of the Future," Trans. R. Dub. Soc. New Ser. Vol. I Mem. 1; "On the Choice of Instruments for Stellar Photography," Mon. Not., Vol. 47 p. 301, April, 1887; "The Development of the Astronomical Telescope," Roy. Inst., May 25th, 1894 and many others. As has been indicated before the writer does not agree entirely with Sir Howard as regards the problems of astronomical photography, but with respect to astrophysical work he is in entire accord with him.

‡ "Investigations in Optics with special reference to the Spectroscope." § 1. "Resolving or Separating Power of Optical Instruments," *Phil. Mag.* Oct. 1879, p. 261.

surface, but in the case of the planet it envelops the whole Earth.

As seen from the center of the Earth, a transit will begin and end when the planet is tangent to the cone which envelopes the Sun and has its vertex at the Earth's centre. For the Earth generally it begins and ends when the planet is tangent to the cone which envelopes the Sun and Earth.

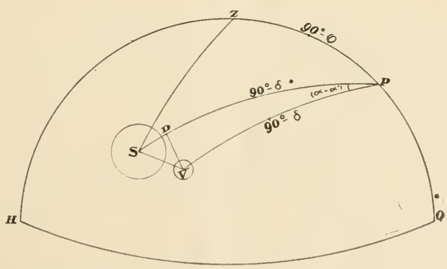
The formulæ for solar eclipses are applicable to transits but in (63) we must use for Venus $k = 0.9975$ and for Mercury $k = 0.3897$, and the rigorous formulæ (49) (50) and (51) must be used for the computation of a, d and g.

As the parallaxes of both planets are very small, it will however be found more convenient to employ the special method of Lagrange as herein modified and improved.

To Find the Times of Ingress and Egress as seen from the Centre of the Earth.

The elements are computed as in the case of eclipses and the parallaxes and semi-diameters of the bodies may be regarded as constant during the period of transit.

Let S represent the centre of the Sun and V that of the planet as seen from the centre of the Earth, and P the north pole, then in the spherical triangle PSV we have $PS = 90° - \delta'$, $PV = 90° - \delta$ and $SPV = \alpha - \alpha'$.



Put $SV = \varDelta$ and $PSV = Q$, then we have

$$\sin \varDelta \sin Q = \cos \delta \sin (\alpha - \alpha')$$
$$\sin \varDelta \cos Q = \sin \delta \cos \delta' - \cos \delta \sin \delta' \cos (\alpha - \alpha') \qquad (184)$$
$$= \sin (\delta - \delta') + 2 \sin \delta' \cos \delta \sin^2 \frac{1}{2} (\alpha - \alpha')$$

or since $\varDelta$, $\delta - \delta'$ and $(\alpha - \alpha')$ are quite small we may substitute the arc for the sine and express them in seconds thus

$$\begin{aligned}\Delta''\sin Q &= (\alpha-\alpha')''\cos\delta \\ \Delta''\cos Q &= (\delta-\delta')'' + \frac{2\sin\delta'\cos\delta\sin^2\frac{1}{2}(\alpha-\alpha')}{\sin 1''} \\ &= (\delta-\delta')'' + \beta'' \quad \text{suppose.}\end{aligned} \tag{185}$$

Taking S as the origin of a system of rectangular coördinates and drawing VD perpendicular to SP we shall evidently have

$$\begin{aligned}\Delta''\sin Q &= x = (\alpha-\alpha')''\cos\delta = VD \\ \text{cos}\ \Delta''\cancel{\sin}\ Q &= y = (\delta-\delta')'' + \beta'' = SD\end{aligned} \tag{186}$$

Now if x and y be computed for several consecutive hours preceding and following conjunction so as to include the entire duration of the transit, their differences will furnish the hourly variations, x' and y', as in the case of solar eclipses.

Let x_0 and y_0 be the values of x and y at an epoch T_0 near conjunction, s' and s the semidiameters of the Sun and planet respectively and let the required time of contact for the centre of the Earth be $T = T_0 + \tau$, for which date $\Delta = s' \pm s$, the upper sign for external and the lower for internal contact, then we shall have

$$\begin{aligned}(s' \pm s)\sin Q &= x_0 + x'\tau \\ (s' \pm s)\cos Q &= y_0 + y'\tau\end{aligned}$$

Put

$$\begin{aligned}m\sin M &= x_0, & n\sin N &= x' \\ m\cos M &= y_0, & n\cos N &= y'\end{aligned} \tag{187}$$

then

$$\begin{aligned}(s' \pm s)\sin Q &= m\sin M + n\sin N.\ \tau \\ (s' \pm s)\cos Q &= m\cos M + n\cos N.\ \tau\end{aligned}$$

which solved in the usual way give

$$\begin{aligned}\sin\psi &= \frac{m\sin(M-N)}{s' \pm s} \\ \tau &= \mp\frac{(s' \pm s)}{n}\cos\psi - \frac{m}{n}\cos(M-N) \\ Q &= N + \psi \\ \text{and}\qquad T &= T_0 + \tau\end{aligned} \tag{188}$$

To Find the Times of Ingress and Egress for a given Place on the Earth's Surface.

For the date T just found compute from the ephemeris the R.A. and Decl. of the Sun and planet and the values of x and y by (186), then the first of (187) will become

$$\begin{aligned}m\sin M &= x \\ m\cos M &= y \\ \text{whence}\qquad m^2 &= x^2 + y^2 = (s' \pm s)^2 \\ \text{or}\qquad m &= (s' \pm s)\end{aligned} \tag{189}$$

Let T' denote the time of ingress or egress for a given place whose zenith is Z, then $T' - T$ is the retardation or acceleration due to parallax and since T' can differ from T by only a few minutes, we may regard m as varying uniformly.

At the time T, the *geocentric* distance $= m = s' \pm s$ and at the time T' the *apparent* distance is also $= s' \pm s$, but at this latter date the geocentric distance has become

$$m' = m + (T' - T)\frac{dm}{dt} \qquad (190)$$

where $\frac{dm}{dt} =$ the change of m in a unit of time.

Differentiating (189) we have

$$\frac{dm}{dt}\sin M + m\cos M\frac{dM}{dt} = \frac{dx}{dt} = x' = n\sin N$$

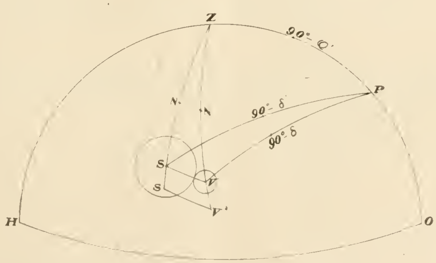
$$\frac{dm}{dt}\cos M - m\sin M\frac{dM}{dt} = \frac{dy}{dt} = y' = n\cos N \quad \text{by (187)}$$

whence $$\frac{dm}{dt} = n\cos(M - N)$$

which substituted in (190) gives

$$m' - m = (T' - T)\,n\cos(M - N) = (T' - T)\,n\cos\psi \qquad (191)$$

because at the date T, $M - N = \psi$ by the first of (188)



If at the time of *geocentric* contact T, the observer were instantly transferred to a point on the surface whose zenith is Z, the bodies would be depressed by parallax to S' and V' respectively and the distance $SV = m = s' \pm s$ would become $S'V' = m'$ and we must now express $m' - m$ in terms of known quantities. Let the true zenith distances of the Sun and planet be denoted by z' and z respectively then $SS' = \rho\pi' \sin z'$ and $VV' = \rho\pi\sin z$ approximately and the spherical triangle ZSV gives

$$\cos VS = \cos ZV\cos ZS + \sin ZV\sin ZS\cos Z$$

or $$\cos m = \cos z\cos z' + \sin z\sin z'\cos Z$$

Differentiating, the angle Z being constant, we get

$$\begin{aligned} \sin m dm &= (\cos z \sin z' - \sin z \cos z' \cos Z)\, dz' \\ &\quad + (\sin z \cos z' - \cos z \sin z' \cos Z)\, dz \\ &= \sin m \cos S\, dz' + \sin m \cos V\, dz \end{aligned}$$

or
$$dm = \cos S\, dz' + \cos V\, dz$$

Now for the differentials dm, dz' and dz we may without appreciable error, substitute the small increments $m' - m$, $\rho\pi' \sin z'$ and $\rho\pi \sin z$ respectively, therefore the last equation becomes

$$m' - m = \rho\pi' \sin z' \cos S + \rho\pi \sin z \cos V \qquad (192)$$

From the triangle ZSV we have

$$\left.\begin{aligned} \cos z &= \cos z' \cos m + \sin m \sin z' \cos S \\ \cos z' &= \cos z \cos m + \sin m \sin z \cos V \end{aligned}\right\} \qquad (193)$$

Eliminating $\sin z' \cos S$ and $\sin z \cos V$ between (192) and (193) we have

$$m' - m = \frac{\rho}{\sin m}(\pi' \cos z - \pi' \cos z' \cos m + \pi \cos z' - \pi \cos z \cos m) \qquad (194)$$

Now if h' and h denote the hour angles ZPS and ZPV of the Sun and planet respectively we shall have from the triangles ZPS and ZPV

$$\begin{aligned} \cos z &= \sin \varphi' \sin \delta + \cos \varphi' \cos \delta \cos h \\ \cos z' &= \sin \varphi' \sin \delta' + \cos \varphi' \cos \delta' \cos h' \end{aligned}$$

and substituting these values of $\cos z$ and $\cos z'$ in (194) we find after putting $\cos m = 1$ and writing m for $\sin m$ which can be done without appreciable error,

$$m' - m = \frac{\pi - \pi'}{m}\left\{\begin{aligned} &\rho \sin \varphi' (\sin \delta' - \sin \delta) \\ &+ \rho \cos \varphi (\cos \delta' \cos h' - \cos \delta \cos h) \end{aligned}\right\} \qquad (195)$$

Let μ be the sidereal time at the first meridian (Greenwich) for the date T, then the hour angles of the Sun and planet at that meridian will be $\mu - \alpha'$ and $\mu - \alpha$ respectively, and the hour angles for any other place whose longitude is λ will be $\mu - \alpha' - \lambda = h'$ and $\mu - \alpha - \lambda = h$ but h' and h are very nearly equal, their difference being $\alpha - \alpha'$ which is, in this case, always quite small, we may therefore use without sensible error the mean value viz:

$\frac{1}{2}(h' + h) = (\mu - \frac{\alpha' + \alpha}{2} - \lambda) = \ominus - \lambda$, that is for $\cos h'$ and $\cos h$ we may write $\cos(\ominus - \lambda)$ in (195) and we have

$$m' - m = \frac{\pi - \pi'}{m} \left\{ \rho \sin \varphi' (\sin \delta' - \sin \delta) \right.$$
$$\left. + \rho \cos \varphi' (\cos \delta' - \cos \delta) \cos (\ominus - \lambda) \right\}$$

Substituting this value of $m' - n$ in (191) and solving for T' we have after some slight reductions

$$T' = T + \frac{2\,\omega\,(\pi - \pi') \sin \frac{1}{2} (\delta' - \delta)}{\sin (s' \pm s)\; n \cos \psi} \left\{ \rho \sin \varphi' \cos \frac{1}{2} (\delta' + \delta) \right.$$
$$\left. - \rho \cos \varphi' \sin \frac{1}{2} (\delta' + \delta) \cos (\ominus - \lambda) \right\} \quad (196)$$

where $\omega = 3600$, to reduce the terms to seconds and

$$\ominus = \mu - \frac{1}{2} (\alpha' + \alpha)$$

The angle Q found from (188) is the position angle or the angular distance of the point of contact measured on the Sun's limb from the north point towards the east, and is nearly constant for all places on the Earth.

Disks of Mercury and Venus. *

The quantities referring to the disk and tabulated for every fifth day in the American Ephemeries, are defined on the same page. The formulae for their computation are easily deduced as follows:

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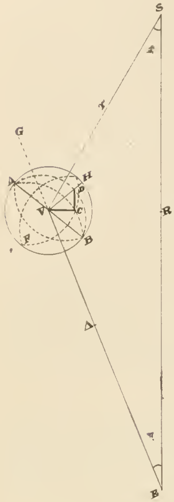
* It was my intention to omit the formulæ for the disks of Venus and Mercury but when I recollected that they proved too difficult for a Professor of Mathematics in the U. S. Navy, I decided to develop them although extremely simple. Some seven years ago Professor W. W. Hendrickson who had been for over 17 years Professor of Mathematics at the Naval Academy, Annapolis, and the head of the mathematical department at that, was assigned to the Nautical Almanac office. He was *de facto* Superintendent of the office, acting as such in the absence of the actual Superintendent and succeeded for a while that official on his retirement last March. Professor H. being desirous of checking these quantities asked me for the formulæ which I did not then have already prepared, but I told him they were very easily deduced and that I always derived them *de novo* whenever I wanted them. This answer gave offense although none was intended. Failing in his attempts to deduce them with all the data in the office at his disposal, he was obliged to write to the regular computer, Mr. Austin of Salt Lake City, for a copy of them At the investigation of the Nautical Almanac office held in July, 1893, he admitted under oath among other things that he was unable to deduce the formulæ, was not an astronomer, knew little or nothing about astronomy, never wrote a paper or made any observations or researches in the science and that his only astronomical achievement up to that time, was the calculation of the Sun's R A. and Decl. from having given the obliquity and longitude, but even then he did not know how to take care of the Sun's latitude. This testimony, with much more of the same tenor, is on file in the Navy Dep't. It was the most astounding admission of ignorance and incompetency on the part of a Professor of Mathematics that ever came under my notice. He has recently been ordered back to Annapolis to assist the cadets in crossing the "pons asinorum" and other similar feats. J. M.

To Find k, the Ratio of the Illuminated to the Entire Disk.

Let S, E and V be the centres of the Sun, Earth and planet respectively, then VS is perpendicular to the plane of the circle $ACBF$ which separates the illumined from the unillumined portion of the planet, and EV is perpendicular to the plane of AHB, therefore the angle CVH = the angle GVS = $E+S$.



The semi-circle ACB when projected on the plane AHB will be the semi-ellipse ADB and $AHBDA$ is the illuminated part of the disk as seen from the Earth.

Let a = the semi-diameter of the planet seen from E then $a\cos(S+E)$ = the semi-axis minor of the ellipse.

$$\frac{1}{2}a^2\pi\cos(S+E) = \text{area of semi-ellipse } ADB,$$

$$\frac{1}{2}a^2\pi = \text{area of the semi-circle } AHB$$

$$\frac{1}{2}a^2\pi\left(1-\cos(S+E)\right) = \text{area of the lune or}$$

crescent $AHBDA$

and $\quad a^2\pi \quad$ = area of the whole disk

therefore

$$k = \frac{\frac{1}{2}a^2\pi\left(1-\cos(S+E)\right)}{a^2\pi}$$

$$= \frac{1}{2}\left(1-\cos(S+E)\right) \qquad (197)$$

$$= \frac{1}{2}(1+\cos i)$$

$$= \cos^2\frac{1}{2}i, \text{ where } i = \text{angle } EVS$$

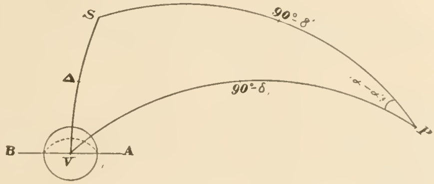
To Find i, which may be called the Parallactic Angle.

The Ephemeris gives for every Greenwich mean noon the logarithms of R, r and $\varDelta$, the three sides of the plane triangle EVS and with these data we have

$$\cos\frac{1}{2}i = \sqrt{\frac{s(s-R)}{\varDelta r}} \quad \text{where } s = \frac{1}{2}(R+r+\varDelta) \qquad (198)$$

To Find θ, the Angle which the Line joining the Cusps makes with the Meridian.

Let P be the north pole, S, the Sun, V, the planet and AVB the line joining the cusps, then $\theta = AVP$. The angle $SPV = \alpha - \alpha'$, the difference of right ascension, $PS = 90^\circ - \delta'$, $PV = 90^\circ - \delta$ and put $VS = \varDelta$ then we have



$$\begin{aligned} \sin \varDelta \cos V &= \sin \delta' \cos \delta - \cos \delta' \sin \delta \cos (\alpha - \alpha') \\ \sin \varDelta \sin V &= \cos \delta' \sin (\alpha - \alpha') \end{aligned}$$

Put $$h \sin H = \cos \delta' \cos (\alpha - \alpha')$$

$$h \cos H = \sin \delta' \qquad \text{that is} \qquad \tan H = \frac{\cos (\alpha - \alpha')}{\tan \delta'}$$

then the above become

$$\begin{aligned} \sin \varDelta \cos V &= h \cos (\delta + H) \\ \sin \varDelta \sin V &= \cos \delta' \sin (\alpha - \alpha') \end{aligned} \tag{199}$$

whence $$\tan V = \frac{\sin H \tan (\alpha - \alpha')}{\cos (\delta + H)} \tag{200}$$

and $$\theta = 90^\circ - V \; (+ 360^\circ \text{ if necessary}) \tag{201}$$

θ being reckoned from 0° to 360°.

The quadrant in which V must be taken is determined by (199), h and $\sin \varDelta$ being of course always positive.

To Find L.

Let a = the geocentric semidiameter in seconds of arc
d = the geocentric distance of the planet
then $\pi d^2 \sin^2 a$ = the area of the disk at distance d
$\pi k d^2 \sin^2 a$ = the area of the visible disk or crescent at same distance
and $\pi \sin^2 1''$ = the area of a disk of radius $1''$ at distance 1
= the unit of light

Now since the intensity of light varies inversely as the square of distance we have

$$\text{Units of light received on crescent at distance } r = \frac{\pi k d^2 \sin^2 a}{\pi r^2 \sin^2 1''} = \frac{k d^2 \sin^2 a}{r^2 \sin^2 1''}$$

and therefore units of light received at distance $d = \dfrac{k \sin^2 a}{r^2 \sin^2 1''}$

or
$$L = \frac{ka''^2}{r^2} \qquad (202)$$

where a and r are given in the ephemeris.

ON NEWTON'S LAW OF GRAVITATION.

PROFESSOR H. SEELIGER.

FOR POPULAR ASTRONOMY.

It is also permissible to bring forward a second example for the case of a constant δ. We have, by (2);

$$X_2 = \delta \int_0^{2\pi} d\varphi \int_0^{\pi} P^1 (\cos \gamma) \sin \gamma \, (R_1 - R_0) d\gamma.$$

$$Z_2 = 2\delta \int_0^{2\pi} d\varphi \int_0^{\pi} P^2 (\cos \gamma) \sin \gamma \log \left(\frac{R_1}{R_0}\right) d\gamma.$$

If we assume here, for the law of increase of R, that m is a magnitude passing gradually to infinity and α a number greater than ½ and further that,

$$\log \frac{R_1}{R_0} = \alpha m + m P^2 (\cos \gamma),$$

We shall have;

$$X_2 = 2\pi\delta \, R_0 \, e^{\alpha m} \, . \int_0^{\pi} \sin \gamma \cos \gamma \, . \, e^{m\left(\frac{3}{2}\cos^2 \gamma - 1\right)} . \, d\gamma = 0$$

and $\quad Z_2 = \dfrac{8\pi\delta}{5} \, m.$

That is, X_2 remains equal to zero and the stress becomes infinite with m.

Other examples may without difficulty, be selected, in which δ is a different function of the place of the point. There can therefore, be no doubt, that the disclosed contradictions must remain for every possible and conceivable distribution of matter, if only δ possess the property emphasized, of having within infinitely great regions of space, finite values different from zero.

Light may of course, be thrown upon the unavoidable contradictions from various directions. They admit also of representation in another form. This will not be done here however, but we shall now draw an inference from the fundamental principles of the theory of the potential.

of this paper. It was a "short" one, the rise beginning 4369, the maximum being passed at 4371.6 and the normal reached 4382, giving the duration of normal light 44 days. This can by no means be considered as an established law of variation but seems to be the best deduction possible with the present data.

EPHEMERIDES OF SATELLITES.*

J. MORRISON, M. A., M. D., PH. D.

FOR POPULAR ASTRONOMY.

When a satellite is carefully observed during an entire revolution, it is found to describe a curve which does not sensibly differ from an ellipse, or to move to and fro in a straight line passing through the centre of the planet. This latter motion can only take place when the Earth is situated in the plane of the satellite's orbit or is passing the node on the ecliptic. With the exception of the three outermost satellites of Saturn, viz.: Iapetus, Hyperion and Titan, the actual orbits of all the other known satellites are regarded as circles. Owing however to the disturbance of other bodies in the solar system, the orbits must approach more nearly to an ellipse than to a circle, but the eccentricity is so exceedingly small that it can be practically disregarded without any appreciable error whatever. The orbits of the three satellites just named are, however, decidedly elliptical with tolerably large eccentricities and must be treated accordingly.

The apparent orbit or that which is actually observed, is the projection of the real orbit on a plane perpendicular to the line of vision and passing through the centre of the planet. When the real orbit is a circle, the apparent orbit is an ellipse with the planet in the centre, and when the real orbit is an ellipse, the apparent one will also be an ellipse but the planet will be in neither the centre nor the focus.

TO DETERMINE THE POSITION OF A SATELLITE IN ITS APPARENT ORBIT AT ANY TIME WHEN THE ELEMENTS OF ITS REAL ORBIT ARE KNOWN.

Let α, δ, Δ be the geocentric polar coördinates of the planet,
α', δ', Δ' the geocentric polar coördinates of the satellite,
and a, d, r the planetocentric polar coördinates of the satellite, referred to the equator and equinox of the epoch for which the ephemeris is to be computed.

* Extracted in part from the author's paper in the Monthly Notices of the Royal Astronomical Society of England, Vol. 44, No. 8, June, 1884.

The general expressions for the rectangular coördinates in terms of the polar are

$$x = \Delta \cos\delta \cos\alpha$$
$$y = \Delta \cos\delta \sin\alpha$$
$$z = \Delta \sin\delta$$

Therefore by equating the rectangular coördinates of the bodies we have

$$\begin{aligned} \Delta' \cos\delta' \cos\alpha' &= \Delta \cos\delta \cos\alpha + r \cos d \cos a \\ \Delta' \cos\delta' \sin\alpha' &= \Delta \cos\delta \sin\alpha + r \cos d \sin a \qquad (203) \\ \Delta' \sin\delta' &= \Delta \sin\delta + r \sin d \end{aligned}$$

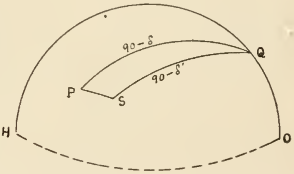
Multiplying the first of this group by $\cos\alpha$, the second by $\sin\alpha$ and adding the results, and again multiplying the first by $\sin\alpha$, the second by $\cos\alpha$ and subtracting we have

$$\begin{aligned} \Delta' \cos\delta' \cos(\alpha' - \alpha) &= \Delta \cos\delta + r \cos d \cos(a - \alpha) \\ \Delta' \cos\delta' \sin(\alpha' - \alpha) &= r \cos d \sin(a - \alpha) \qquad (204) \\ \Delta' \sin\delta' &= \Delta \sin\delta + r \sin d \end{aligned}$$

We have now to consider the spherical triangle formed by the north pole of the equator, the planet and the satellite; thus let Q be the north pole, P the planet and S the satellite, then in the spherical triangle PQS we have $PQ = 90° - \delta$, $QS = 90° - \delta'$ and $PQS = \alpha' - \alpha$

Put $PS = s$, the apparent distance of the satellite from the planet as seen from the Earth

and $QPS = p$, the angle of position of the satellite reckoned from the north toward the east and from 0° to 360°.



By the fundamental formulæ of spherical trigonometry we have

$$\begin{aligned} \sin s \sin p &= \cos\delta' \sin(\alpha' - \alpha) \\ \sin s \cos p &= \cos\delta \sin\delta' - \sin\delta \cos\delta' \cos(\alpha' - \alpha) \qquad (205) \\ \cos s &= \sin\delta \sin\delta' + \cos\delta \cos\delta' \cos(\alpha' - \alpha) \end{aligned}$$

Multiplying by Δ' and substituting from (204) this group becomes

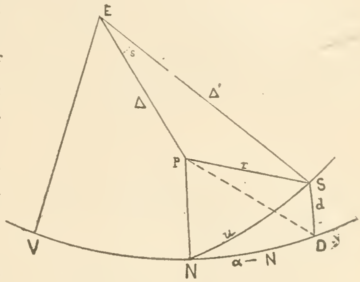
$$\begin{aligned} \Delta' \sin s \sin p &= r \cos d \sin(a - \alpha) \\ \Delta' \sin s \cos p &= r [\cos\delta \sin d - \sin\delta \cos d \cos(a - \alpha)] \qquad (206) \\ \Delta' \cos s &= \Delta + r[\sin\delta \sin d + \cos\delta \cos d \cos(a - \alpha)] \end{aligned}$$

from which we must eliminate a, d and Δ'.

Let VD represent an arc of the equator, NS an arc of the orbit

of the satellite, E the Earth, P the planet and S the satellite; draw SD perpendicular to VD and let EV be the direction of the vernal equinox. Then N will be the position of the ascending node of the orbit of the satellite on the equator; VD and SD the planetocentric right ascension and declination of the satellite which we have denoted by a and d respectively.

Put $VN = N$, the R. A. of the ascending node;
$SND = i$, the inclination of the orbit to the plane of the equator;
$NS = u$, the angular distance of the satellite from the ascending node;
then $ND = a - N$, and from



the right angled spherical triangle NSD whose center is P, we have

$$\begin{aligned} \cos d \cos(a - N) &= \cos u \\ \cos d \sin(a - N) &= \sin u \cos i \\ \sin d &= \sin u \sin i \end{aligned} \qquad (207)$$

Multiplying the first of this group by $\cos(\alpha - N)$, the second by $\sin(\alpha - N)$ and adding the results; also multiplying the first by $\sin(\alpha - N)$ and the second by $\cos(\alpha - N)$ and subtracting we have

$$\begin{aligned} \cos d \cos(a - \alpha) &= \sin u \cos i \sin(\alpha - N) + \cos u \cos(\alpha - N) \\ \cos d \sin(a - \alpha) &= \sin u \cos i \cos(\alpha - N) - \cos u \sin(\alpha - N) \\ \sin d &= \sin u \sin i \end{aligned}$$

and substituting these in (206) we have after some simple reductions

$$\begin{aligned} \varDelta' \sin s \sin p &= r \sin u \cos i \cos(\alpha - N) - r \cos u \sin(\alpha - N) \\ \varDelta' \sin s \cos p &= r \sin u \left(\cos\delta \sin i - \sin\delta \cos i \sin(\alpha - N)\right) \\ &\quad - r \sin\delta \cos u \cos(\alpha - N) \qquad (208) \\ \varDelta' \cos s &= \varDelta + r \sin u \left(\sin\delta \sin i + \cos\delta \cos i \sin(\alpha - N)\right) \\ &\quad + r \cos\delta \cos u \cos(\alpha - N) \end{aligned}$$

from which $\varDelta'$ must be eliminated.

In the plane triangle EPS formed by joining the Earth, planet and satellite, let σ denote the supplement of the angle EPS, then we shall evidently have

$$\begin{aligned} \varDelta' \sin s &= r \sin \sigma \\ \varDelta' \cos s &= r \cos \sigma + \varDelta \end{aligned} \qquad (209)$$

which being substituted in (208) give after reduction

$$\begin{aligned}\sin\sigma\sin p &= \sin u\cos i\cos(\alpha-N)-\cos u\sin(\alpha-N)\\ \sin\sigma\cos p &= \sin u\left(\cos\delta\sin i-\sin\delta\cos i\sin(\alpha-N)\right)\\ &\qquad -\cos u\sin\delta\cos(\alpha-N)\\ \cos\sigma &= \sin u\left(\sin\delta\sin i+\cos\delta\cos i\sin(\alpha-N)\right)\\ &\qquad +\cos u\cos\delta\cos(\alpha-N)\end{aligned} \tag{210}$$

These equations are perfectly rigorous and determine σ and p without ambiguity, but as the angle σ can not be compared directly with observation, it will be more convenient to express σ in terms of s which can be observed; thus from (209) we have

$$\tan s = \frac{r\sin\sigma}{r\cos\sigma+\Delta} \tag{211}$$

and since s is always very small, we may substitute the arc for the tangent and develop the second member into a series.

Hence we have

$$s = \frac{r}{\Delta}\sin\sigma - \frac{1}{2}\frac{r^2}{\Delta^2}\sin 2\sigma + \frac{1}{3}\frac{r^3}{\Delta^3}\sin 3\sigma - \text{etc.} \tag{212}$$

Now r is always very small compared with Δ, and therefore the terms containing the square and higher powers of $\frac{r}{\Delta}$ may be neglected without sensibly affecting the result.

We shall then have

$$s'' = \frac{r}{\Delta\sin 1''}\sin\sigma \tag{213}$$

and if a'' be the value of $\frac{r}{\sin 1''}$ at distance unity, then $\frac{a''}{\Delta}$ will be its value at distance Δ, therefore the last equation may be written thus

$$s'' = \frac{a''}{\Delta}\sin\sigma \qquad \text{whence} \qquad \sin\sigma = \frac{\Delta}{a''}\,.\,s'' \tag{214}$$

which substituted in the first two equations of (210) gives

$$\begin{aligned} s''\sin p &= \frac{a''}{\Delta}\left\{\sin u\cos i\cos(\alpha-N)\cos u\sin(\alpha-N)\right\}\\ s''\cos p &= \frac{a''}{\Delta}\left\{\sin u\,(\cos\delta\sin i-\sin\delta\cos i\sin(\alpha-N)\right.\\ &\qquad \left.-\cos u\sin\delta\cos(\alpha-N)\right\}\end{aligned} \tag{215}$$

The error committed in s'' by omitting the second term of (212)

is
$$\frac{1}{2}\frac{r^2}{\Delta^2}\frac{\sin 2\,\sigma}{\sin 1''}$$

which is a maximum when $\sigma = 45°$ or $135°$.

Put $\frac{1}{2}\frac{r^2}{\Delta^2}.\frac{1}{\sin 1''} = \eta$ then $\frac{r}{\Delta} = \sqrt{\eta \sin 2''}$, but $\frac{r}{\Delta}$ is the tangent of the angle of greatest elongation as seen from the earth, hence putting

$$\frac{r}{\Delta} = \tan \varepsilon \text{ we have } \eta = \frac{\tan^2\varepsilon}{\sin 2''}$$

when
$$\begin{array}{ll} \varepsilon = 0'\ 50'' & \eta = 0''.006 \\ \varepsilon = 1\ \ 50 & \eta = 0.\ 029 \\ \varepsilon = 3\ \ 35 & \eta = 0.\ 112 \\ \varepsilon = 5\ \ 46 & \eta = 0.\ 290 \\ \varepsilon = 9\ \ 45 & \eta = 0.\ 829 \end{array}$$

Thus we see, that for the satellites of Mars, Saturn, Uranus, Neptune and the three inner satellites of Jupiter whose greatest angle of elongation does not exceed 6′, the error committed in s'', is less than 0″.3 and for Callisto the fourth satellite of Jupiter whose elongation is never more than 9′ 45″, the error can never exceed 0″.83. The formulæ (213) may therefore be regarded as practically exact.

In order to adapt (210) and (215) to logarithmic computation put,

$$\begin{aligned} \sin f \cos F &= \cos i \cos (\alpha - N) \\ \sin f \sin F &= - \sin (\alpha - N) \\ \cos f &= - \sin i \cos (\alpha - N) \end{aligned} \tag{216}$$

$$\begin{aligned} \sin g \cos G &= \cos \delta \sin i - \sin \delta \cos i \sin (\alpha - N) \\ \sin g \sin G &= - \sin \delta \cos (\alpha - N) \\ \cos g &= \cos \delta \cos i + \sin \delta \sin i \sin (\alpha - N) \end{aligned} \tag{217}$$

$$\begin{aligned} \sin h \cos H &= \sin \delta \sin i + \cos \delta \cos i \sin (\alpha - N) \\ \sin h \sin H &= \cos \delta \cos (\alpha - N) \\ \cos h &= \sin \delta \cos i - \cos \delta \sin i \sin (\alpha - N) \end{aligned} \tag{218}$$

then (210) and (215) become

$$\begin{aligned} \sin \sigma \sin p &= \sin f \sin (F + u) \\ \sin \sigma \cos p &= \sin g \sin (G + u) \\ \cos \sigma &= \sin h \sin (H + u) \end{aligned} \tag{219}$$

and
$$\begin{aligned} s'' \sin p &= \frac{a''}{\Delta} \sin f \sin (F + u) \\ s'' \cos p &= \frac{a''}{\Delta} \sin g \sin (G + u) \end{aligned} \tag{220}$$

The last group which is known as Bessel's formulæ, gives s'' and

p for any given date and also enables us to compare the computed with the observed place. When s and p are required for a series of dates, it will be most convenient to compute the values of the auxiliary angles f, F, g and G for the mean noon of several consecutive days and then interpolate for any intermediate date.

The auxiliaries h and H will not be required unless we desire to check the value of σ in (219).

From the position angle and the apparent distance as found by observation we may compute the radius rector and the value of u or the distance of the satellite from the node and compare these with the values deduced from the elements of the orbit.

Resuming equations (220) and putting $r = \dfrac{a''}{\Delta}$, we have

$$s \sin p = r \sin f \sin (F + u)$$
$$s \cos p = r \sin g \sin (G + u)$$

from which to find r and u, s and p being known from observation and f, F, g and G from (216) and (217).

Multiply the first of these by $\sin g \sin G$ and the second by $\sin f \sin F$, subtract the second product from the first, factor and divide by $\sin f \sin g \sin (G - F)$ and we obtain

$$s\left(\frac{\sin G}{\sin f \sin (G - F)} \sin p - \frac{\sin F}{\sin g \sin (G - F)} \cos p\right) = r \sin u$$

Put $\dfrac{\sin G}{\sin f \sin (G - F)} = f' \cos F'$ and $\dfrac{\sin F}{\sin g \sin (G - F)} = f' \sin F'$

and the above becomes

$$s f' \sin (p - F') = r \sin u \tag{221}$$

Similarly, by multiplying the first by $\sin g \cos G$ and the second by $\sin f \cos F$, subtracting etc. and putting

$$\frac{\cos G}{\sin f \sin (F - G)} = g' \cos G' \text{ and } \frac{\cos F}{\sin g \sin (F - G)} = g' \sin G'$$

we get
$$s g' \sin (p - G') = r \cos u \tag{222}$$

whence
$$\tan u = \frac{f' \sin (p - F')}{g' \sin (p - G')}$$

and
$$r = \frac{s f' \sin (p - F')}{\sin u} = \frac{s g' \sin (p - G')}{\cos u} \tag{223}$$

which determine u and r without ambiguity.

To Find the Time of Greatest Eastern or Western Elongation, the Position Angles and the Distance of the Satellite from the Planet.

Let us conceive three planes to be passed as follows:—

The *first* through the centre of the planet and the poles of the equator and orbit of the satellite; the *second* through the centre of the planet, the pole of the orbit of the satellite and the centre of the Earth; the *third* through the centre of the planet, the pole of the equator and the centre of the earth.

These three planes will determine on the celestial sphere a spherical triangle having its centre at the centre of the planet.

Let AV represent an arc of the equator, NS an arc of the orbit of the satellite, Q the pole of the equator, K the pole of the orbit of the satellite, N the position of the ascending node, P the planet and E the Earth. Join PE, PK and PQ, and in the plane KPE draw PB perpendicular to PK and PC perpendicular to PE; then PB will lie in the plane of the orbit and PC in the plane of the apparent orbit which is the projection of the former on the plane passing through P and perpendicular to the line PE joining the Earth and planet.

Put $\theta =$ the angle EPB.

$q =$ the angle of position of the minor axis of the apparent orbit,

$p_0 = q - 90°$, the angle of position of the major axis,

$u_0 =$ the angular distance of the end of the major axis from the ascending node measured on the orbit of the satellite,

a and $b =$ the semi major and minor axes respectively of the apparent orbit;

then in the spherical triangle EKQ we shall have

$KQ = i$, the inclination of the orbit of the satellite,

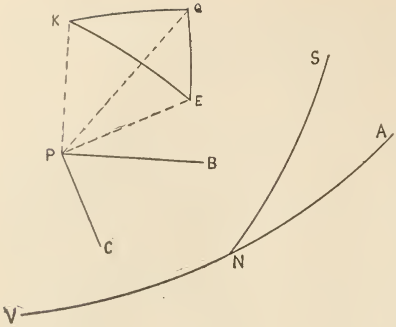
$EQ = \delta + 90°$, the polar distance of the Earth as seen from the planet, and

$EQK =$ the difference between the R. A. of the Earth and the R A. of the pole of the orbit of the satellite,

$= (\alpha - 180°) - (N - 90°)$

$\alpha - N - 90°$;

hence the remaining three parts of the triangle can be found.



Since the planes PKE and PKQ pass through the pole of the orbit, they are perpendicular to its plane and the former intersects the orbit in those points which, as seen from the Earth, are nearest to the planet, or in other words, the intersection of the plane PKE with the apparent orbit, determines the minor axis, while the plane PKQ being perpendicular to the planes of the equator and of the orbit of the satellite, intersects the latter at a point 90° distant from the node. The angle EKQ being the inclination of these two planes is measured by the arc of the orbit intercepted between these two points of intersection; now the side KQ will intersect the orbit in a point whose position is $90° + N$; and if we denote the position of the intersection of the side KE with the orbit by β, we shall have

$$\text{the angle } EKQ = 90° + N - \beta.$$

But since the plane PKE intersects the apparent orbit in its minor axis, the position at that point of the orbit at which the satellite, as seen from the Earth, will be at its greatest eastern elongation will be $\beta - 90°$, and therefore the distance of this point from the node will be

$$\beta - 90° - N$$

therefore we have $\quad u_0 = \beta - 90° - N.$

But we have just shown that the

EKQ

$$\text{angle } KEQ = -\beta + 90° + N$$
$$= -u_0.$$

The angle KEQ is the angle of position of the minor axis of the apparent orbit; hence we have

KEQ

$$\text{angle } EKQ = q = p_0 - 90°.$$

The side KE is the angular distance of the Earth from the pole of the orbit, as seen from the planet and is equal to the angle BPC; therefore we have

$$KE = 90° - \theta,$$

where θ is always to be taken between $+ 90°$ and $- 90°$. When

θ is *positive* the Earth will be on the *north* side of the plane of the orbit and the position angle will *increase*, and where θ is *negative* the Earth will be on the *south* side and the position angle will *decrease*.

The fundamental formulæ of spherical trigonometry being applied to the triangle EKQ, we have

$$\begin{aligned}
\cos EK &= \cos KQ \cos EQ + \sin KQ \sin EQ \cos KQE \\
\sin EK \cos EKQ &= \sin KQ \cos EQ - \cos KQ \sin EQ \cos KQE \\
\sin EK \sin EKQ &= \sin EQ \sin KQE \\
\sin EK \cos KEQ &= \sin EQ \cos KQ - \cos EQ \sin KQ \cos KQE \\
\sin EK \sin KEQ &= \sin KQ \sin KQE
\end{aligned}$$

But we have shown that

$$KQ = i,\ EQ = \delta + 90^\circ \text{ and } KQE = \alpha - N - 90^\circ$$

which are known; and

$$EK = 90^\circ - \theta,\ EKQ = -u_0 \text{ and } KEQ = p_0 - 90^\circ$$

which are to be found.

Substituting these values of the sides and angles in the preceding formulæ, we have

$$\begin{aligned}
\sin \theta &= -\cos i \sin \delta + \sin i \cos \delta \sin (\alpha - N) \\
\cos \theta \cos u_0 &= -\sin i \sin \delta - \cos i \cos \delta \sin (\alpha - N) \\
\cos \theta \sin u_0 &= \cos \delta \cos (\alpha - N) \qquad (224) \\
\cos \theta \sin p_0 &= \cos i \cos \delta + \sin i \sin \delta \sin (\alpha - N) \\
\cos \theta \cos p_0 &= \sin i \cos (\alpha - N)
\end{aligned}$$

for the determination of θ, u_0 and p_0. The agreement of the two values of $\cos \theta$ with the one value of $\sin \theta$ will serve to check the accuracy of the computation. It now remains to transform these equations for logarithmic calculation.

If we put

$$\begin{aligned}
c \sin C &= \cos i \\
c \cos C &= \sin i \sin (\alpha - N) \qquad (225)
\end{aligned}$$

and

$$\begin{aligned}
c' \sin C' &= \sin i \\
c' \cos C' &= -\cos i \sin (\alpha - N)
\end{aligned}$$

and remembering that for the greatest western elongation $-u_0$ becomes $-u_0 + 180^\circ$ and p_0 becomes $p_0 + 180^\circ$, we shall have

$$\begin{aligned}
\sin \theta &= c \cos (C + \delta) \\
\cos \theta \cos u_0 &= \pm c' \cos (C' + \delta) \\
\cos \theta \sin u_0 &= \pm \cos \delta \cos (\alpha - N) \qquad (226) \\
\cos \theta \sin p_0 &= \pm c \sin (C + \delta) \\
\cos \theta \cos p_0 &= \pm \sin i \cos (\alpha - N)
\end{aligned}$$

where the upper sign applies to the greatest eastern and the lower to the greatest western elongation. Now if u denote the distance of the satellite from the ascending node at the date T and if t denote the time of greatest elongation we shall have

$$(t - T)\,\mu = u_0 - u$$

whence
$$t = T + \frac{u_0 - u}{\mu} \tag{227}$$

where μ is the daily (or hourly) motion in orbit. We also have

$$\begin{aligned} b'' &= a'' \cos KE \\ &= a'' \sin \theta \end{aligned} \tag{228}$$

which determines the semi minor axis of the apparent orbit.

The British Nautical Almanac employs the following formulæ for determining q and θ, viz.:

$$\tan q = -\frac{\sin Q \cot (\alpha - N)}{\cos (Q - \delta)}$$

and
$$\tan \theta = \tan (Q - \delta) \cos q$$

where
$$\tan Q = \tan i \sin (\alpha - N)$$

of which the demonstration is as follows:—Resuming the first and last two equations of (224), remembering that $p_0 = q + 90°$, we have

$$\begin{aligned} \sin \theta &= - \cos i \sin \delta + \sin i \cos \delta \sin (\alpha - N) && (a) \\ \cos \theta \cos q &= \cos i \cos \delta + \sin i \sin \delta \sin (\alpha - N) && (b) \\ \cos \theta \sin q &= - \sin i \cos (\alpha - N) && (c) \end{aligned}$$

Dividing (c) by (b) and putting $\tan Q = \tan i \sin (\alpha - N)$ we get

$$\begin{aligned} \tan q &= -\frac{\sin i \cos (\alpha - N)}{\cos i \cos \delta + \sin i \sin \delta \sin (\alpha - N)} \\ &= -\frac{\tan i \cos (\alpha - N)}{\cos \delta + \sin \delta \tan Q} \\ &= -\frac{\tan i \cos (\alpha - N) \cos Q}{\cos (Q - \delta)}, \text{ but } \tan i = \frac{\tan Q}{\sin (\alpha - N)} \\ &= -\frac{\sin Q \cot (\alpha - N)}{\cos (Q - \delta)} \end{aligned} \tag{229}$$

Again dividing (a) by (b) we have after reducing

$$\frac{\tan \theta}{\cos q} = \frac{-\sin \delta + \cos \delta \tan Q}{\cos \delta + \sin \delta \tan Q} = \tan (Q - \delta)$$

therefore
$$\tan \theta = \tan (Q - \delta) \cos q \tag{230}$$

A check on these formulæ may be deduced thus:—

$$\text{since } \tan Q = \tan i \sin (\alpha - N)$$

we have
$$\sin i \sin (\alpha - N) = \frac{\cos i \sin Q}{\cos Q}$$

and dividing this by (b) we get

$$\frac{\sin i \sin(\alpha - N)}{\cos\theta\cos q} = \frac{\cos i \sin Q}{\cos Q\big(\cos i\cos\delta + \sin i \sin\delta\sin(\alpha - N)\big)}$$

$$= \frac{\sin Q}{\cos Q\big(\cos\delta + \sin\delta\tan i\sin(\alpha - N)\big)}$$

$$= \frac{\sin Q}{\cos Q(\cos\delta + \sin\delta\tan Q'}$$

$$= \frac{\sin Q}{\cos(Q - \delta)} \tag{231}$$

an equation which involves all the known and unknown quantities and which will be satisfied only when they are all correct.

GEOCENTRIC CONJUNCTION.

At the instant of geocentric conjunction $p = 0$ or 180°, $\sin p = 0$ and $\cos p = \pm 1$.

therefore the first of (220) gives

$$\sin(F + u) = 0$$

or $\qquad F + u = 0$ or 180°

therefore $\qquad u = -F$ or $u = 180^\circ - F$

and from (216) we get

$$\tan F = -\frac{\tan(\alpha - N)}{\cos i}$$

and therefore in both cases we have

$$\tan u = \frac{\tan(\alpha - N)}{\cos i} \tag{232}$$

hence u and $(\alpha - N)$ will be in the same quadrant. This formulæ results also from the right spherical triangle whose base is $(\alpha - N)$ and one of its angles i.

From the second of (220) we have when $p = 0$

$$s = \frac{a}{\varDelta}\sin g \sin(G + u)$$

$$= A\sin(G - F) \qquad \text{where} \qquad A = \frac{a}{\varDelta}\sin g$$

and when $p = 180$

$$-s = A\sin(G + 180^\circ - F) = -A\sin(G - F)$$

and therefore in both cases

$$s = A\sin(G - F) \tag{233}$$

When the real orbit is an ellipse as in the case of Titan, Hyper-

ion and Iapetus, the values of r and u for any date, are obtained from the elements of the satellite's orbit thus:

$$u = \text{mean anomaly} + \text{equation of the centre} + \text{distance from node to periplaneten}$$

$$= M + E + \omega \qquad (234)$$

The values of r and E (the equation of the centre) are furnished by tables prepared for each satellite, or they may be computed from the following formulæ, for the demonstration of which see the author's paper in the Monthly Notices of the Royal Astronomical Society of England, Vol. 43, No. 7, May 1883.

$$\begin{aligned}
\log \frac{r}{a} = {} & \frac{e^2}{4} + \frac{e^4}{32} + \frac{e^6}{96} + \ldots \\
& + \left(-e + \frac{3}{8} e^3 + \frac{e^5}{64} + \ldots\ldots\right) \cos M \\
& + \left(-\frac{3}{4} e^2 + \frac{11}{24} e^4 + \frac{3}{64} e^6 + \ldots\right) \cos 2M \\
& + \left(-\frac{17}{24} e^3 + \frac{77}{128} e^5 + \ldots\ldots\right) \cos 3M \\
& + \left(-\frac{71}{96} e^4 + \frac{129}{160} e^6 + \ldots\ldots\right) \cos 4M \\
& + \ldots\ldots\ldots\ldots
\end{aligned} \qquad (235)$$

and

$$\begin{aligned}
E = {} & \left(2e - \frac{e^3}{4} + \frac{5}{96} e^5 + \ldots\ldots\right) \sin M \\
& + \left(\frac{5}{4} e^2 - \frac{11}{25} e^4 + \frac{17}{192} e^6 + \ldots\right) \sin 2M \\
& + \left(\frac{13}{12} e^3 - \frac{43}{64} e^5 + \ldots\ldots\right) \sin 3M \\
& + \left(\frac{103}{96} e^4 - \ldots\ldots\right) \sin 4M \\
& + \ldots\ldots\ldots
\end{aligned}$$

Where e is the eccentricity, M the mean anomaly and a the satellite's mean distance or the semi-axis major of its orbit.

The values of s and p and also of u_0 and p_0 can then be computed on the supposition of a circular orbit of radius r as above found.

Since the angular velocity in an elliptic orbit varies inversely as the square of the radius vector, if μ be the angular velocity (per minute or hour) at the mean distance a, then the angular velocity at distance r will be $\mu_1 = \frac{\mu}{r^2}$ which is to be used for μ in (227) in computing the correction to T. If u_0 differs considerably from u,

the value of t thus found will be only an approximation, and a recomputation for this approximate time will be necessary.

For the time of conjunction, u is determined by (232) which must also satisfy (234) subject to the condition that $p = 0°$ or 180° and the distance from the centre of the planet by (233).

It now remains to make an application of the preceding formulæ and for this purpose we will compute the apparent position and date of greatest elongation of Deimos for 1899 Jan'y 18*d* Greenwich M. T.

The elements of the orbit of Deimos corrected by the observations made at the Lick Observatory in 1888 and reduced to the equator and equinox of the above date are as follows:

EPOCH 1899 JAN'Y $18^d.0$ G. M. T.

Period	$30^h\ 17^m\ 9026$
μ	$285°.163634$
a	$32''.36$ (at distance 1)
i	$35°\ 34'.7$
N	$48\quad 16$
u	$300\ .63887$

Suppose we want the apparent position at Jan'y 18*d* 18^h G. M. T.

The motion in 18 hours $= 213°.87273$, therefore for the above date $u = 154°.5116$. We also have $\alpha = 121°\ 43'$, $\delta = +\ 24°43'.3$, $\log \Delta = 9.81400$ and $\alpha - N = 73°\ 27'$. Then by (216) and (217) we find $F = 283°\ 35'\ 15''$, $\sin f = 9.99395$, $G = 329°\ 31'\ 35''$, $\sin g = 9.37089$, $F + u = 78°\ 5'\ 57''$, $G + u = 124°\ 2'\ 17''$, $\log \frac{a''}{\Delta} = 1.69601$ or $\frac{a}{\Delta} = 49''.66$ the semi-major axis of the apparent orbit. From (220) we easily find $p = 78°\ 35'\ 45''$ and $s = 48''.885$. Comparing the values of $\frac{a}{\Delta}$ and s we see that the satellite is not far from its greatest elongation, *i. e.* near the extremity of the major axis of its apparent orbit.

For the position angle of the major axis and the date of greatest elongation we find from

$$
(225)\qquad
\begin{array}{rlrl}
C = & 55°\ 33'\ 40'', & \log c & = 9.99395 \\
C' = & 143\quad 16\quad 0 & \log c' & = 9.98802 \\
C + \delta = & 80\quad 16\quad 59 & C' + \delta & = 167°\ 59'\ 19''
\end{array}
$$

Then by (226) we easily find
$\sin \theta = 9.22126$, $\theta = 9°\ 34'\ 51'' =$ the elevation of the Earth *above* the plane of the orbit.

We also find	$u_0 = 164°\ 47'\ 10''$,	$\cos \theta = 9.99391$
and	$p_0 = \ 80\quad 19\quad 25$	$\cos \theta = 9.99391$
also	$b = 8''.266$ and	$\theta = 9°\ 34'\ 51''$
semidiam. of ♂	$= 7\ .7$	

From the values of u_0 and u and of p_0 and p we see that the satellite has not quite arrived at its greatest eastern elongation.

Then by (227) we have

$$\begin{aligned} t = T + \frac{u_0 - u}{\mu} &= \text{Jan. } 18d\ 18^{\text{h}} + \frac{10.2755}{11.8818} \\ &= \text{Jan. } 18d\ 18^{\text{h}} + 0^{\text{h}}.865 \\ &= \text{Jan. } 18d\ 18^{\text{h}}.865 \text{ G. M. T.} \\ \text{or} \qquad t &= \text{Jan. } 18d\ 13^{\text{h}}.729 \text{ Wash. M. T.} \end{aligned}$$

If we add one period or $1d\ 6^{\text{h}}.3$ we shall obtain the date of the next greatest eastern elongation or Jan. $19d\ 20^{\text{h}}.03$.

The ephemeris gives Jan. 19 21.1 which is more than an hour in error. It was probably computed from the uncorrected elements of 1879 Nov. 5, G. M. T.

The position angle and distance are about correct but the apparent orbit is not correctly delineated for the date of opposition. The satellite grazes the southern limb of the planet, the semi-minor axis being only half a second greater than the semi-diameter of the planet.

PLANET NOTES FOR MAY.

H. C. WILSON.

Mercury, having just passed inferior conjunction, will be invisible during the first part of May, but during the last days of the month may be seen toward the southeast about a half hour before sunrise. Mercury will be at greatest elongation, west from the Sun 24° 45′, on the morning of May 28.

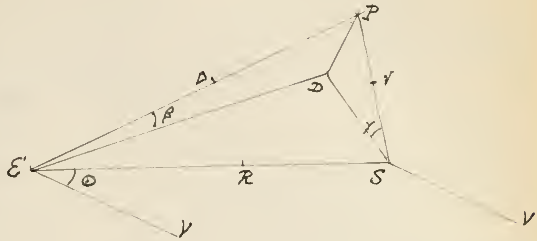
Venus is now becoming conspicuous in the evening twilight. Venus will pass between the Pleiades and Hyades during the first days of the month and on the 18th will pass two degrees and a third north of Neptune. On the 22d, at about noon by Central Standard time, Venus will be in conjunction with the Moon, 51′ south of the latter, and to observers in high northern latitudes an occultation might be witnessed with the aid of a telescope. The phase of Venus is gibbous, nearly full, and her brightness less than one-third of that at her maximum.

Mars comes to the meridian at about half past nine o'clock in the morning and so is visible only a short time before sunrise and then in an unfavorable position.

Jupiter is now at his best for evening observation, on the meridian shortly after 9 o'clock and at his greatest northern declination for the year during May. The belts are quite vivid in coloring and the great equatorial belt, which sometimes has been nearly white, is now filled with mottled patches of light red color. The "great red spot" is seen with great difficulty and can be recognized only by the bend in the white belt north of it and the notch in the dark red belt. Its southern portion is either covered by, or merged into, the red belt which formerly has made a detour to the south as it passed by the "great red spot."

Saturn may be observed about midnight in Scorpio. This planet will be at

Transformation of Heliocentric Coordinates



Let the plane of the paper represent the plane of the Ecliptic, S the sun, E the earth and P a planet. draw PD perpendicular to the plane of the ecliptic, join DS, DE, PS and PD and let S♈ and E♈ be the direction of the vernal equinox: the we shall have

♈SD = λ, the planets heliocentric Longitude
PSD = γ, " " Latitude
PS = r, " radius vector.

also, ES = the Suns distance or the Earths radius Vector, R
♈ES = the Suns Longitude and
♈SE = ⊙ suns longitude + 180°
= the Earths Heliocentric longitude

all of which are derived from the Planetary and Solar tables.

♈ED = ψ, the planets geocentric Long.
PED = β. " " Latitude
and PE = Δ " " distance
which are to be found.

The triangle DES lies on the plane of the ecliptic and its sides are R, $r\cos\gamma$ and $\Delta\cos\beta$, the angle

$$DSE = VSE - VSD$$
$$= \odot + 180 - \lambda$$
$$DES = VED - VES$$
$$= \psi - \odot$$

Therefore we have from these data

$$\Delta\sin\beta = r\sin\gamma \quad \text{- - -} \quad (1)$$
$$\Delta\cos\beta\sin(\psi - \odot) = r\cos\gamma\sin(\odot - \lambda + 180) \quad (2)$$
and
$$\Delta\cos\beta\cos(\psi - \odot) = R + r\cos\gamma\cos(\odot - \lambda + 180) \quad (3)$$

from which ψ, β and Δ can be found

The geocentric coordinates of a planet are never tabulated in an Ephemeris; their only use is in the computation of the orbit of a planet or a comet and in that case they are computed from the observed Right Ascension and Declination.

Rectangular Coordinates.

If three planes perpendicular to one another be passed through the centre of the Earth, their inter-sections will form three rectangular axes. The position of a point in space - a planet for instance -

will be determined by its perpendicular distance from these three axes. In the case now considered the planes are the plane of the Earth's orbit, the plane of the solstitial colure and a plane perpendicular to both of these, the axis of x lies on the plane of the earth's orbit and joins the equinoctial points, the axis of y perpendicular to it and the axis of z perpendicular to both. If x, y, z be the rectangular coordinates referred to the ecliptic or Earth's orbit, we shall evidently have

$$x = r \cos\gamma \cos\lambda$$
$$y = r \cos\gamma \sin\lambda$$
$$z = r \sin\gamma$$

Now, if this ~~system~~ be revolved around the axis of X through an angle ω, the obliquity of the ecliptic, the coordinate x will remain unchanged and y and ~~be~~ z will be referred to the plane of the equinoctial as the fundamental plane, and if x', y' and z' be the new coordinates we shall have

$$x' = x$$
$$y' = y \cos\omega - z \sin\omega$$
$$z' = y \sin\omega + z \cos\omega$$

in which substitute the values of x, y and z and we have

$$x' = r\cos\gamma\cos\lambda$$
$$y' = r\cos\gamma\sin\lambda\cos\omega - r\sin\gamma\sin\omega$$
$$z' = r\cos\gamma\sin\lambda\sin\omega + r\sin\gamma\cos\omega$$

Put $\quad r\cos\gamma\sin\lambda = h\cos\theta$

and $\quad r\sin\gamma = h\sin\theta$

that is $\quad \tan\theta = \dfrac{\tan\gamma}{\sin\lambda}$ and $h = \dfrac{r\sin\gamma}{\sin\theta}$

And we have.

$$x' = r\cos\gamma\cos\lambda$$
$$y' = h\cos(\theta+\omega) = \frac{r\sin\gamma}{\sin\theta}\cos(\theta+\omega)$$
$$z' = h\sin(\theta+\omega) = \frac{r\sin\gamma}{\sin\theta}\sin(\theta+\omega)$$

which are the rectangular coordinates of the planet referred to the equinoctial or plane of the equator as the fundamental plane, and since they are respectively parallel to the Sun's Coordinates to the same plane, therefore by adding them algebraically we have

$$x' + X = \Delta\cos\delta\cos\alpha$$
$$y' + Y = \Delta\cos\delta\sin\alpha$$
$$z' + Z = \Delta\sin\delta$$

in which α and δ are the R.A. and declination, Δ the distance.

Whence we get for the true Right Ascension and Declination

$$\tan\alpha = \frac{y'+Y}{x'+X}.$$

$$\tan\delta = \frac{z'+Z}{y'+Y}\sin\alpha$$

$$= \frac{z'+Z}{x'+X}\cos\alpha$$

And $$\Delta = \frac{z'+Z}{\sin\delta}.$$

The two values of $\tan\delta$ will serve to check the accuracy of the work.

If m and n denote the hourly motion in R.A. and Dec we shall have

Aberration in R.A. = $[9.140776]\, m\,\Delta$

Aberration in Dec. $[9.140776]\, n\,\Delta$

which applied with the proper sign to the true place will give the apparent place as tabulated in the Ephemeris. bearing always in mind that the true place is always in advance of the apparent,

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THE SUN DIAL OF AHAZ.

J. MORRISON, M.D., M.A., PH.D.*

FOR POPULAR ASTRONOMY.

Time like space cannot be defined; we conceive it to be unlimited, continuous, homogeneous and divisible without end, but these characteristics convey to us no idea of its nature or essence. It is, however, unlike ~~time~~ in that it has only one dimension. As space/ regards continuous duration we can only look backward to the past and forwards to the future. It can therefore be represented graphically by a straight line extending in opposite directions to infinity.

Space on the other hand has three dimensions, length, breadth and depth or height, each extending in opposite directions to infinity. Plane or Euclidean geometry makes us acquainted with the properties of space of one and two, and solid geometry with those of three dimensions; and notwithstanding its apparent impossibility, there may be a fourth dimension, or what is equivalent thereto, of which we have not and can not have, from our present standpoint, any consciousness while in our present state of existence.

In the future life we may possibly live in four dimensional or hyperspace, but however this may be, we are certain of one thing, viz.: we shall exist independently of matter and probably also of time and space as we now understand them. The discussion of these quantities in the abstract involves the consideration of the infinite, which our finite minds cannot grasp and about which we cannot reason intelligently.

The determination of a scientific unit for the measurement of time, has been not only a very difficult problem but one which has required centuries for its solution. With space it is quite otherwise, for we can apply an arbitrary unit such as a yard measure as often as necessary to find how many yards a given length contains. Our unit too will remain invariable while the temperature is constant. In the case of time however we can

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not keep such a measure by us, nor can we repeat the measurement over and over again because time once passed is irretrievably lost forever. Neither will an appeal to our senses justify any result of our measurements. The recurrence of certain astronomical phenomena first furnished man with the natural units of time; thus, the Sun in his annual and diurnal motions marked out times and seasons, days and years, but it was by no means an easy matter to determine their precise beginning and ending. The subsequent artificial subdivisions of the day into hours, minutes and seconds is of great antiquity and is due to the Chaldeans. It was soon discovered that the year and day are not of constant length and that the interval from noon to noon varied from day to day. The only invariable unit of time we have is furnished by the diurnal motion of the Earth; the time it takes to turn once on its axis or through 360° is assumed to be always the same and is called a sidereal day, hence in 1 hour it turns through 15°, in one minute 15′ and so on. We are however by no means certain that this unit is absolutely invariable. Geodetic measurements have shown that the Earth is not exactly an oblate spheroid of revolution, nor does it appear that the density is the same at equal depths, from which circumstances it may result that the axis of revolution does not coincide with the geometric axis. These, combined with the increase of mass due to the fall of meteoric matter together with tidal friction, may and probably do cause the length of the sidereal day to vary, and there is rter good reason to believe that it is now about $\frac{1}{60}$ of a second ~~longer~~ than when the Egyptians were laying the foundations of their loftiest pyramid.

The sidereal day can only be determined by refined astronomical observations and is therefore not a suitable unit for the ordinary purposes of civil life. The apparent or true solar day is the time the Earth makes one revolution in regard to the Sun and it is apparent or true noon when the Sun is on the meridian. Owing to the motion of the Earth in its orbit or to the apparent motion of the Sun eastward, the true or apparent solar day is about four minutes longer than the sidereal. The apparent solar days are unequal and the mean or average of them all during a year is called a mean solar day which is also divided in hours, minutes and seconds, and these are now the units employed in the ordinary affairs of life in all civilized countries.

Mean time is comparatively a recent invention and was not introduced into France until the year 1816.

Formerly there was no common agreement as to when the day

began. In all civilized countries, the day is now regarded as beginning at midnight, as was first suggested by Hipparchus.

Ptolemy and the ancient Egyptians counted the 24 hours from noon and this practice is still followed by astronomers. There is also other ancient authority for the same method, thus we read in Genesis "The evening and the morning were the first day," and so on, from which language we infer that the day was regarded as beginning at noon.

The earliest devices for subdividing the day into hours, of which we have any positive knowledge, were the gnomons or styles employed in Ancient Egypt and Asia Minor. These consisted at first of a vertical pillar or style erected at the centre of a series of concentric circles described on a horizontal plane and located at such distances from each other that the extremity of the shadow of the pillar or style passed from one to another during approximately equal intervals of time. Of course such a device could be accurate only on certain days of the year. It was only an approximation and a rough one at that.

Subsequently the style or gnomon was inclined or directed to the pole about which, it was observed, all the heavenly bodies appeared to revolve. In order to give it stability, it was made in the form of a right-angled triangle whose plane coincided with the meridian and whose hypothenuse had an elevation above the horizontal plane equal to the latitude of the place. Such an instrument when accurately constructed and correctly adjusted, is capable of indicating true or apparent time throughout the entire year. Owing to atmospheric refraction which always elevates the Sun in a vertical direction, dial time is a trifle too fast in the forenoon and too slow in the afternoon, but the effect is too small to be considered in the construction of dials which even when most carefully made, do not admit of any great degree of accuracy.

The earliest record of sun-dials which we possess dates from about 712 B. C. and is found in II Kings, Chap. 20, v. 9-11, which are as follows:

"And Isaiah said, This sign shalt thou have of the Lord, that the Lord will do the thing that he hath spoken: shall the shadow go forward ten degrees or back ten degrees?"

"And Hezekiah answered, It is a light thing for the shadow to go down ten degrees: nay, but let the shadow return backward ten degrees. And Isaiah, the prophet, cried unto the Lord, and he brought the shadow ten degrees backward by which it had gone down in the dial of Ahaz."

And as if this were not enough to impress all future generations, the prophet Isaiah himself referring to the same subject in his Book of prophecy, Chap. 38, v. 8, says, "Behold, I will bring again the shadow of the degrees, which is gone down in the sun-dial of Ahaz, ten degrees backward. So the Sun returned ten degrees, by which degree it was gone down."

And here we may ask, why the sun-dial of Ahaz? Would not any other sun-dial have served the same purpose?

This is regarded by most people as a miracle and a first class one at that, equalling if not surpassing the alleged performance of Joshua; he only made the Sun stand still but Isaiah not only made the Sun come to a standstill but also made it go backwards to such an extent that the shadow on the sun-dial of Ahaz retrograded ten degrees, then brought it to a standstill again and allowed it to resume its ordinary course. This extraordinary movement of the Sun implies, as we now know, corresponding changes in the diurnal motion of the Earth—charges which did not and could not take place without disastrous consequences to our globe. There was no suspension of the laws of motion, nor did the Earth experience the slightest disturbance. Moreover, we have no evidence that the Creator has ever violated or could violate His own laws. These and similar passages are accepted by many people "on faith" which, in many cases, is very elastic. Theologians and pulpit orators push them aside, not being able to offer any rational or intelligent explanation. In the hands of sceptics and infidels, they have contributed not a little to discredit the Scriptures in many quarters.

In this paper we purpose to discuss the subject mathematically and to show by the rigorous formulæ of mathematics, that this apparently extraordinary conduct of the sun-dial of Ahaz, is not only possible from an astronomical point of view but that the statement of the prophet is, without the shadow of a doubt, literally and absolutely true.

The sacred record does not inform us as to the form of the dial or the latitude of the place for which it was constructed. It will therefore, be necessary for a better understanding of the subject to refer briefly to the history of dials and to their form and construction in general.

The Chaldean astronomer, Berosus, who lived about 540 B. C., was the first to construct hemispherical dials. These consisted of a hollow hemisphere, with its rim placed truly horizontal, and a small bead or globule supported in any way at its centre. When the Sun was above the horizon, the shadow would fall on the in-

side of the hemisphere and during the course of the day, would describe an approximately circular arc which could be divided into any number of equal parts, which indicated what were then called *temporary hours*. At the equinoxes the shadow would describe a great circle and at all other times, arcs of small circles, and of these a large number would evidently be required, owing to the variable declination of the Sun. This form of dial was very difficult to construct and could only furnish a rough approximation at the best. It is in every respect inferior to the ordinary and more simple horizontal or declining dial, with which the ancients were also familiar, and of which we shall speak presently.

About the middle of the last century (1746-1762), four hemispherical dials were discovered in Italy, one of which probably belonged to Cicero as he states in one of his letters that he sent a sun-dial to his villa at Tusculum, now Tivoli, where it was found. Another was unearthed at Pompeii, and an eminent German authority (Martini), who carefully examined it, says that it was constructed for the latitude of Memphis in Egypt. This fact is important as showing that in ancient times dials were erected in latitudes differing by many degrees from those for which they were constructed, an adjustment to the original latitude being in such cases always necessary. About the year 290 B. C., Papirius Cursor erected the first sun-dial at Rome. This dial had been taken from the Samnites, who probably obtained it from Greece or Egypt. In the year 261 B. C., Valerius Messala brought a dial from Catania in Sicily and erected it in the forum. Here is another instance of a dial being erected in a locality about five degrees north of the place for which it was constructed. This also was, no doubt, the work of a foreign artist, for a dial was not actually constructed at Rome until the year 164 B. C.

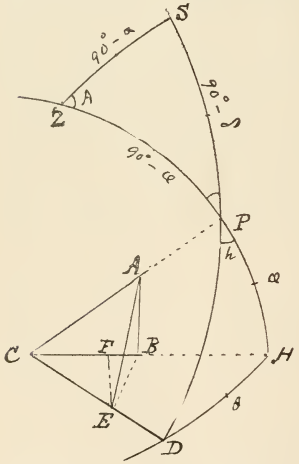
Herodotus informs us that the ancient Greeks obtained their knowledge of sun-dials from the Babylonians, and the extensive acquaintance of geometry which the former possessed, enabled them to construct dials of great excellence, some of which are still in existence. One of the most interesting monuments of antiquity is the Tower of the Winds at Athens; its form is that of a regular octagon, on the faces or sides of which eight different dials are shown—four facing the cardinal points and the other four the intermediate points. Meton erected the first sun-dial in Athens in B. C. 433, and portable dials are said to have been in use in China in very ancient times.

Ptolemy, in his Syntaxis, describes the construction of ordinary horizontal and vertical dials by means of his analemma, an instrument which he invented and which enabled him to solve graphically many astronomical problems.

The Arabians, in both ancient and mediæval times, were very expert in the construction of dials, surpassing even the Greeks themselves.

The horizontal dial is the simplest, the most easily constructed and adjusted, and doubtless the most ancient, consisting at first of a vertical style or pillar erected on a horizontal plane, at the centre of a series of concentric circles as already stated, and subsequently of an inclined style placed in the plane of the meridian and having an elevation equal to the latitude of the place—a form which admits of no further improvement or simplification. The sun-dial of Ahaz must have been of this kind as no other form could produce such apparently extraordinary results.

When we stand on the south side of an ordinary horizontal dial with our face towards the north, we observe the shadow of the style moving from left to right, that is, in the same direction as the hands of a clock or watch. In fact the early clock makers took the sun dial for their model and arranged the mechanism so that the hands moved in the same direction.



We will first determine the locus of the extremity of the shadow on the horizontal plane. In the diagram, let ABC be the style and CE the boundary line of the shadow on the plane of the dial, B be the origin, BH the initial line, and x, y the coördinates of E; draw EF perpendicular to BH, and let the angle $EBF = A$, the Sun's azimuth, $AEB = \alpha$ the Sun's altitude and $AB = a$, the perpendicular height of the style.

Then $\cos A = \dfrac{x}{\sqrt{x^2 + y^2}}$, $\sin \alpha = \dfrac{a}{\sqrt{x^2 + y^2 + a^2}}$, $\cos \alpha = \sqrt{\dfrac{x^2 + y^2}{x^2 + y^2 + a^2}}$

and from the triangle ZSP in which $SP = 90 - \delta$, $ZP = 90 - \varphi$ and $ZS = 90 - \alpha$, we have

$$\sin \delta = \sin \alpha \sin \varphi + \cos \alpha \cos \varphi \cos A$$

and substituting the above values of $\cos A$, $\sin \alpha$ and $\cos \alpha$ we have after some easy reductions

$$(\sin^2\delta - \cos^2\varphi)\, x^2 + \sin^2\delta . y^2 - a \sin 2\varphi . x + a^2 (\sin^2\delta - \sin \varphi) = 0 \quad (1)$$

which is the equation of a conic section, and therefore when

$$\delta + \varphi \begin{cases} > 90^\circ, & \text{the locus is an ellipse} \\ = 90\ , & \text{“ \quad “ \quad parabola} \\ < 90\ , & \text{“ \quad “ \quad hyperbola} \end{cases}$$

We may here note some special cases

1. If $\varphi = 0$ or the place is on the equator, we shall have

$$\cos^2\delta . x^2 - \sin^2\delta . y^2 = a^2\sin^2\delta$$

which is a hyperbola.

2. If $\varphi = 90^\circ$ or the pole, we have

$$x^2 + y^2 = a^2\cot^2\delta$$

which is a circle.

3. If $\delta = 0$ or the Sun is on the equator we easily find

$$x = - a \tan \varphi,$$

which represents a straight line parallel to the axis of y.

4. If $\delta = \varphi = \omega$, the obliquity of the ecliptic, we find

$$\cos 2\omega . x^2 - \sin^2\omega . y^2 + a \sin 2\omega . x = 0$$

which also represents a hyperbola.

If we conceive the three planes ABC, CBE and ACE to be produced indefinitely they will determine on the celestial sphere a right spherical triangle PHD, in which PH is the latitude of the place and the angle HPD, the hour angle. The arcs HP and DP produced will pass through the zenith and the Sun respectively; join ZS, and let $HP = \varphi$, $HPD = h$, HD which measures the angle $HCD = \theta$ and PZS, the Sun's azimuth $= A$.

Then from the right spherical triangle PHD we have

$$\tan \theta = \sin \varphi \tan h \qquad (2)$$

This equation enables us to determine the boundary line CE of the shadow for every hour, half hour, quarter hour and so on.

If the plane HCD is not horizontal but inclined to the horizon by an angle ψ and still perpendicular to the plane of the meridian HPC, we shall then have $PH = \varphi - \psi$ and the angle HPD remaining as before formula (2) becomes

$$\tan \theta = \sin (\varphi - \psi) \tan h \qquad (3)$$

This is in fact the formula for computing a horizontal dial for a

latitude equal to $(\varphi - \psi)$; therefore if a horizontal dial constructed for a given place, be carried to any other place, it will be an inclined dial for the latter place and its inclination to the horizon will be equal to the difference of the latitudes of the two places; thus suppose a horizontal dial constructed for Lat. 18° *N* be carried to a place in Lat. 38° *N*, it must be placed at an inclination of 20° to the horizon in order that the elevation of the style may be equal to 38°, that is to say, the south side of the dial must be tilted downwards through an angle of 20°, the difference of the latitudes, the plane of the style coinciding of course with the plane of the meridian of the place. The ancients were acquainted with this form of dials, for we have already cited two such cases; the first a dial found in Pompeii but constructed for the latitude of Memphis and the second one erected in the Roman Forum but made for the latitude of Catania some 5° south.

From the spherical triangle *PZS* we have

$$\cot A = \frac{\tan \delta \cos \varphi - \sin \varphi \cos h}{\sin h}$$

$$= \tan \delta \cos \varphi \operatorname{cosec} h - \sin \varphi \cot h \qquad (4)$$

Now if during the day, the Sun's azimuth has a maximum value, the second member of the last equation must be a minimum, and to determine whether this is the case or not we differentiate equation (4) and put the differential co-efficient equal to zero, δ and φ being regarded as constant, thus

$$\frac{d. \cot A}{dh} = - \tan \delta \cos \varphi \cot h \operatorname{cosec} h + \sin \varphi \operatorname{cosec}^2 h = 0$$

Cancelling the common factor and solving we obtain

$$\cos h = \frac{\tan \varphi}{\tan \delta} \qquad (5)$$

which is possible only when $\delta > \varphi$ or when the Sun crosses the meridian *north* of the zenith of the dial.

Equation (5) also shows that when the azimuth attains its maximum value, the triangle *P Z S* is right angled at the Sun, or in other words, the Sun is at his greatest elongation east or west as the case may be.

If then the Sun transits north of the zenith, the shadow of the style will move forwards from sunrise to the instant of greatest elongation; it will remain stationary for a moment and then move backwards until the Sun attains his greatest western elongation, when it will again advance.

Now the latitude of Jerusalem where the dial of Ahaz was erected, is about 32° N or about 8½° north of the Tropic of Cancer and therefore the Sun could never transit north of its zenith.

The dial of Ahaz must therefore have been an inclined one constructed for some place whose latitude was less than 23½°, so that when erected at Jerusalem, its zenith was, about the time of the summer solstice, between the equator and the point of the Sun's transit and consequently the Sun's azimuth would have two maximum values between sunrise and sunset, the shadow would therefore go backwards on the dial during the interval in which the Sun was passing from his eastern to his western elongation, and he would also apparently move backwards during the same interval, because from sunrise to the eastern elongation, his motion would be eastward, then westward to the western elongation and then eastward again till sunset.

Of course when the declination of the Sun became less than the latitude of the zenith of the dial, the shadow would move forward during the whole day; the phenomenon could therefore only take place at or near the summer solstice.

The Hebrews never devoted themselves to the study of mathematics and astronomy, nor in fact to any of the other physical sciences. The dial of Ahaz was no doubt the work of a foreign artist in Egypt or Southern India, most probably the latter. If constructed in Egypt it must have been south of Syen or in upper Egypt on the border of Nubia, but we have no evidence that the people of that region were at that date far enough advanced in the arts and sciences to be able to construct such instruments. The ancient Hindoos however have left behind them abundant evidence of their ability as mathematicians and astronomers and were doubtless fully competent to construct any form of dial.

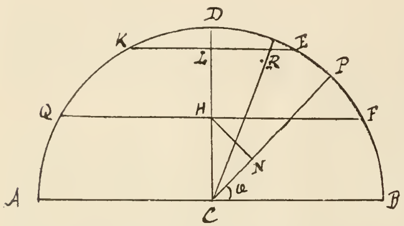
Although Descriptive Geometry is regarded as a comparatively recent invention—the work of the eminent French Geometer Monge—yet it is very probable that the Hindoo Geometers were able to solve the triangle *PHD* graphically and to lay down the lines of the shadow for the several hours, with all the accuracy necessary, and when a dial was once accurately constructed it could be copied indefinitely and the duplicates exported to foreign countries with instructions for their adjustment in other latitudes. In this way the dials of India, Babylon and Egypt found their way to Palestine, Greece and Italy for at the date referred to and for many centuries before, an extensive commercial intercourse existed between the former countries and those bordering on the Mediterranean.

Let ADB be a semi-circle, divide the quadrant DB into six equal parts of 15° each—we have only divided it into three equal parts of 30° each to avoid an excessive number of lines—take PCB equal to the latitude φ; draw the chords KE, QF, etc. cutting DC in L, H, etc.; draw HN perpendicular to PC and make LR equal to CN, join CR and produce it to meet the semi-circle, then CR will be the two o'clock hour-line of the shadow.

For HC being the sine of 30° or $\sin h$ we have

$$\begin{aligned} \sin h \sin \varphi &= CN \\ &= LR \\ &= LC \tan LCR \\ &= \cos h \tan \theta \end{aligned}$$

whence
$$\tan \theta = \sin \varphi \tan h$$
which is Eq. 1.



This construction is quite simple and completely avoids the more complicated processes of Descriptive Geometry. In fact, the hour angles may be laid down by means of the accompanying construction, involving only the principles of Elementary Geometry.

To determine the angle through which the shadow went backwards in terms of the latitude and declination we must eliminate h from (2) and (5); thus from (2) we have

$$\begin{aligned} \tan \theta &= \sin \varphi \tan h \\ &= \frac{\sin \varphi \sin h}{\cos h} \\ &= \frac{\sin \varphi \sin h}{\tan \varphi \cot \delta} \qquad (6) \\ &= \frac{\cos \varphi \sin h}{\cot \delta} \end{aligned}$$

whence
$$\sin h = \frac{\tan \theta \cot \delta}{\cos \varphi} \qquad (7)$$

Squaring (5) and (7) and adding, we find

$$\frac{\tan^2\theta + \sin^2\varphi}{\cos^2\varphi \tan^2\delta} = 1$$

whence

$$\tan^2\theta = \cos^2\varphi \tan^2\delta - \sin^2\varphi$$

$$= \frac{\cos^2\varphi - \cos^2\delta}{\cos^2\delta}$$

$$= \frac{(\cos\varphi + \cos\delta)(\cos\varphi - \cos\delta)}{\cos^2\delta}$$

$$= \frac{\sin(\delta+\varphi)\sin(\delta-\varphi)}{\cos^2\delta}$$

and
$$\tan\theta = \pm\frac{\sqrt{\sin(\delta+\varphi)\sin(\delta-\varphi)}}{\cos\delta} \qquad (8)$$

which is possible only when $\delta > \varphi$, or when the Sun crosses the meridian north of the zenith of the dial, φ being in all cases the latitude of the place for which the dial was constructed.

It will also be observed that θ has two values by reason of the double sign, that is the shadow will go back through double of the angle whose tangent is $\frac{\sqrt{\sin(\delta+\varphi)\sin(\delta-\varphi)}}{\cos\delta}$, and from (2) we have

$$\tan h = \frac{\tan\theta}{\sin\varphi}$$

$$= \frac{\sqrt{\sin(\delta+\varphi)\sin(\delta-\varphi)}}{\cos\delta\sin\varphi} \qquad (9)$$

which gives half the interval during which the shadow moved backwards, or the interval from either elongation to noon.

The conduct of the dial of Ahaz was therefore not miraculous but the result of natural causes.

The prophet as well as Hezekiah himself, was no doubt ignorant of this property of sun-dials and his question was doubtless prompted by a divine influx or impulse for which he was not responsible. The astronomical phenomenon however served the purpose of a sign to Hezekiah that the promise made would be fulfilled.

If they had observed the dial for several successive days they would have probably found the same thing which would have further confirmed the sign; and if observed some weeks or months afterwards they would have found the shadow constantly moving forward—a circumstance which would have created no doubt in their minds since a further confirmation of the sign was now no longer necessary.

The reader who may not be able to follow the mathematical proof just given, can obtain a clear view of the phenomenon by considering the case of the diurnal motion of a circumpolar

star—one in Ursa Major or the Great Dipper, for instance. Suppose the star is at its lower transit or below the pole, it is then due north of the observer. As it ascends the celestial vault it moves eastward until it attains a point where it appears for a moment to move straight up—the point of greatest eastern elongation—it then changes its apparent course and moves westward, crossing the meridian north of the zenith and continuing its westward motion until it arrives at its greatest western elongation, when it now for a moment appears to move straight down and then again moves eastward until it arrives at its greatest eastern elongation. Now if it were brilliant enough to cast a shadow we would evidently find that the shadow of a vertical pillar or style would first move forward on a horizontal plane until the star arrived at its greatest elongation, when it would remain stationary for a moment and then move back until the greatest western elongation, when it would again change its direction and move forwards. This is just what occurred with the sun-dial of Ahaz—the Sun taking the place of the star, that is to say, the Sun crossed the meridian north of the zenith of the dial. There was, therefore, no miracle but simply an ordinary astronomical phenomenon, which at certain times and under certain circumstances can happen anywhere. This is not the only astronomical phenomenon alluded to in the Bible, capable of a scientific explanation but we cannot discuss them just now. There are, however, many things recorded therein which cannot be so explained. They admit, however, of a rational and intelligent treatment when considered from the proper standpoint, and although an Astronomical Journal is not the most appropriate place for the discussion of miracles in general, the following brief allusion to them may not be out of place.

The human body was made for a temporary dwelling place of the soul or spirit, and not the reverse. The former is "of the Earth, earthy," returns to the earth and will never again be resumed, but the latter will enjoy a perpetual existence in the spiritual world; it must be of the same form, possess all the senses in an eminently exalted degree and perform all the functions which it did when temporarily residing in the body; it must be so, otherwise we would lose our identity and be no longer the same individuals we are here. While in the body it exercised the functions of the senses—sight, hearing, etc.—through material organs by which its powers were immeasurably diminished. Here we see as if "through a glass darkly," but there are numerous instances recorded in the Scripture where this material organ has been

pushed or set aside, as it were, and the spiritual function allowed to display its full activity.

The burning bush seen by Moses, the scene witnessed by Elijah and his servant, Joshua's stationary Sun, Saul's experience with the witch of Endor, the Transfiguration, the Star of Bethlehem and the scenes described in Chap. 27 v. 53 of Matthew and Chap. 21 of John, the circumstances attending the conversion of St. Paul and all the scenes alluded to in the Book of Revelation, were all seen or heard by the spiritual faculties and not through the agency of the material organs. They were witnessed or heard in the spiritual world and not in the material. The change was in the individuals and not in the things seen or heard. The experience of Elijah's servant proves this, and if the record is true and we believe it is, one such statement is just as conclusive as a hundred thousand. Moreover, the Apostle John states again and again in the Apocalypse that when he saw and heard the things there described, he was "in the spirit," that is, seeing and hearing as to his spiritual senses, and he frequently asserts that they are true for he says, "I, John, saw and heard them." There are also numerous well authenticated instances of persons obtaining glimpses or visions of the spiritual world just prior to their departure from this. These manifestations are usually regarded by most persons, as hallucinations or the results of disordered mental faculties, but we really know nothing about them. This reminds us of the practice of chemists half a century or more ago; when they found a phenomenon which they could not explain—and this happened very frequently—they referred it to catalytic action, the result of catalysis, whatever that may be.

The so-called miracles are all capable of a rational explanation—an explanation too which does no violence to the laws regulating matter, but which is in strict accord either with well known scientific principles or with what has been revealed to us of the spiritual nature of man.

1757 P St. N. W., Washington, D. C.
Nov. 5, 1898.

THE PROPER MOTION OF CERTAIN STARS AS A CRITERION OF THEIR DISTANCE.

J. G. PORTER.

For Popular Astronomy.

There is perhaps no problem in practical astronomy much more difficult than the determination of the distance of the stars:

hence any consideration which will tend even indirectly to enlarge our knowledge of this subject is to be welcomed. For the suggestion that I here make I claim no originality; it is in fact so simple that it must occur to any one dealing with the subject; but I am not aware that it has ever been put to any practical test.

In the ordinary determination of stellar parallax we employ the Earth's orbital motion to shift us from one point of view to another. Since the direction and rate of the solar journey through space are now approximately known, why can we not utilize this motion to furnish us our base-line? In other words is it not possible to tell something from the direction and amount of a star's proper motion as to how far away it is likely to be?

It has already been shown in several recent investigations that the proper motion of the stars is a much better criterion of their distance than brightness. In fact we may assume with considerable certainty that the average distance of large groups of stars is inversely proportional to the amount of their motion. My present suggestion is as to whether it may not be possible to extend this principle to smaller groups, and even with some probability to individual stars.

We find in those regions of the heavens which are approximately 90° from the apex of the Sun's way, a good many stars whose motion appears to be parallactic, that is, they are drifting almost directly away from the point in the sky towards which the solar system is moving. This must impress anyone who looks through an extensive catalogue where the proper motions are given. Two objections may be urged against making use of this drift to determine the distance of these stars. In the first place it may be said that it is reasoning in a circle to discover the solar motion through the movements of the stars, and then use this motion to measure their distances. But while this is true to a certain extent, I do not apprehend that it will seriously vitiate our reasoning, since the solar motion is equally shown by other classes of stars. A more serious objection is that it is impossible in the case of any given star to know surely that its motion is purely parallactic, and not partly at least due to the *motus peculiaris* of the star itself. Manifestly the only way to arrive at certainty on this point is to measure the star's parallax in the ordinary manner; but at the same time it is to be remarked, that since no general law governing the movements of the stars has yet been discovered, the probability that many stars in the same region of the sky would be moving directly away from the apex

our government with a chance for the methods of the practical politician I am not so sure."

So far as we know, this opinion is very generally endorsed by astronomers everywhere in this land. It is emphatically the reasonable view of the case, especially if we do not give the politician too prominent a place in making or hindering a wise administration of a worthy plan of scientific work. It seems to us that the politician is not so much to be feared in this matter as its unwise and designing friends are, who make use of the politician to carry forward plans and scheming ready-made to his hand. The average politician does not know much about scientific work or its real needs. He is generally ready to do his duty as a legislator, if not posted, when it is made plain to him, and he will surely look after the interests of his friends. What is wanted to reach the root of the matter, in our judgment is, agreement among astronomers generally as to what is best, the formation of a plan to carry out what is adopted as a wise course and finally an organization of astronomers that shall have representation in the United States wide enough to make its influence felt in effective ways.

It is to be expected that the officers of the Navy in charge of the Observatory will oppose any change of plan of administration; that the committee on naval affairs in Congress will do the same and that all employees at the Naval Observatory, if they do anything, will likewise oppose such a movement. As we recall the history of previous efforts on the part of outside friends and astronomers to secure a needed change in the administration of the Observatory, these three sources of influence against the change were strong enough to defeat it at critical points of its progress. The naval officers interested were outspoken and very active, their influence in Congress was considerable. The professors and employees of the Observatory who best knew the conditions of things in detail were silent spectators in the main, possibly in deference to the views of their superiors, and possibly in view of a tenure of office that should hold to an honorable retirement. Unfortunate for the success of previous attempts as it was, we could not greatly blame the able professors of the Observatory for their inactivity for now we well know what it might have cost them if such a course had been pursued, especially if they had expressed their views in favor of a change at all prominently.

This state of things even now does not make any needed change in the administration of the Observatory impossible.

Whatever the opposition is it can be overcome. Something of an attempt has already begun in the meeting of astronomers and astrophysicists, held at Harvard College Observatory last summer. The beginning seems to be a good one, but how much will come of it will depend on how much real work is put into forwarding a wise plan, sustained by a wide and general interest that means work for many friends. Committees and resolutions only will accomplish very little.

THE SUN DIAL OF AHAZ.

J. MORRISON, M. D., Ph D.

FOR POPULAR ASTRONOMY.

My paper in the December number of this Journal has been criticised by Messrs. Saunders and Easton who have, in recent issues, set forth their views in such a manner that I feel justified in referring again to the matter in dispute. These gentlemen have been wrestling with equations (2) and (3) which are practically identical, and because they cannot determine from them a maximum value of θ in terms of h (the hour angle) they jump at the conclusion that I am wrong and that the shadow cast by AC (see diagram) must always advance and can never recede under any condition. A mere inspection of these equations is sufficient to show that they do not admit of the treatment which Messrs. Saunders and Easton propose, for the simple reason that h increases continually and φ is constant. The change of the direction of motion of CE and BE—the shadows cast by AC and AB respectively—depends on the Sun's *azimuth* and not on the hour angle, and the azimuth can have a maximum value only when the Sun culminates north of the zenith of the dial—the observer being in the northern hemisphere. Equation (5) gives the value of the hour angle when the azimuth is a maximum and equation (8) gives the maximum value of θ, all of which is explicitly shown in my paper. Since the hour angle is the great difficulty which these gentlemen encounter, let us remove it by the following process. They will, I presume, admit that equations (2) and (4) exist at the same time; eliminate h between them and we obtain

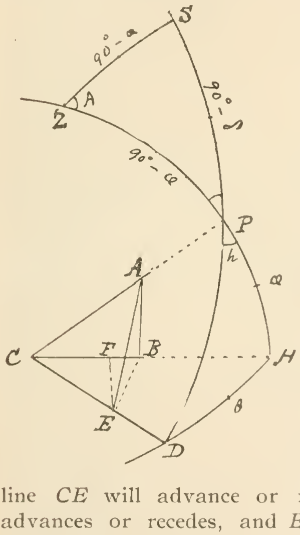
$$\cot A = \tan \delta \cos \varphi \sqrt{1 + \sin^2 \varphi \cot^2 \theta} - \sin^2 \varphi \cot \theta$$

from which we find in the usual manner

$$\tan^2\theta = \frac{\cos^2\varphi - \cos^2\delta}{\cos^2\delta}$$

or $$\tan\theta = \pm\frac{\sqrt{\sin(\delta+\varphi)\sin(\delta-\varphi)}}{\cos\delta}$$

which is identical with equation (8) deduced by an easier and simpler process. Mr. Saunders makes a slight transformation of equation (2) and obtains a result which is useless, if not absurd, and proves nothing. He also misrepresents me by quoting me as saying that the shadow can never go backwards except within the torrid zone. I have not published any such statement in my paper or anywhere else. Since these gentlemen do not follow my work correctly, I will now try to give the steps to take another view of the matter. Referring again to the diagram, *CE* is the shadow cast by the hypothenuse *AC*; the point *A*, common to both *AC* and *AB*, casts its shadow at E, and if we take any number of points in *AC* and through these pass a saw parallel to *AB*, thus cutting the stylus into a number of perendicular laminae whose heights vary from 0 at *C* to *AB* at *A*, the shadow of the vertex of everyone of these perpendiculars, will fall on the line *CE*, and if we remove any number of them, for instance *AB* and those adjacent to it, or some of the intermediate ones, the effect will be the same, *i. e.*, the line joining the shadow of their vertices will be *CE*, no matter what the number of these perpendiculars may be. The point *C* being stationary, the line *CE* will advance or recede according as the point *E* advances or recedes, and *E* is the shadow cast by the vertex of the perpendicular *AB*. Now suppose we place a stick perendicular to a horizontal plane—a level and smooth piece of ground will do, if nothing better is at hand—at any place north of the tropic of Cancer; its shadow at noon will fall due north and will *always* advance or move in the direction of the hands of a watch. If we incline the stick towards the north or place it parallel to the Earth's axis,



i. e., direct it to the pole, its shadow at noon will also fall north and will always advance. This latter is the case of the common sun dial. But suppose we incline the stick towards the south, that is to say, direct it to a point on the meridian *south* of the Sun, the shadow at noon will now fall south and not north as before and as the Sun moves westward across the meridian, the shadow of the vertex will move eastward or retrograde not only on the horizontal plane but on the plane perpendicular to the stick. We may erect any number of such sticks all parallel to the first but shorter so that the line joining their vertices may be directed to the pole, the shadows of the vertex of each when joined will form the line *CE* (see diagram) as already shown and the motion of this line around *C* as a centre is controlled or governed by the azimuth of the Sun and not by the hour angle as I have already stated. From sunrise until the Sun attains its greatest elongation with regard to the point of the meridian to which the stick is directed, the shadow or the line *CE* will advance; at the moment of greatest elongation it will remain stationary for an instant, then recede or go backwards until the Sun arrives at its greatest western elongation when it will for an instant again become stationary and then advance until sunset. So far as the motion of the line *CE* is concerned it makes no difference whatever whether the plane on which the shadow falls is horizontal or perpendicular to the stick; the latter position is the case of the Sun dial of Ahaz. The amount of retrogression of the shadow depends, as I have shown in the paper referred to, on the latitude of the place and declination of the Sun; if these are nearly equal ($\delta > \varphi$) the angle of retrogression will be small as was the case in question, viz., 10°, that is 5° on each side of the meridian. We do not know whether or not those degrees were identical with ours, but this is not a material factor in the case, all that is necessary for us to show, is that it is possible for the shadow to recede, or, in other words, the phenomenon was purely astronomical and one which any person can easily verify by the instructions I have just given, or we can proceed as follows: Take a circular piece of card board to represent the plane of the dial and at or near its centre erect a right angled triangle of the same material and having the angle *ACB* (see diagram) equal to 20° suppose, then at any place north of the tropic of Cancer—Iowa City, for instance—adjust it as near as practicable to the meridian and latitude of the place, when it will be seen that the shadow of the hypotenuse *AC* will, near the time of the summer solstice, behave as I have just stated.

In a former issue another gentleman asks to what extent I claim priority for showing that the shadow can recede. He might just as well have asked if I claim equation (4) as my own discovery.

Unfortunately I can claim nothing at all in this discussion for the reason that the direction and motion of the shadow of a perpendicular or inclined stick, have been known from time immemorial; they were known to the ancient Hindoos, Egyptians, Babylonians and Greeks, but are at the present day unknown to my learned critics. I never saw the work of Nonius to which the gentleman refers, nor have I even heard of Hutton's edition of the same. They are not well known, are now rare books and probably out of print long ago.

Another of my correspondents asks: "Does the shadow of the style of a sun dial recede for several days every year at every place within the torrid zone?"

To which I answer, no. If the dial is properly constructed the shadow always advances at every place on the Earth's surface, but on the Equator the line CE instead of revolving around C as a centre, moves parallel to the meridian but always towards the east.

The dial of Ahaz was not constructed for Jerusalem; it was probably brought from India by some person who presented it to the king; it required no extra graduated curves, as the shadow receded for a few days only near the summer solstice. The phenomenon which was then regarded as a miracle was perhaps observed for the first time by the prophet and may never have been again but if it was, it would only be regarded as a confirmation of the sign.

A POPULAR ASTRONOMICAL OBSERVATORY.*

GEORGE E. LUMSDEN.

The last instance to which your attention shall be invited is that of the Gesellschaft Urania of Berlin. This scientific society was founded in 1888 as a joint stock company, with a capital of 300,000 marks, about $100,000. In 1896 there were nearly 400 shareholders, who possessed a fine two-story building, fitted with all sorts of astronomical and physical apparatus, and erected in one of the public parks on a site presented by the Government. Shareholders have free admission to all the spectacular

* Continued from page 317.

and special exhibitions and receive a copy of the magazine, "Heaven and Earth," published by the society. The success of this purely scientific undertaking has been phenomenal and, notwithstanding the necessarily great initial outlay, and some difficulties, not anticipated by the founders, is now on a paying basis and in possession of a fine property. From the report for the year ending March 31st, 1894, the last in possession of the writer, it may be gathered that 25,210 family tickets were issued (5,043 to new applicants), and that the public attendance, which reached 117,617 paying visitors, included 26,400 visitors on special occasions and 10,000 pupils from the city schools. The income derived from all sources was more than $30,000. The astronomical equipment includes one reflector and the following refracting telescopes; one 12-inch, one 6-inch, one 5-inch, and one 4-inch. Complete details may be obtained from the literature issued by the Managing Committee.

We see what is being done in Europe and in the United States by public spirited men and women. Their example can and ought to be followed in Canada. In the United States, from the very public schools of the higher grades to the most ambitious colleges, there is rivalry in this field. The great universities are vieing, each with the other, in the splendor of their astronomical and astro-physical equipments, due, in some part, to the open-handed generosity of opulent men desirous to see in their own day some of the results of munificence wisely directed into popular educational channels. Munificence of this kind is royal, whether the patron be prince or private citizen. Science owes much to munificence, and nobly are her professional votaries striving to repay the debt. Her scrolls are rapidly filling with names illustrious by gifts. The glory of the Ptolemies is associated with patronage of literature and science. When all else that he did shall have been forgotten, it will be remembered that Frederic II. of Denmark, was the friend and protector of Tycho Brahé, and while the name of the astronomer lives, so shall that of his king. The name "Lick" will never be blotted from the page of Astronomy. If munificence becomes princes, how much more does it become rich men who, in giving, are giving that which is theirs by right of their own energy and thrift.

So far, munificence has done nothing in Canada for Astronomy. Not a public telescope pierces the Canadian skies, pure as those of more favored lands. But the day is coming when this reproach will be wiped away. When once they understand the needs of learning, wealthy men will come forward with substan-

THE SIDEREAL DAY.

J. MORRISON. M.D; M.A., Ph. D.*

FOR POPULAR ASTRONOMY.

From time immemorial, the sidereal day has been classed among the so-called Astronomical Constants, that is, the time during which the Earth performs one absolute revolution on its axis, has been regarded as *invariable*. We purpose to show in this short paper, that, owing to the nature of the circumstances on which it depends, this assumption cannot be true. In order to obtain a clear view of the causes, which are continually operating to produce a variation in the duration of the sidereal day, we must go back a long way and briefly review the geologic history of our planet as revealed to us in the structure and conformation of the rocky pages of its crust or surface. There is good reason to believe that the matter composing the Earth once existed in a gaseous condition, having been thrown off or detached from the solar globe when its volume extended or filled the Earth's present orbit. This is a well founded assumption in harmony with the Nebular Hypothesis and is abundantly confirmed by all the structural and dynamical features of the solar system.

The irregular gaseous mass detached from the solar globe—and which subsequently became the Earth—would of course receive a motion of translation from the parent mass and also a motion of rotation around an axis passing through its center of gravity. While in this gaseous condition it abandoned or set free a portion of its own mass which became the Moon, but as this comparatively small mass rapidly cooled by the radiation of heat, the surface soon became too viscid to repeat the process. By the mutual attraction of its own particles, this gaseous mass would finally assume an approximately spherical figure and the duration of its axial rotation could not then have differed much, if any, from the Moon's present sidereal period, viz. 27 days 7 hours. As the cooling proceeded rapidly by the radiation of heat, contraction of volume would ensue, and the axial rotation would be accelerated in accordance with well-known dynamical principles which will be explained presently.

At the very high temperature which must have prevailed during this long period while in a gaseous condition, all the chemical elements of which the mass is composed—about 67 in number—were dissociated or existed in a free or uncombined state, but when the temperature declined to a certain point, chemical affinity would assert its power and combinations of the various elements would ensue. Oxygen and hydrogen would unite to form water in the form of vapor or steam; oxygen and carbon to form carbon dioxide; calcium and other metals would combine with oxygen to form oxides of these metals, and so on. In process of time as the mass cooled, the aqueous vapor would condense and showers of rain would be precipitated on the hot surface to be sent back again and again as steam. By a repetition of this

process, the temperature would eventually become lowered to such a degree that water would collect on the surface and thus would be formed a universal ocean of warm water highly charged with carbon dioxide and holding in solution carbonate of lime and other chemical substances which would be precipitated to the bottom and thus a solid crust of limestone would be formed. As the cooling proceeded now however more slowly than formerly, the gaseous interior would contract or shrink away from the crust forming extensive cavities bridged over, just as the water of a frozen river or pond falls away from the ice on its surface. When the crust increased in thickness by the deposition of sedimentary matter from the super-incumbent ocean, a tremendous strain would be produced in it, and when it could no longer be borne, extensive rents or cracks would be made, collapse of the crust on the molten matter beneath would occur and enormous masses of molten or igneous rock would exude through the cracks, technically called "faults" and thus mountains were born. This condition of things is well illustrated in the Laurentian Mountains in northern Canada, which extend in the form of a circular arc from the eastern coast of Labrador to near the mouth of the Mackenzie River and form for themostpart, a water-shed for the streams flowing south into the St. Lawrence, the Great Lakes and the Mississippi and those flowing north into Hudson Bay and the Arctic Sea.

The eastern end of this chain is a continuous solid wall of granite of immense magnitude, but the western portion consists for the most part of rounded off hills and isolated peaks the result of extensive erosion in subsequent geologic ages.

This mighty wall of granite was pushed up in a molten condition through the comparatively thin overlying crust of limestone which still rests, in some places, against its sides like the sloping roof of a house. These ancient limestone strata show evidence of having been subjected to a very high temperature. This extensive mass of igneous rock is the oldest portion of our continent and furnishes today, the most positive evidence of the condition of our globe which we have just described. Concurrent with this mighty upheaval which occurred during the Azoic period, that is, ages before the dawn of animal or vegetable life—there must have been a sinking or settling down of the entire surface or in other words a shortening of the axes of the Earth with consequent accelerated axial rotation.

After the lapse of an immensely long period to be counted only by millions of years and comprising the greater part, if not the

whole, of what is called the Palæozoic Period during which enormous deposits of sedimentary rock were laid down at the bottom of the ocean there occurred another tremendous upheaval when the Appalachian Mountains were born of the deep—an upheaval which must have shaken the Earth to the very center.

This mighty range extends under different names, from near the mouth of the St. Lawrence to northern Alabama and was at first of great height—so high that in some places it toppled over, thus burying the recently deposited strata far below the more ancient.

Another geologic age rolls by comprising nearly the whole of the Mesozoic Period when another tremendous convulsion shattered the crust for thousands of miles and the Rocky and Andes Mountains emerged from beneath the briny waves. These great seismic disturbances were the result of foldings or crumplings of the crust which had bridged over enormous cavities formed by the shrinking of the molten or gaseous mass beneath, and consequently there must have been at the same time a great subsidence of the general surface, but by reason of the great thickness of the crust no molten or igneous rock appears to have been protruded as was the case when the Laurentian Mountains were upheaved. Again after another long interval of time had rolled away, the glacial period arrived during the passing away of which, the Laurentian and Appalachian Mountains were planed down to their present dimensions; the debris of the former consisting of sand, gravel and boulders, was distributed over the southern portion of Canada and the adjacent portions of the northern States, and that of the latter spread out to the south and east to lay the foundation of what subsequently by another seismic disturbance, became the Atlantic coast states. These crumplings of the crust which resulted in the birth of the Appalachian, Rocky and Andes systems of mountains, must have produced very extensive cracks or faults in the crust and it is along these that seismic disturbances occur. One of these is believed to extend from New England along the Atlantic coast to the West India Islands and probably into South America, and another along the Pacific coast of Canada, the United States, Mexico, Central and South America to Tierra del Fuego in southern Chile. The Charleston earthquake of 1886 was on the former and the recent earthquakes of San Francisco and Valparaiso of 1906 along the latter. Another very extensive fault, no doubt, runs through the Alaskan peninsula, the Aleutian Islands, Kamchatka, the Kurile Islands, Japan, the Philippines, Formosa,

Java and Sumatra, for this region is notorious for its seismic disturbances which have been not only very numerous but also very destructive to life and property. Southern Europe is traversed by another fault extending through southern Portugal, Spain, Italy, Greece, Turkey and probably on through Asiatic Turkey, southeastern Russia, Persia and India. Hot springs, active and extinct volcanoes abound along these faults.

Beginning on the east coast of Greenland we have Jan Mayen which has been in a violent state of activity ever since its discovery, Hecla and numerous boiling springs in Iceland, several active volcanoes in the Alaskan peninsula and the Aleutian Islands, while Kamchatka, the Kurile Islands, Japan, the Philippines, Java and Sumatra fairly bristle with these fiery outlets. Europe has three active volcanoes, Vesuvius, Stromboli and Aetna. Mexico, Central America and the entire chain of the Andes are alive with volcanic activity, so also are the West India Islands, the Sandwich Islands, New Zealand, the Canaries, and the numerous islands in the Pacific Ocean and lastly we have two terrific volcanoes in the south frigid zone, viz., Mounts Erebus and Terror which have been in a violent state of activity since their discovery by Captain Ross in 1841.

These numerous outlets of subterranean heat must produce a decided effect in lowering the temperature of the molten interior with corresponding contraction of volume. Cavities are thus formed beneath the crust which finally collapses and a folding or overlapping of the edges of the fault produces an oblique downward motion sufficient sometimes to twist or wreck the foundations of the strongest structures man can erect. The fall of even a few inches of the solid crust which may vary probably from seventy to a hundred miles or more in thickness, would produce a shock or vibration which would doubtless be felt for a thousand miles or more.

Ever since the Spanish occupation of South America, seismic disturbances of a more or less violent character have occurred especially along the Pacific coast—the last being that of August, 1906. In 1730 Valparaiso was almost completely ruined by an earthquake of unusual violence; in 1822 Santiago was partially destroyed and a long portion of the coast of Chile permanently raised, and in 1829 the same city was again visited and the elevated coast depressed several feet below its normal level. In 1835, 1849 and 1851 violent earthquakes occurred in Chile, the last being especially destructive in Valparaiso in which over four hundred houses were wrecked and several lives lost. In 1880,

Iliapil, near Valparaiso was destroyed and over two hundred persons perished. In 1885, the islands of Santa Maria and Concepcion off the coast of Chile were uplifted and subsequently depressed eight feet below their former position. This earthquake was felt for more than a thousand miles along the coast. It is very probable, nay almost certain, that not only the portion of the coast, and the above-mentioned islands, but also a large portion of the Andes and a large area of the floor of the ocean were depressed at the same time. On August 13—15, 1892, Peru was visited by one of the most destructive earthquakes on record, four cities and several towns were wholly or partially destroyed and over twenty-five thousand persons perished. The entire Pacific coast of South America has frequently been more or less submerged by tidal waves the result of submarine eruptions which are very probably three or four times more numerous than on land one having recently taken place near Hawaii where vast numbers of cooked or boiled fish were washed ashore; another off the coast of Alaska where a new island has been formed, thus increasing the domains of Uncle Sam, and it is reported that the island of Juan Fernandez, some five hundred miles west of Valparaiso, has disappeared by the subsidence of the floor of the ocean.

In the early part of the last century, about 1834, a new island, Graham's Island, was heaved up in the Mediterranean southeast of Malta, and remained for a year or more when it gradually sank out of sight, an extensive shoal still remaining.

In June 1773, Santiago, Guatemala, was completely wiped out together with nearly all its inhabitants and quite recently the volcano Santa Maria in the same republic, violently erupted and destroyed an immense amount of property in its vicinity.

When faults extend under oceans, water percolates down to the heated interior where it is converted into superheated steam which ultimately bursts out at the point of least resistance with terrific violence. The explosions of Krakatoa, in 1882, and of Mount Pelée in Martinique which recently wiped out the city of St. Pierre, were of this kind and probably all submarine disturbances are of the same character.

Another region notorious for seismic disturbances, is the south of Europe extending from the south of Portugal to Asiatic Turkey and in fact on through the Caucasus and Persia to Hindostan. Vesuvius, Stromboli and Aetna have wrought frightful destruction of life and property during the last three thousand years. The fate of Pompeii and Herculaneum is well-known. On February 26, 1531, Lisbon, the capital of Portugal, was partially

destroyed, fifteen hundred houses, were more or less wrecked and about thirty thousand people perished, and again on November 1, 1755 a large portion of the same city sank and about sixty thousand persons were engulfed beneath the Atlantic. This collapse of a portion of the Earth's crust produced a shock or vibration which was felt five thousand miles away. The same catastrophe befell Port Royal, in Jamaica, West Indies during the early part of the last century and ships now sail over where a portion of Lisbon once stood as well as over Port Royal.

In 1851 the south of Italy was visited by an earthquake which caused the death of about nineteen thousand people, and again in December 1857, and also quite recently, several towns in Italy were partially ruined and ten thousand persons perished. During the last century numerous shocks or vibrations have been felt in Greece, Turkey, the Caucasus region, Asiatic Turkey, India and China which were attended with great destruction of property and loss of life. Antioch on the Orontes, in Asiatic Turkey was almost ruined during the early part of the last century and about the same time an earthquake occurred in southeastern Missouri in the neighborhood of New Madrid causing a subsidence of the ground and the formation of several small lakes which still remain. While this paper is being written, news is received of slight earthquake vibrations in Maine. In short, there is no part of the Earth's surface absolutely free from seismic disturbance. The three regions most noted for these phenomena, are the south of Europe, the west coast of South America and the east coast of Asia. A Japanese observer has recorded several hundred seismic vibrations during the last two or three years. These were all the result of the slow settling down of the Earth's crust in that particular region. Among other noted depressions of portions of the Earth's surface may be mentioned that which occurred in Western Asia during the time of the Patriarch Abraham. The plain on which the cities of Sodom and Gomorrah stood, sank, the Dead Sea was formed and the River Jordan arrested in its course. This historic river formerly flowed into the Gulf of Akaba which forms the northeastern extremity of the Red Sea through a channel still well defined. The Dead Sea, a considerable tract of country around it, and the entire valley of the Jordan are now more than a thousand feet below the level of the Mediterranean. A similar condition of things exists in the case of the Caspian and Aral Seas. There is also a large area of depression several feet below the Caspian, in southeastern Russia.

Several monuments and columns which stood high above the

Mediterranean in Pliny's time, are now wholly or partially submerged, the coast for many leagues having sunk several feet. From the facts already mentioned there is good reason to believe that large areas of the floor of the Pacific Ocean are slowly settling down.

Seismic disturbances of great magnitude appear to be going on in northern Canada. The eastern shore of Hudson Bay is rising and the western shore sinking, new islands have appeared and the Bay is becoming shallower—all within the memory of persons now living on its shores.

Eminent geologists say that the north shore of Lake Superior is rising and the southern shore subsiding and that in eight hundred or a thousand years the Great Lakes on our northern borders will find an outlet in the Mississippi. According to researches carried on by the U. S. Coast and Geodetic Survey the North American continent is floating on an elastic sea of molten or gaseous material. All these movements appear to indicate that the continent or at least the northeastern portion of it is canting over to the southwest.

Certain distinguished scientists entertain the opinion that the interior of the Earth must be "solid and as rigid as steel". In a gaseous condition it certainly was when detached from the parent body which subsequently became the Sun, and in a liquid or semi-gaseous state when the Laurentian Mountains were thrown up, for the upheaved matter penetrated all the fissures and cracks in the upturned overlying strata. As a matter of fact, volcanoes still pour forth *liquid rock* or lava which is probably gaseous at great depths. Owing to the tremendous pressure at the depth of even a thousand miles below the surface, the temperature must be very great, far surpassing any temperature we can produce, and therefore all the chemical elements must be dissociated. Now there is a temperature common to all gases *above which they can not be condensed by any pressure*; this is the *critical temperature*. Hydrogen, oxygen, methane, carbon dioxide and other gases have been subjected to a pressure of 3,600 atmospheres without condensing them, because the temperature employed was far above the critical temperature belonging to that particular gas. Again, all liquids when heated above the critical temperature, are transformed into gases although subjected to intense pressure. A pressure of 74 atmospheres and a temperature of 31 degrees C., are required to liquefy carbon dioxide, but *above this temperature* no pressure whatever would condense it. The critical temperature of the metals in a gaseous condition must of course be vastly

higher but far below the temperature which must exist in the interior of the Earth. In consequence then of these critical conditions of matter, the interior must be gaseous, condensed probably to the consistence of tar, but liquid when emitted by volcanoes; it can not then be solid, much less as rigid as steel.

By reason of these subsidences of portions of the Earth's crust, embracing, no doubt, large areas of continents and oceans and especially such as have occurred at various times on the west coast of South America, the position of the center of gravity of the Earth must undergo a slight change, and as the axis of rotation must pass through this point, the position of the Earth's poles can not be permanent but must also be subject to a slight movement and therefore a slight variation in the latitude of places must take place—a variation which has already been established by direct astronomical observations at several observatories. The shifting of the poles through a distance of 101 feet would produce a change of latitude which would attain a maximum value of $\pm 1''$. Moreover the subsidence of the general surface will reduce the length of a degree of latitude and also increase the weight of bodies on the surface. It is quite evident then that an exceedingly slight change in the position of the center of gravity would be fully sufficient to account for the variation of latitude which has actually been found to have taken place in recent times amounting to about $\pm 0.5''$ in some places.

But these very slight variations resulting from the partial or general subsidence of the Earth's crust, are practically of little or no importance and utterly insignificant in comparison with another variation, viz., that of the shortening of the length of the sidereal day, which, as we shall see, must constitute a serious disturbing factor in our lunar, solar and planetary tables.

As a result of the seismic disturbances already mentioned—and these are but a small portion of those which have actually taken place within the dates recorded—it is not perhaps too much to assume that the Earth's radius has been reduced by, say, four feet during the last century, and for the sake of illustration we will take four feet.

Let R = the Earth's radius in, say, 1800
r = " " " " " 1906
F = acceleration of gravity in 1800
f = " " " " 1906
V = velocity per second in 1800
v = " " " " 1906
T = duration of one absolute revolution in 1800
t = " " " " " " 1906

where R, r, F, f, V, and v are expressed in feet and T and t in seconds.

Then we shall have by a well-known formula in dynamics:

$$F = \frac{V^2}{R} \qquad \text{and } TV = 2\pi R$$

$$f = \frac{v^2}{r} \qquad \text{and } tv = 2\pi r$$

$$\text{Whence } \frac{F}{f} = \frac{V^2}{v^2} \cdot \frac{r}{R} \quad (1), \text{ and } \frac{V}{v} = \frac{R}{r} \cdot \frac{t}{T} \qquad (2)$$

By the theory of attraction we have

$$f : F : : R^2 : r^2 \quad \text{whence } \frac{F}{f} = \frac{r^2}{R^2} \qquad (3)$$

From (1), (2) and (3) we easily find

$$\frac{t}{T} = \left(\frac{r}{R}\right)^{3/2}$$

$$\text{whence } t = \left(\frac{r}{R}\right)^{3/2} T \qquad (4)$$

From the Clark Spheroid of 1866, adopted by the U. S. Coast and Geodetic Survey we have:

$R = 20926062$ feet, hence $r = 20926058$ feet and $T = 86400$ sidereal seconds.

The coefficient of T in (4) is .999,999,784,957,059 and therefore $t = 86399.9814202898976$ seconds and $T - t = .0185797$ seconds, or the sidereal day in 1906 is very nearly one fifty-fourth of a second shorter than it was in the year 1800.

This assumption of a reduction of four feet in the radius during a century may be, and probably is, too large. A reduction of two feet in the radius will shorten the day about one one-hundred-twenty-fifth of a second.

This slight variation in the length of the sidereal day is certainly the disturbing factor in our lunar and other astronomical tables.

Hansen's lunar tables prepared about half a century ago gave the Moon's place very accurately for several years, but during the last two or three decades the discrepancy between the computed and observed places of our satellite, is increasing, and to such an extent that one astronomer has prepared and published an "*Empirical Correction*" to be applied to the computed place to make it agree more closely with the observed or actual place— a species of astronomical tinkering not only vicious in principle bnt also inconsistent and unscientific.

In Hansen's tables the time was taken as the independent variable, but as the sidereal day most certainly varies it will not be very difficult to make an adjustment in accordance therewith and when such is judiciously made, it will doubtless be found

that the tables are as accurate and trustworthy now as when first issued.

The variation of the sidereal day will also operate to produce a discordance in the case of the solar and the planetary tables. Again, the recorded dates of ancient eclipses—that of Thales for example—can not be even approximately verified on the assumption that the sidereal day is a constant and invariable interval. During the time of Thales the sidereal day may have been a minute or even more longer than at present. In remote geologic ages when the radiation of heat from the Earth must have been immensely greater than at any subsequent time vast areas, perhaps entire continents, may have settled down or become deeply submerged, and this irregular or partial subsidence may have produced a great change in the position of the center of gravity and consequently great movements in the poles—ten or twenty degrees or more—resulting in a great variation of climate which we know once existed in high northern latitudes. There can be no doubt but that seismic disturbances have always, and are still, slowly operating to produce great changes in both the structural and dynamical circumstances of our globe, and thus it is, that "change and decay in all around we see" thoughout the entire realm of creation.

THE ORIENTATION OF THE FIELD OF VIEW OF A TELESCOPE.

E. T. WHITELOW.

FOR POPULAR ASTRONOMY.

Several times I have been asked by members of the Manchester Astronomical Society, who are users of reflectors, for hints as to how to determine the cardinal points of the field of view which appear to change so erratically when the telescope is shifted from one side of the stand to the other, or when rotated in its cradle. Similar questions arose with users of solar or stellar reflecting eyepieces and spectroscopes. Troublesome as the subject is to beginners in practical observation (and even to some of us who are hardly beginners) there does not seem to be any reference to it in text-books or manuals. The recent correspondence in the *English Mechanic* shows the matter to be of somewhat wide interest.

Mr. Whitmell's excellent summary, in the *English Mechanic*, of the varied aspects of the field of view is complete and correct for the position he has chosen, viz., a horizontal telescope pointing

GENERAL PERTURBATIONS AND THE PERTURBATIVE FUNCTION.

J. MORRISON, M. A., M. D., PH. D.

FOR POPULAR ASTRONOMY.

It was first proved by Sir Isaac Newton in his immortal work The Principia, that when a particle moves around a centre of force which varies directly as the mass and inversely as the square of the distance, the path or orbit described is a conic section with the centre of force in the focus, and the radius vector describes equal areas in equal times. In the case of a planet moving around the Sun, the orbit is an ellipse with the Sun in one of the foci. It is evident that the planet will move most rapidly in perihelion and most slowly in aphelion. The mean motion or that which it would have, if it described a circular orbit, is evidently equal to 360° or 2π divided by the periodic time T or $\frac{2\pi}{T}$. From perihelion to aphelion the true place of the planet will be in advance of the mean place; at aphelion the mean and true places will coincide, and from aphelion to perihelion the mean place will be in advance of the true until perihelion is reached when they again coincide and so on.

The angular distance between the true and mean places or to express it more technically, between the true and mean anomalies, is called the elliptic inequality or 'the equation of the centre,' and it is the only correction to be applied to the mean to obtain the true anomaly in the case we are now considering.

If however we suppose another planet to be added to the system, the circumstances of the motion of both planets, become much more complicated; each disturbs the motion of the other; the equable description of areas which obtained in the case of a single planet now no longer exists, and the computation of the true place of either planet is a work of prodigious difficulty. It is the famous "problem of three bodies" which has severely taxed the ingenuity and analytical skill of mathematicians since the discovery of the law of universal gravitation.

In this and subsequent papers we purpose to develop as clearly and as briefly as the difficulties of the problem will admit, the formulae for undisturbed and disturbed motion, so as to enable the reader to understand the more abstruse and elaborate developments of LaPlace, LeVerrier and others.

Let x, y, z be the coördinates of a planet referred to the centre

of gravity of the Sun S, as the origin and r its radius vector, also let m denote the ratio of the mass of the planet (m) to that of the Sun or $m = \frac{\text{mass of planet}}{\text{mass of Sun}} = \frac{\text{mass of planet}}{k^2}$ where k^2 is the well known Gaussian constant of solar attraction whose value will be determined farther on, then the mass of the planet $= mk^2$.

Let x', y', z' be the coördinates, r' the radius vector and $m'k^2$ the mass of a second planet (m') and similarly for other bodies of the system and let ρ, ρ_1, ρ_2 be their mutual distance, or mm', mm'' etc., then we shall have

$$\begin{aligned}
& r^2 = Sm^2 = x^2 + y^2 + z^2 \\
& r'^2 = Sm'^2 = x'^2 + y'^2 + z'^2 \\
& r''^2 = Sm'' = x''^2 + y''^2 + z''^2, \text{ etc.} \qquad (1) \\
\text{and} \quad & \rho^2 = mm'^2 = (x' - x)^2 + (y' - y)^2 + (z' - z)^2 \\
& \rho_1^2 = mm''^2 = (x'' - x)^2 + (y'' - y)^2 + (z'' - z)^2, \text{ etc.}
\end{aligned}$$

Considering only three bodies, the Sun S, the disturbed planet (m) and the disturbing planet (m') and putting for the sake of brevity $k^2 + k^2 m$ or $k^2 (1 + m) = \mu$, it is evident that in the relative motion of (m) around S, it will be acted on by the three forces $-\frac{\mu}{r^2}, \frac{k^2 m'}{\rho^2}$ and $-\frac{k^2 m'}{r'^2}$ respectively directed along the lines mS, mm' and $m'S$, and since the cosines of the angles which the directions of each of these forces make with the axis of x, are respectively

$$\frac{x}{r}, \quad \frac{x' - x}{\rho} \text{ and } \frac{x'}{r'}$$

We shall have for the components of these forces parallel to the same axis a/

$$-\frac{\mu x}{r^3}, \quad \frac{k^2 m' (x' - x)}{\rho^3} \text{ and } -\frac{k^2 m' x'}{r'^3} \text{ respectively,}$$

the first and third being negative because the force tends to diminish the coördinates x, y and z.

For the components of these same forces parallel to the axis of Y we shall have in a similar manner

$$-\frac{\mu y}{r^3}, \quad \frac{k^2 m' (y' - y)}{\rho^3} \text{ and } -\frac{k^2 m' y'}{r_1^2}$$

and for those parallel to the axis of Z

$$-\frac{\mu z}{r^3}, \quad \frac{k^2 m' (z' - z)}{\rho^3} \text{ and } -\frac{k^2 m' z'}{r^3}.$$

The sum of these three components for each direction is evi-

dently the total force parallel to this direction, which acts on the planet (m) and it must be equal to the accelerations $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}$ and $\frac{d^2z}{dt^2}$ respectively. If then we extend these results to a number of bodies m'', m''', etc., we shall have for the equations of the relative motion of (m) around S.

$$\frac{d^2x}{dt^2}+\frac{\mu x}{r^3}=k^2\sum m'\left(\frac{x'-x}{\rho^3}-\frac{x'}{r'^3}\right)$$

$$\frac{d^2y}{dt^2}+\frac{\mu y}{r^3}=k^2\sum m'\left(\frac{y'-y}{\rho^3}-\frac{y'}{r'^3}\right) \qquad (2)$$

$$\frac{d^2z}{dt^2}+\frac{\mu z}{r^3}=k^2\sum m'\left(\frac{z'-z}{\rho^3}-\frac{z'}{r'^3}\right)$$

where the symbol $\sum$ indicates that each mass k^2m'', k^2m''', introduces a term similar to that which results from the action of m' on m — a term which we obtain by simply changing in the second members m', ρ, x' and r' into m'', ρ', x'' and r'' respectively, and so on.

To facilitate the solution of the problem, the second members of (2) are put into the following form.

From (1) we obtain

$$\frac{d}{dx}.\frac{1}{\rho}=\frac{x'-x}{\rho^3}. \qquad \frac{d}{dy}.\frac{1}{\rho}=\frac{y'-y}{\rho^3}, \qquad \frac{d}{dz}.\frac{1}{\rho}=\frac{z'-z}{\rho^3}$$

and the terms $\frac{x'}{r'^3}, \frac{y'}{r'^3}, \frac{z'}{r'^3}$ are derivatives of $\frac{xx'+yy'+zz'}{r'^3}$ in regard to the variables x, y and z. If then we put

$$\Theta=\frac{m'}{1+m}\left(\frac{1}{\rho}-\frac{xx'+yy'+zz'}{r'^3}\right)$$
$$+\frac{m''}{1+m}\left(\frac{xx''+yy''+zz''}{r''^3}\right)+\text{etc.} \qquad (3)$$

we shall have for the partial differential coefficients with respect to x, y and z

$$(1+m)\left(\frac{d\Theta}{dx}\right)=\sum m'\left(\frac{x'-x}{\rho^3}-\frac{x'}{r'^3}\right)$$

$$(1+m)\left(\frac{d\Theta}{dy}\right)=\sum m'\left(\frac{y'-y}{\rho^3}-\frac{y'}{r'^3}\right)$$

$$(1+m)\left(\frac{d\Theta}{dz}\right)=\sum m'\left(\frac{z'-z}{\rho^3}-\frac{z'}{r'^3}\right)$$

Substituting in (2) we have

$$\frac{d^2x}{dt^2} + \frac{\mu x}{r^3} = k^2\,(1+m)\left(\frac{d\Theta}{dx}\right)$$

$$\frac{d^2y}{dt^2} + \frac{\mu y}{r^3} = k^2\,(1+m)\left(\frac{d\Theta}{dy}\right) \qquad (4)$$

$$\frac{d^2z}{dt^2} + \frac{\mu z}{r^3} = k^2\,(1+m)\left(\frac{d\Theta}{dz}\right)$$

which are the differential equations of disturbed motion. The force whose components are expressed by the second members of (4) is the disturbing force. It expresses the difference between the action of the bodies m', m'', &c , on m and on the Sun, resolved parallel to the coördinate axes, and the quantity Θ from which the second members of (4) are derived is called the Perturbative Function, the development of which into an infinite series, constitutes the chief difficulty in the solution of this problem. The integration of the equations in group (4) has never been effected except by a series of approximations.

We must first consider the case of a single planet moving round the Sun, subject only to the reciprocal action of the two bodies.

This is the case of *undisturbed* motion; the disturbing force will then be zero and group (4) becomes

$$\frac{d^2x}{dt^2} + \frac{\mu x}{r^3} = 0$$

$$\frac{d^2y}{dt^2} + \frac{\mu y}{r^3} = 0 \qquad (5)$$

$$\frac{d^2z}{dt^2} + \frac{\mu z}{r^3} = 0$$

which are the differential equations of undisturbed motion and their integration will introduce six arbitrary constants which determine the circumstances of the orbit which the planet describes round the Sun under their mutual influence. The integration of (5) presents no great difficulty and is given in most works on Physical or Theoretical Astronomy, but as the student may not have any such works by him and in order to render these short papers as complete as practicable, we will give the integration here.

Multiply the first of (5) by y and the second by x and subtract the second product from the first, integrate the result and we have

$$\frac{xdy}{dt} - \frac{ydx}{dt} = h \qquad \text{(a constant)}$$

similarly $$\frac{zdx}{dt} - \frac{xdz}{dt} = h' \qquad \text{``} \qquad (6)$$

and $$\frac{ydz}{dt} - \frac{zdy}{dt} = h'' \qquad \text{``}$$

If we multiply these by z, y and x respectively and add the products we obtain

$$hz + h'y + h''x = 0 \qquad (7)$$

which is the equation of a plane passing through the origin of coördinates, that is, through the center of the Sun.

Let dv be the angle described by the radius vector r in the time dt, then the area described by this radius on the plane of the orbit will be $\frac{1}{2}r^2dv$, and if i, i_1 and i_2 denote the angles which the orbit makes with the coördinate planes, xy, xz and yz respectively we shall have

$$\begin{aligned} r^2dv \cos i &= h\,dt \\ r^2dv \cos i_1 &= h'\,dt \qquad (8) \\ r^2dv \cos i_2 &= h''\,dt \end{aligned}$$

because the first members of (6) are the projections of double the area described by the radius vector during the instant dt, on the planes of xy, xz and zy.

Squaring, adding and reducing we have

$$\begin{aligned} r^2\,dv &= (h^2 + h'^2 + h''^2)^{\frac{1}{2}}\,dt \\ &= H\,dt \end{aligned}$$

or $$r^2\frac{dv}{dt} = H \qquad (9)$$

where for brevity we put $H^2 = h^2 + h'^2 + h''^2$.

Let ☊ denote the angle which the common intersection of the plane of reference and the plane of the orbit, makes with the axis of x, or the longitude of the node, then by spherical trigonometry we shall have

$$\cos i_1 = -\sin i \cos ☊ \text{ and } \cos i_2 = \sin i \sin ☊$$

which combined with (8) and (9) give

$$\begin{aligned} h &= H \cos i \\ h' &= -H \sin i \cos ☊ \qquad (10) \\ h'' &= H \sin i \sin ☊ \end{aligned}$$

which determine i and ☊ when the arbitrary constants h, h' and h'' are known, thus they give

$$\tan \Omega = -\frac{h''}{h'} \text{ and } \tan i = \frac{\sqrt{(h'^2 + h''^2)}}{h}. \tag{11}$$

If we multiply equations (5) by $2dx$, $2dy$ and $2dz$ respectively, add and integrate we shall find

$$\frac{dx^2 + dy^2 + dz^2}{dt^2} + 2\mu \int \frac{xdx + ydy + zdz}{r^3} = 0$$

and from the first of (1) we have

$$rdr = xdx + ydy + zdz$$

which substituted in the above and completing the integration, gives

$$\frac{dx^2 + dy^2 + dz^2}{dt^2} - \frac{2\mu}{r} - C = 0 \tag{12}$$

where C is an arbitrary constant.

But $dx^2 + dy^2 + dz^2$ is the square of the space described by the body in the time dt, and therefore in polar coördinates.

$$\frac{dx^2 + dy^2 + dz^2}{dt^2} = \frac{dr^2 + r^2 dv^2}{dt^2}$$

which substituted in (12) becomes

$$\frac{dr^2}{dt^2} + \frac{r^2 dv^2}{dt^2} - \frac{2\mu}{r} - C = 0 \tag{13}$$

Eliminating dt between (9) and (13) we obtain

$$\frac{dr}{dv} = \frac{r\sqrt{C r^2 + 2\mu r - H^2}}{H} \tag{14}$$

But at the extremities of the major axis we must have $\frac{dr}{dv} = 0$, and therefore (14) enables us to determine the maximum and minimum values of r which we know to be $a(1+e)$ and $a(1-e)$ respectively. Equating the second member to zero and reducing we find

$$Cr^2 + 2\mu r - H^2 = 0$$

By the theory of quadratics, the sum of the two roots is equal to $-\frac{2\mu}{C}$ and their product to $-\frac{H^2}{C}$, we therefore have

$$2a = -\frac{2\mu}{C} \text{ and } a^2(1 - e^2) = -\frac{H^2}{C}$$

whence $$C = -\frac{\mu}{a} \text{ and } H = \sqrt{a\mu(1 - e^2)}$$

These values of C and H being substituted in (14) we have

$$dv = \frac{dr}{r} \frac{\sqrt{a\mu\,(1-e^2)}}{\sqrt{-\frac{\mu}{a} r^2 + 2\mu r - a\mu\,(1-e^2)}}$$

$$= \frac{dr}{r^2} \frac{\sqrt{a\mu\,(1-e^2)}}{\sqrt{-\frac{\mu}{a} + \frac{2\mu}{r} - \frac{a\mu\,(1-e^2)}{r^2}}}$$

and dividing numerator and denominator by $\sqrt{a\mu\,(1-e^2)}$ and writing $-d\frac{1}{r}$ for $\frac{dr}{r^2}$ we have

$$dv = \frac{-d\frac{1}{r}}{\sqrt{-\frac{1}{a^2\,(1-e^2)} + \frac{2}{ar\,(1-e^2)} - \frac{1}{r^2}}}$$

$$= \frac{-d\frac{1}{r}}{\sqrt{\frac{1}{a^2}\left\{\frac{1}{(1-e^2)^2} - \frac{1}{1-e^2}\right\} - \left\{\frac{1}{r} - \frac{1}{a\,(1-e^2)}\right\}^2}}$$

$$= \frac{d\left\{\frac{a\,(1-e^2)}{re}\right\}}{\sqrt{1 - \left\{\frac{\frac{a}{r}\,(1-e^2) - 1}{e}\right\}^2}}$$

If we put $\qquad \dfrac{\frac{a}{r}\,(1-e^2) - 1}{e} = x \qquad$ we have

$$dv = -\frac{dx}{\sqrt{1-x^2}}$$

and $\qquad v = \int \dfrac{-dx}{\sqrt{1-x^2}} = \omega + \cos^{-1} x$

$$= \omega + \cos^{-1}\left\{\frac{\frac{a}{r}\,(1-e^2) - 1}{e}\right\}$$

ω being an arbitrary constant.

Whence we have

$$\frac{a}{r}\,(1-e^2) - 1 = e\cos\,(v-\omega)$$

or

$$r = \frac{a\,(1-e^2)}{1 + e\cos\,(v-\omega)} \qquad (15)$$

the polar equation of the ellipse.

The angles v and ω are the angles which the radius vector and line of apsides, make with a fixed axis situated in the plane of the orbit or the true longitude of the planet and the longitude of the perihelion respectively. The difference $v - \omega$ determines the angular distance of the planet from perihelion and is called the true anomaly.

If the angle $v - \omega$ is reckoned from the perihelion then $\omega = 0$ and the above equation becomes

$$r = \frac{a\,(1 - e^2)}{1 + e \cos v}. \tag{16}$$

If instead of eliminating dt between equations (9) and (13), we eliminate dv, we shall have

$$\frac{dr^2}{dt^2} + \frac{H^2}{r^2} = \frac{2\mu}{r} + C$$

whence
$$dt = \frac{rdr}{\sqrt{C + \frac{2\mu}{r} - \frac{H^2}{r^2}}}$$
and substituting the values of C and H, we find

$$dt = \sqrt{\frac{a}{\mu}} \cdot \frac{rdr}{\sqrt{2ar - r^2 - a^2\,(1 - e^2)}}$$

$$= \sqrt{\frac{a}{\mu}} \cdot \frac{rdr}{\sqrt{a^2e^2 - (a - r)^2}}$$

which is easily integrated by introducing an auxiliary variable u connected with r by the relation

$$r = a\,(1 - e \cos u) \tag{17}$$

whence
$$dr = ae \sin u du$$

and
$$dt = \sqrt{\frac{a^3}{\mu}}\,(1 - e \cos u)\, du$$

an expression whose integral is

$$nt + c = u - e \sin u \tag{18}$$

where $n = \sqrt{\frac{\mu}{a^3}}$ and c is a constant.

The angle u is called *eccentric* anomaly, nt the *mean* anomaly of the planet (m) and n is the mean motion. When we take for the origin of time the instant the planet is in perihelion, $c = 0$ and we have

$$nt = u - e \sin u \tag{19}$$

If however we reckon the time to commence at any other instant after passing the perihelion nt must be increased by the constant $\varepsilon - \omega$, where ε denotes the mean longitude of the planet at the origin of the time or the longitude at the epoch, and then we shall have

$$nt + \varepsilon - \omega = u - e \sin u \tag{20}$$

Let T be the duration of the sidereal revolution of the planet (m) and a its mean distance from the Sun, then in putting $u = 2\pi$ in (19) we have $nT = 2\pi$

whence
$$T = \frac{2\pi}{n} = 2\pi \sqrt{\frac{a^3}{\mu}} \tag{21}$$

and also
$$\mu = 4\,\pi^2.\,\frac{a^3}{T^3}$$

Eliminating r between (15) and (17) we have

$$\cos u = \frac{e \cos (v - \omega)}{1 + e \cos (v - \omega)}$$ +/

whence
$$\frac{1 - \cos (v - \omega)}{1 + \cos (v - \omega)} = \frac{1 + e}{1 - e}.\frac{1 - \cos u}{1 + \cos u}$$

and
$$\tan \frac{1}{2} (v - \omega) = \sqrt{\frac{1 + e}{1 - e}}.\tan \frac{1}{2} u \tag{22}$$

If the angle v to be reckoned from the perihelion we have $\omega = 0$ and the above becomes

$$\tan \frac{1}{2} v = \sqrt{\frac{1 + e}{1 - e}}.\tan \frac{1}{2} u \tag{23}$$

a formula which enables us to determine the true anomaly in terms of u and also of nt the mean anomaly.

To determine the nature of the conic section described by the body (m) in its motion round the sun, let $V =$ the velocity in the orbit, then by (12) and (13) we have

$$V^2 = \frac{2\mu}{r} + C$$

$$= 2\left(\frac{\cancel{2}}{r} - \frac{1}{a}\right) \text{ since } C = -\frac{\mu}{a} \tag{24}$$ μ/

we also have
$$\frac{1}{a} = \frac{2}{r} - \frac{V^2}{\mu}$$

and as a is positive in the ellipse, infinite in the parabola and negative in the hyperbola we conclude that the orbit is an ellipse, a parabola or a hyperbola according as

$$V^2 < \frac{2\mu}{r}, = \frac{2\mu}{r} \text{ or } > \frac{2\mu}{r}$$

We have next to determine the value of k which is constant for the solar system.

From (21) we have $$T^2 = 4\pi^2 \frac{a^3}{\mu}$$

$$= 4\pi^2 \frac{a^3}{k^2 (1+m)}$$

whence $$k = \frac{2\pi}{T} \cdot \frac{a^{\frac{3}{2}}}{\sqrt{1+m}} \tag{25}$$

In the case of the Earth, $a = 1$, $T = 365.25638435$ days and the combined mass of the Earth and Moon is $\frac{1}{354710} = m$.

Therefore we easily find

$$\log k = 8.2355814$$

and in seconds of arc

$$\log k'' = 3.5500066$$

or $$k = 3548''.19 = 59' \;\; 8''.19$$

The quantity $\frac{2\pi}{T}$ is the mean angular motion in a mean solar day and since $\sqrt{1+m}$ differ very little from unity, k is very nearly equal to the mean daily motion of the Earth—a result which might have been anticipated, because the mass of the Sun is just such as will communicate to the Earth the mean daily motion which it actually has.

There is another method of arriving at the equation of the orbit described by the planet, which we will now give for the purpose of making the subject as clear and explicit as practicable. We have already shown (7) that the plane of the orbit passes through the centre of the Sun and if we take this as the plane of reference or the fundamental plane, then $z = 0$, and the general equations (5) of undisturbed motion are

$$\begin{aligned} \frac{d^2x}{dt^2} + \frac{\mu x}{r^3} &= 0 \\ \frac{d^2y}{dt^2} + \frac{\mu y}{r^3} &= 0 \end{aligned} \tag{26}$$

and (6) reduces to

$$x \frac{dy}{dt} - y \frac{dx}{dt} = h \tag{27}$$

which transformed into polar coördinates becomes

$$r^2 \frac{dv}{dt} = h \tag{28}$$

where h is twice the area described in the time dt.

In polar coördinates we have

$$x = r \cos v \text{ and } y = \sin v$$ r/

then we have

$$\frac{d}{dt}\left(\frac{x}{r}\right) = -\sin v \frac{dv}{dt}$$

$$= -\frac{r \sin v}{r^3} \cdot r^2 \frac{dv}{dt}$$

$$= -\frac{y}{r^3} h$$

whence $$\frac{y}{r^3} = -\frac{1}{h} \cdot \frac{d}{dt}\left(\frac{x}{r}\right)$$

Similarly $$\frac{x}{r^3} = \frac{1}{h} \cdot \frac{d}{dt}\left(\frac{y}{r}\right) \tag{29}$$

which substituted in (26) give

$$\frac{d^2x}{dt^2} = -\frac{\mu}{h} \cdot \frac{d}{dt}\left(\frac{y}{r}\right)$$

$$\frac{d^2y}{dt^2} = \frac{\mu}{h} \cdot \frac{d}{dt}\left(\frac{x}{r}\right) \tag{30}$$

By the integration of these we obtain

$$\frac{dx}{dt} = -\frac{\mu}{h}\left(\frac{y}{r} + \beta\right)$$

$$\frac{dy}{dt} = \frac{\mu}{h}\left(\frac{x}{r} + \alpha\right)$$

where a and β are constants.

Substituting in (27) we have

$$\frac{\mu x}{h}\left(\frac{x}{r} + \alpha\right) + \frac{\mu y}{h}\left(\frac{y}{r} + \beta\right) = h$$

or $$\alpha x + \beta y + r = \frac{h^2}{\mu} = \gamma. \text{ (suppose)} \tag{31}$$

and eliminating r by the relation $r^2 = x^2 + y^2$ and reducing we get

$$(1 - \alpha^2)\, x^2 - 2\alpha\,\beta\, xy + (1 - \beta^2)\, y^2 \\ + 2\alpha\gamma x + 2\beta\gamma y - \gamma^2 = 0 \qquad (32)$$

which is the general equation of the conic sections, the origin of coördinates being evidently at the focus.

The significance of the constants α, β and γ are now to be determined, If we denote by ω the angle which the axis of the conic makes with the axis of x, and by p the distance from the focus to the directrix, the general equation of the conic is

$$(1 - e^2 \cos^2 \omega)\, x^2 - 2e^2 \sin \omega \cos \omega .\, xy + (1 - e^2 \sin^2 \omega)\, y^2 \\ + 2e^2\, p \cos \omega .\, x + 2e^2\, p \sin \omega .\, y - e^2 p^2 = 0, \qquad (33)$$

and comparing the coefficients of these two equations we easily find

$$e = \sqrt{\alpha^2 + \beta^2}$$

$$\tan \omega = \frac{\beta}{\alpha} \qquad (34)$$

If $\omega = 0$, the axis of x coincides with the axis of the curve, $\beta = 0$ and then (32) becomes

$$(1 - \alpha^2)\, x + y^2 + 2\alpha\gamma x - \gamma^2 = 0$$

and if $x = 0$, $\qquad y = \gamma$

$$= \text{the semi-latus rectum}$$

therefore $\qquad \gamma = \dfrac{h^2}{\mu} \qquad (35)$

When $\beta = 0$, $\alpha = e$ and (31) becomes

$$ex + r = \frac{h^2}{\mu}$$

a/ or in polor coördinates

$$r = \frac{\dfrac{h^2}{\mu}}{1 + e \cos v} \qquad (36)$$

the well known polar equation of the conic sections.

In the preceding formulae (16), (17) and (23) both r and v are connected with nt,—the mean anomaly which we shall henceforth denote by M — through the transcendental equation (19), and in the integration of the equations of disturbed motion it will be necessary to express r and v as functions of M.

Writing M for nt we have from (19)

$$u = M + e \sin u$$

from which we develop cos u, cos $2u$, etc., by Lagrange's theorem, thus

$$F(u) = F(M) + e \left\{ f(M) \frac{dF(M)}{dM} \right\} + \frac{e^2}{1.2} \frac{d}{dM}$$

$$\left\{ (f(M))^2 \frac{dF(M)}{dM} \right\} + \frac{e^3}{1.2.3} \frac{d^2}{dM^2} \left\{ (f(M))^3 \frac{dF(M)}{dM} \right\} + \text{etc.}$$

Put $F(u) = \cos u$, $F(M) = \cos M$, $f(M) = \sin M$, then we shall have

$$f(M) \frac{dF(M)}{dM} = -\sin^2 M = \frac{\cos 2M - 1}{2}$$

$$\frac{d}{dM} \left\{ (f(M))^2 \frac{dF(M)}{dM} \right\} = \frac{d}{dM} \left\{ -\sin^3 M \right\} =$$

$$-3 \cos M + 3 \cos^3 M$$

$$= \frac{3}{4} \cos 3M - \frac{3}{4} \cos M$$

$$\frac{d^2}{dM^2} \left\{ (f(M))^3 \frac{DF(M)}{dM} \right\} = \frac{d}{dM} \left\{ -\sin^4 M \right\}$$

$$= -2 \cos 2M + 2 \cos 4M, \text{ etc.}$$

Substituting in the above we find

$$\cos u = \cos M + \frac{e}{1} . \frac{\cos 2M - 1}{2} + \frac{e^2}{1.2} \left(\frac{3}{4} \cos 3M - \frac{3}{4} \cos M \right)$$

$$+ \frac{e^3}{1.2.3} (-2 \cos 2M + \cos 4M) + \text{etc.}$$

$$= \cos M + \frac{e}{2} \cos 2M - \frac{e}{2} + \frac{3e^2}{2^3} \cos 3M - \frac{3e^2}{2^3} \cos M$$

$$+ \frac{e^8}{3} \cos 4M - \frac{e^3}{3} \cos 2M$$

$$\begin{aligned} = -\frac{e}{2} &+ \left(1 - \frac{3e^2}{2^3} + \ldots\ldots \right) \cos M \\ &+ \left(\frac{e}{2} - \frac{e^3}{3} + \ldots\ldots\ldots \right) \cos 2M \\ &+ \left(\frac{3e^2}{2^3} - \ldots\ldots\ldots\ldots \right) \cos 3M \\ &+ \left(\frac{e^3}{3} - \ldots\ldots\ldots\ldots \right) \cos 4M \\ &+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned} \qquad (37)$$

In a similar manner we find

$$\begin{aligned}\cos 2u = &\left(\quad - e + \frac{e^3}{2^2.3} \;.\;.\;.\;.\;.\;.\;.\;.\right)\cos M \\ &+ \left(1 - e_3 + \frac{5e^4}{2_3 3} \;.\;.\;.\;.\;.\;.\;.\;.\right)\cos 2\,M \\ &+ \left(e - \frac{3^2e^3}{2^3} + \;.\;.\;.\;.\;.\;.\;.\;.\;.\right)\text{ccs } 3\,M \qquad (38) \\ &+ \left(e^2 - \frac{2^2e^4}{3} + \;.\;.\;.\;.\;.\;.\;.\;.\;.\right)\cos 4\,M \\ &+ (\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;)\end{aligned}$$

and so on.

From (17) and (37) we easily find

$$\begin{aligned}\frac{r}{a} = &\left(1 \;\; + \frac{e^2}{2}\right) \\ &+ \left(- e + \frac{3}{2^3}\,e^3 \;\;.\;.\;.\;.\;.\;.\;.\;.\right)\cos\,M \\ &+ \left(- \frac{e^2}{2} + \frac{e^4}{3} - \;\;.\;.\;.\;.\;.\;.\;.\;.\;.\right)\cos 2\,M \qquad (39) \\ &+ \left(- \frac{3e^3}{2^3} + \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\right)\cos 3\,M \\ &+ (\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;)\end{aligned}$$

which expresses r in terms of M.

From (28) we have $\dfrac{dv}{dt} = \dfrac{h}{r^2}$

where h is twice the area described in a unit of time, therefore

$$\begin{aligned}Th &= 2 \times \text{area of the ellipse} \\ &= 2\pi a^2 \sqrt{1 - e^2} \\ &= \frac{dM}{dt} \cdot \frac{a^2}{r^2} \sqrt{1 - e^2)}\end{aligned}$$

and

$$h = \frac{2\pi}{T} \cdot a^2 \sqrt{1 - e^2}$$

and

$$\begin{aligned}\frac{dv}{dt} &= \frac{2\pi}{T} \cdot \frac{a^2}{r^2} \cdot \sqrt{1 - e^2} \\ &= \frac{dM}{dt} \cdot \frac{a^2}{r^2} \sqrt{1 - e^2}\end{aligned}$$

or

$$\begin{aligned}\frac{dv}{dM} &= \frac{\sqrt{1 - e^2}}{(1 - e\cos u)^2} \quad \text{by (17)} \\ &= (1 - e^2)^{\frac{1}{2}} (1 - e\cos u)^{-2}\end{aligned}$$

Developing the second member by the binomial theorem and changing the powers of cos u to the cosines of multiples of u we have

$$\begin{aligned}
\frac{dv}{dM} &= (1-e^2)^{\frac{1}{2}}\,(1-e\cos u)^{-2} \\
&= (1+e^2+e^4+\ldots\ldots\ldots\ldots) \\
&\quad + 2\,(e+e^3+e^5+\ldots\ldots\ldots\ldots)\cos u \\
&\quad + \left(\frac{3}{2}e^2+\frac{7}{2^2}e^4+\ldots\ldots\ldots\ldots\right)\cos 2u \\
&\quad + \left(e^3+\frac{11}{2^3}e^5+\ldots\ldots\ldots\ldots\right)\cos 3u \\
&\quad + (\ldots\ldots\ldots\ldots\ldots\ldots)\ldots
\end{aligned}$$

Substituting the values of cos u, from (37) (38), etc., multiplying by dM and integrating we obtain

$$\begin{aligned}
v = M &+ \left(2e-\frac{e^3}{2^2}+\frac{5e^5}{3.2^5}+\ldots\ldots\ldots\right)\sin M \\
&+ \left(\frac{5e^2}{2^2}-\frac{11e^4}{3.2^3}+\ldots\ldots\ldots\right)\sin 2M \qquad (40) \\
&+ \left(\frac{13e^3}{3.2^2}-\frac{43e^5}{2^6}+\ldots\ldots\ldots\right)\sin 3M \\
&+ \ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}$$

No constant is added because $v \doteq 0$ when $M = 0$.

The quantity $v - M$ is the equation of the centre.

For a more elaborate development of these formulæ extended to the 12th power of e and to the 12th multiple of u and M, see my paper in Monthly Notices of the Royal Astronomical Society, Vol. 43, No. 7.

The formulæ just derived enable us to arrive at an approximate integration of the formulæ (4) for disturbed motion.

REPRESENTATIVE STELLAR SPECTRA BY SIR WILLIAM HUGGINS AND LADY HUGGINS.

W. W. PAYNE.

As announced before in this magazine, we have received Vol. 1 of the publications of Sir William Huggins' Observatory at Tulsehill, London, England. The full title of this noble volume is "An Atlas of Representative Stellar Spectra from λ 4870 to λ 3300. There is added a discussion of the evolutional order of the stars,

and an interpretation of their spectra, preceded by a short history of the Observatory and its work. The book is a large quarto in form, printed on fine, heavy plate paper, with some titles and initial letters in red ink. It contains 165 pages with 13 full page plates of representative and most interesting stellar spectra. The plates are beautiful and perfect half-tones in almost every particular and the large scale in which they appear makes them most valuable for comparison or for general reference. The book as a whole is a superb specimen of printing, and is surely very creditably to the publishers, Messrs. Wm. Wesley & Son of London, one of the oldest and best known publishing houses in the world.

On account of its real scientific value, this book ought to find ready sale in America; for certainly scientists, Observatories, scientific libraries and the best general libraries will want a late, authoritative work on stellar spectroscopy by an author who is recognized everywhere as one of the first, if not the leading one in his chosen field of work.

Dr. Huggin's Observatory at Tulse-hill is widely known among astronomers as the pioneer institution in applying the spectroscope to astronomy. It was early in the sixties that Dr. Huggins began this work, and the rapid discoveries which he made have served as the foundation of the new astronomy which has since grown so rapidly and which still promises more and more by the aid of celestial photography. The broad outlook in science thus opened appears to be limitless in the way of better observation, permanent record and more refined and satisfactory measurement. These are the great factors of real progress to which the practical astronomer must ever look with increasing dependence as the limits of his field of labor and discovery are widened and removed farther and farther from the well beaten paths of knowledge.

When Dr. Huggins began to apply the spectroscope to the study of the stars, the difficulties which he met and was obliged to overcome were enormous. These are briefly set out in the first chapter of this volume which, in this regard, is especially good reading for young people who are anxious to do something in new, or comparatively new lines of work, and imagine the task will be an easy one when they once get at it. The experiences of such pioneers who have worked most industriously for nearly two scores of years must contain lessons of great value for those who would walk in like illustrious paths.

It was the announcement of Kirchoff in 1858, that he had discovered the true nature and the chemical conditions of the Sun,

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Prof: J. B. Cherriman, M.A. Cantab.
University College, Toronto.
From about 1852 to about 1877.

John Couch Adams

PLATE I.

Popular Astronomy.

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HISTORICAL EULOGY ON LAPLACE.

BY BARON FOURIER*

(Translated by T. J. J. See)

FOR POPULAR ASTRONOMY.

Gentlemen, the name of Laplace has resounded in all parts of the world where the sciences are honored; but his memory could not receive a more appropriate homage than the unanimous tribute of admiration and regret of the illustrious body in whose labors and glories he has shared. He has consecrated his life to the study of the greatest objects which can occupy the human mind.

The wonders of the heavens, the profound questions of natural philosophy, the ingenious and subtle combinations of mathematical analysis, all the laws of the universe, have been presented to his mind for nearly sixty years, and his efforts have been crowned by immortal discoveries.

It was observed from his earliest studies that he was endowed with a prodigious memory; for him all intellectual occupations were easy. He rapidly acquired a very extensive knowledge of the ancient languages, and cultivated diverse branches in literature. Everything interests the born genius, everything can reveal it. His first successes were in theological studies; he treated with talent and extraordinary sagacity the most difficult points of controversy. We do not know by what fortunate circumstance Laplace passed from scholasticism to sublime geometry. This last science which admits of no division attracted and fixed his attention. From that time he abandoned himself without reserve to the impulses of his genius, and perceived clearly that a sojourn at the capital had become to him a necesity. D'Alembert was then enjoying all the splendor of his renown. It fell to his lot to inform the Court of Turin that its Royal Academy possessed a mathematician of the first order, Lagrange, who, by default of this noble recognition, had been compelled to remain long ignored. D'Alembert had announced to the King of Prussia

* Delivered in the public session of the Royal Academy of Sciences, June 15, 1829.

that only one man in Europe could replace at Berlin the illustrious Euler, who, recalled by the Russian Government, consented to return to St. Petersburg. I find, in the unpublished letters in possession of the Institute of France, the details of this glorious negotiation which fixed Lagrange in residence at Berlin. About the same time Laplace began that long career which he was soon to distinguish.

He presented himself to D'Alembert, preceded by numerous recommendations which were believed to be very powerful. But his attempts proved unsuccessful; he was not even introduced. Then he addressed to the one whose support he solicited a very remarkable letter on the general principles of mechanics, of which M. Laplace has several times shown me diverse fragments. It was impossible that so great a geometer as D'Alembert could fail to be struck by the singular profundity of this writing. The same day he summoned the author of the letter, and said to him, in his own words: "Sir, you see that I attach little importance to recommendations; you have no need for them; you have made yourself better known; this satisfies me; my support is due you."

A few days later he caused Laplace to be nominated Professor of Mathematics at the Military School of Paris. From this time on, devoted exclusively to the science which he had chosen, he gave to his work a fixed and never-deviating direction; for imperturbable constancy of purpose has always been the principal trait of his genius. He touched already the known limits of mathematical analysis, he possessed all that was ingenious and powerful in this science at the time, and no one was more capable than he to extend its domain. He had solved a leading question of theoretical astronomy. He formed the project of consecrating his life to this sublime science; he was destined to perfect it, and able to comprehend it in all its extent. He meditated profoundly his glorious design; he has spent his whole life in accomplishing it with a perseverance which is perhaps unexampled in the history of the sciences.

The immensity of the subject flattered the just pride of his genius. He undertook to compose the *Almagest* of his age, a monument which he has left us under the name of the *Mécanique Céleste*; and his immortal work transcends that of Ptolemy as much as the analysis of the moderns surpasses the elements of Euclid. Time, which alone justly dispenses literary glory, which consigns to oblivion all contemporaneous mediocrities, perpetuates the memory of great works. These alone transmit to posterity the character of each century. Thus will the name of

Laplace live through all ages. But I hasten to remark that enlightened and faithful history will not separate his memory from that of the other successors of Newton. It will associate the illustrious names of D'Alembert, of Clairaut, of Euler, of Lagrange and of Laplace. I restrict myself here to citing the great geometers whom the sciences have lost, and whose researches have had for a common aim the perfection of physical astronomy. To give a correct idea of their works it is necessary to compare them; but space obliges me to reserve a portion of this discussion for our memoirs.

Next to Euler, Lagrange has contributed most to the founding of mathematical analysis. In the writings of these two great geometers it has become a distinct science which alone of mathematical theories can be said to be completely and rigorously demonstrated. It alone among all these theories is sufficient in itself and throws light upon all the rest; to them it is so necessary, that, deprived of its aid, they could very imperfectly exist.

Lagrange was born to invent and to extend all the sciences of analysis. In whatever condition fortune might have placed him, whether peasant or prince, he would have been a great geometer; he would necessarily have done it and without any effort; the same cannot be said of all who have excelled in this science, even in the first ranks. Had Lagrange been a contemporary of Archimedes and of Conon, he would have divided the glory of the most memorable discoveries. At Alexandria he would have been a rival of Diophantus.

The distinctive trait of his genius consists in the unity and grandeur of his views. He always fixes attention on an idea which is simple, just and very exalted. His principal work, the *Mécanique Analytique*, might be named the *Mécanique Philosophique*; for he reduced all the laws of equilibrium and of motion to a single principle; and what is not less admirable he subjected them to a single analytical method of which he himself was the inventor. All his mathematical compositions are remarkable for symmetry of form, generality of method, and, if we may so speak, for perfection of analytical style.

Lagrange was no less a philosopher than a great geometer. He proved this through the whole course of his life by the moderation of his desires, his immutable attachment to the general interests of humanity, by the noble simplicity of his manners and elevation of character, and finally by the fitness and profundity of his scientific works.

Laplace had received from nature all the force of genius which

an immense enterprise could exact. Not only has he collected in his *Almagest of the 18th Century* what the mathematical and physical sciences had already invented, and which constitute the foundation of astronomy; but he added to this science capital discoveries of his own which had escaped all of his predecessors.

He has solved, either by his own methods or by those whose principles Euler and Lagrange have pointed out, the most important and certainly the most difficult of all the problems which have been considered before him. His constancy triumphed over all obstacles. When his first efforts were not successful he renewed them under the most ingenious and the most diverse forms.

Thus there was observed in the motion of the Moon an acceleration whose cause could not be discovered. It had been held that this effect could arise from the resistance of the ethereal medium where the heavenly bodies move. If this were true, the same cause, affecting the course of the planets, would tend to change little by little the primitive order. These bodies would be incessantly disturbed in their courses and finally precipitated on the mass of the Sun. It would be necessary that the creative power intervene anew to prevent or to repair the immense disorder which the lapse of time would produce.

This cosmological question is assuredly one of the greatest which the human intelligence can propose; today it is solved. The first researches of Laplace upon the invariability of the dimensions of the solar system and his explanation of the secular equation of the Moon, have led to this solution.

He first examined if the acceleration of the lunar motion could be explained by supposing that the action of gravity is not instantaneous but subject to successive transmissions like light. From this point of view he could not discover the true cause. Finally a new research served his genius better. On March 19th, 1787, he presented to the Academy of Sciences a clear solution free from this capital difficulty. He proved very distinctly that the observed acceleration is a necessary effect of universal gravitation.

This great discovery subsequently cleared up the most important points of the system of the world. As a result the same theory shows that, if the action of gravitation upon the heavenly bodies is not instantaneous, we must suppose that it is propagated more than fifty million times faster than light, whose velocity is well known to be seventy thousand leagues per second. He further concluded from his theory of the Moon's motion that

the medium in which the heavenly bodies move offers to the course of the planets a resistance which is practically insensible; for this cause would effect especially the motion of the Moon, and here the effect produced could not be observed.

The discussion of the motion of this body is rich in remarkable results. We conclude from it, for example, that the rotatory motion of the Earth on its axis is invariable. The length of the day has not changed one-hundredth part of a second during two thousand years. It is remarkable that an astronomer would have no need to leave his Observatory to measure the distance from the Earth to the Sun. It suffices for him to observe closely the variation of the lunar motion; from this he could conclude the distance with certainty.

A consequence still more striking is that which relates to the figure of the Earth; for even the form of the terrestrial globe is impressed upon certain inequalities of the course of the Moon. These inequalities would not exist if the Earth were perfectly spherical. We can determine the amount of terrestrial flattening by observation of the lunar motions alone and the results deduced from it agree with the actual measures yielded by the great geodetic voyages to the equator, in the northern regions, in India and numerous other countries. It is to Laplace above all others that we owe this astonishing perfection of modern theories.

I cannot here undertake to indicate his works one after another and the discoveries to which they have led.

This enumeration alone, however brief it might be, would exceed the limits which I was compelled to prescribe for myself. Besides his researches on the secular equation of the Moon, and the no less difficult and no less important discovery of the long inequality of Jupiter and Saturn, we could cite his admirable theorems on the libration of Jupiter's satellites. We should mention his analytic treatment of the tides of the sea, and show the immense extension he has given this subject.

There is no important point of physical astronomy which was not for him an object of study and profound discussion; he subjected to calculation the greater part of the physical conditions which his predecessors had omitted. In the already very complex question of the form and the rotatory motion of the Earth, he considered the effect of the presence of water distributed between the continents, the compression of the interior layers and the secular diminution of the dimensions of the globe.

In this collection of researches we should notice especially those which relate to the stability of great phenomena: no object is

more worthy of the meditation of philosophers. Thus we have recognized that the causes, either accidental or constant, which disturb the equilibrium of the oceans are subjected to limits which cannot be passed. The specific gravity of water being much less than that of the solid Earth it follows that the oscillations of the ocean are always confined between very narrow limits; which would not be the case if the liquid spread over the globe were of much greater density. In general nature holds in reserve conservative and ever-present forces which act as soon as the trouble begins and augment in proportion as the disturbance is greater. They quickly re-establish the accustomed order. We find in all parts of the universe this preservative power. The forms of the great planetary orbits and their inclinations vary and oscillate in the course of ages; but these changes are limited. The principal dimensions are maintained and this immense assemblage of heavenly bodies oscillates about a mean position toward which it is always carried. Everything is adjusted for order, perpetuity and harmony.

In the primitive and liquid state of the terrestial globe, the heaviest matter settled towards the center; and this condition has determined the stability of the sea.

Whatever may have been the physical cause of the formation of the planets, it imparted to all the bodies a projectile motion in a common direction about an immense globe; in this way the solar system has been rendered stable. The same effect is produced in the systems of satellites and of rings. Order is there maintained by the power of the central mass. Hence, there is not, as Newton himself and Euler had suspected, an adventitious force which should one day repair or prevent the disturbance which time had wrought. This is the law of gravitation itself, which rules all, is sufficient for all, and maintains order and variety. Having proceeded once only from supreme wisdom, it has presided since the origin of time and renders all disorder impossible. Newton and Euler did not yet recognize all the perfections of the universe. In general whenever any doubt had arisen in regard to the exactness of the Newtonian law, and, for the explanation of apparent irregularities, we have proposed the accession of a foreign cause, it has always happened, after a profound examination, that the original law has been verified. It explains to-day all known phenomena. The more precise the observations the better do they conform to theory. Of all the great mathematicians, Laplace is the one who has penetrated most profoundly these great questions; he has, so to speak, settled them.

We cannot affirm that it was given to him to create an entirely new science as did Archimedes and Galileo; to give to mathematical theories original and immensely extended principles, as did Descartes, Newton and Leibnitz; or like Newton to transfer for the first time to the heavens and to extend throughout the universe the terrestrial dynamics of Galileo; but Laplace was born to perfect all, to penetrate everything, to extend all limits, to solve what was believed to be insoluble. He would have completed the science of the skies, if this science could be completed.

We find this same character in his researches on the analysis of probability, a science entirely modern and of immense extent, whose object, often misconceived, has given rise to the most erroneous interpretations, but whose applications will one day embrace the whole domain of human knowledge, a happy supplement to the imperfection of our nature. This art arose from a single trait of the clear and prolific genius of Pascal; it has been cultivated, since its origin, by Fermat and Huyghens. A philosophical geometer, John Bernouilli, was its principal founder.

A singularly happy discovery of Stirling, the researches of Euler, and above all an ingenious and important application due to Lagrange, have perfected this doctrine; it was cleared up by the objections of D'Alembert and by the philosophical views of Condorcet: Laplace collected and fixed its principles. It has become a new science of prodigious extent, subject to a single analytical method. Fruitful in ordinary applications, it will one day illuminate every branch of natural philosophy. If it is permissible to express here a personal opinion we shall add that the solution of one of the principal problems, which the illustrious author has treated in the tenth chapter of his work, does not appear to be exact; and yet considered in its totality this work is one of the most precious monuments of his genius.

After having cited such brilliant discoveries it would be useless to add that Laplace belonged to all the great academies of Europe. I could, and perhaps ought, to recall also the high political dignities with which he was clothed; but this enumeration would belong only indirectly to the object of this discourse. It is the great mathematician whose memory we celebrate. We have separated the immortal author of the *Mécanique Céleste* from all the accidental facts which concern neither his glory nor his genius.

Indeed, gentlemen, of what concern is it to posterity, who will have so many other details to forget, whether Laplace was at one time minister of a great nation? What concerns us most are

the eternal truths he discovered; these are the immutable laws of the stability of the world, and not the rank which he occupied some years in the senate called conservator. What concerns us, gentlemen, perhaps still more than his discoveries, is the example which he set to all those to whom the sciences are dear; it is the spirit of this incomparable perseverence which has sustained, directed and crowned so many glorious efforts.

I shall therefore omit the accidental circumstances and, so to speak, fortuitous incidents, particulars which have no relation to the perfection of his works. But I may say that in the first body of the state, the memory of Laplace was celebrated by an eloquent and friendly voice, which important services rendered to the historical sciences, to letters and to the state have for a long time distinguished.* I shall recall especially that literary ceremony which attracted the attention of the Capital. The French Academy, concurring in its recognition with the acclamations of the country decided that it would acquire a new glory in crowning triumphs of eloquence and of political virtue.† At the same time it chose to return as a successor of Laplace an academician illustrious for more than a title, who united in literature, in history, in public administration, all the qualities of superiority.‡

Laplace enjoyed an advantage which fortune has not always accorded to great men. From his early youth he was deservedly appreciated by illustrious friends. We have in hand some letters, still un-edited, which show us all the zeal with which D'Alembert introduced him to the military school of Paris, to prepare for him, if that were necessary, a better position at Berlin. President Bochard de Saron printed his first works. All the testimonials of friendship which were given him recall his great works and his great discoveries; but nothing could contribute more to the progress of all physical knowledge, than his relations with the illustrious Lavoisier, whose name, consecrated in the history of the sciences, has become an enduring object of respect and grief.

These two celebrated men united their efforts. They undertook and carried out very extended researches for measuring one of the most important elements in the physical theory of heat. They made also, about the same time, a long series of experiments on the dilatation of solids. The works of Newton have made sufficiently known the value which this great mathematician at-

* The Marquis de Pastoret.
† M. Royer-Collart.
‡ The Count Daru.

tached to the special study of the physical sciences. Laplace is, of all his successors, the one who has made the greatest use of his experimental method; he was almost as great a physicist as a geometer. His researches on refraction, on capillarity, barometric measures, on static properties of electricity, velocity of sound, molecular actions, properties of gases, attest that nothing in the investigation of nature could be foreign to him. He always desired especially the perfection of instruments; he caused to be constructed, at his own expense, by a celebrated artisan, a very precise astronomical instrument and has given it to the Observatory of France.

All kinds of phenomena were perfectly known to him. He was connected by an old friendship with two celebrated physicists whose discoveries have thrown light on all the arts and all the theories of chemistry. History will connect the names of Berthollet and Chaptal with that of Laplace. He liked to associate with them and their conferences have always had for an object and end the increase of knowledge most important and most difficult to acquire. The gardens of Berthollet at his house d'Arcueil were not separated from those of Laplace. Great memories and great sorrows have distinguished this enclosure. It was here that Laplace received celebrated strangers, influential men, from whom the sciences had received or might expect benefactions, but, especially those who were drawn by sincere zeal to the sanctuary of the sciences. Some were just commencing their careers, others would soon bring theirs to a close. He entertained them with extreme politeness. He even carried it so far that he allowed those who did not yet know of the full extent of his genius to believe that he might himself gain knowledge from their conversations.

In citing the mathematical works of Laplace we ought especially to point out the depth of his researches and the importance of his discoveries. His works are moreover, distinguished by another quality which all readers have appreciated. I refer to the literary merits of his compositions. The work entitled *Système du Monde* is remarkable for elegant simplicity of style, and for purity of language. There was no other example of this species of production; but we could form of it an approximate idea if we remember that it is possible to acquire a knowledge of the phenomena of the heavens in agreeable writings. The suppression of the signs required in the calculus can not contribute to clearness or render the reading easier. The work is a perfectly regular exposition of the results of profound study; it is an in-

genious resumé of the principal discoveries. The precision of style, the choice of methods, the grandeur of the subject, give a singular interest to this vast picture; but its real utility is to recall to geometers theorems whose demonstrations are known to them. Properly speaking it is a condensation of the material of a mathematical treatise.

The purely historical works of Laplace have a different object. They present to geometers with admirable talent the march of the human mind in the discovery of the sciences. Consequently the most abstract theories have a beauty of expression which is all their own; the same thing is observed in several treatises of Decartes, in certain pages of Galileo, of Newton, and of Lagrange. The novelty of the views, the elevation of thought, their relations to the great objects of nature, charm and fill the mind. It suffices that the style be pure and of a noble simplicity; such is the kind of literature which Laplace has chosen, and it is certain that he has here gained a place in the first rank. If he wrote the history of great astronomical discoveries, it became a model of elegance and of precision. No principal feature escaped him; his expression is neither obscure nor pretentious. All that he called great is great indeed; all that he omitted did not deserve to be cited.

M. Laplace preserved to a very advanced age that extraordinary memory which was the subject of remark in his earliest years; a precious gift which is not genius but which serves it to acquire and to retain. He never cultivated the fine arts, but he appreciated them. He was fond of Italian music and of the verses of Racine, and often took pleasure in reciting from memory various passages from this great poet. Paintings of Raphael ornamented his apartments. They were to be found by the side of the portraits of Descartes, of Francois Viète, of Newton, of Galileo, and of Euler.

Laplace always had the habit of taking very light nourishment; he diminished the quantity more and more even to excess. His very delicate eye-sight required continual precautions; but he managed to preserve it without alteration. These precautions for himself had but a single aim, that of reserving all his time and all his energy for intellectual work. He lived for the sciences; the sciences have rendered his memory eternal.

He contracted the habit of excessive concentration of thought, so injurious to the health, so necessary to profound study; and yet he underwent a sensible weakening only in the last two years of his life. At the commencement of the disease to which he suc-

cumbed there was noticed with alarm a symptom of delirium. The sciences occupied him still. He spoke with an unaccustomed ardor of the motions of the stars, and then of an experiment in physics, which he said was capital, announcing to the persons whom he believed to be present that he would soon entertain the Academy with these questions. His strength left him little by little. His physician*, who merited all his confidence by superior talents and by the care which friendship alone could inspire, watched near his bed. M. Bouvard, his co-worker and friend, never left him for an instant. Surrounded by a beloved family, under the eyes of a wife whose devotion had aided in supporting the sorrows inseparable from life, whose amenities and graces had made known to him the prize of domestic happiness, he received from his son, M. Marquis de Laplace, express testimonials of the most touching devotion. He gave signs of gratitude for the repeated tokens of interest shown him by the King and the Dauphin. The persons who assisted at his last moments recalled to him the titles of his glory and his brilliant discoveries. He answered: "That which we know is little, that of which we are ignorant is immense." This at least is, so for as we have been able to ascertain, the sense of his last words, which were spoken with difficulty. Moreover, we have often heard him express these thoughts and almost in the same terms. He passed away without pain. His supreme moment had arrived; the powerful spirit which had long animated him separated from the mortal envelope and returned to the skies.

The name of Laplace honors one of our provinces already prolific in great men, ancient Normandy.

He was born March 23, 1749; he died in his seventy-eighth year, May 5th, 1827, at nine o'clock in the morning.

I shall recall to you, gentlemen, the sad gloom which spread over this palace like a cloud when the new fatality was announced. This was the day and the hour of your accustomed meeting. Each of you maintained a mournful silence; each felt the sad blow with which the sciences had been smitten. Every respect was shown in this place which he had so long occupied among you. One thought only was present; all other meditation had become impossible. You adjourned with a unanimous resolution, and this time only were your usual labors interrupted.

It is undoubtedly beautiful, it is glorious, it is worthy of a powerful nation, to ordain brilliant honors to the memory of its celebrated men. In the country of Newton the chiefs of state

* M. Magendie.

wished to have the mortal remains of this great man solemnly interred among the royal tombs. France and Europe have offered to the memory of Laplace an expression of their grief less showy doubtless, but perhaps more touching and more true.

He has received an unaccustomed homage; he received it from his own people, in the midst of a learned company which alone could appreciate all of his genius. The voice of the weeping sciences has made itself audible in every country of the world where philosophy has penetrated. We have in hand much correspondence from all parts of Germany, England, Italy, New Holland, the English possessions in India, the two Americas; and we find everywhere the same sentiments of admiration and of regret. Certainly this universal lamentation of the sciences, so nobly and so freely expressed, has no less truth and splendor than the sepulchral pomp of Westminster.

Let me be permitted, before closing this discourse, to reproduce here a thought which presented itself when I recalled in this place the great discoveries of Herschel, but which is applicable still more directly to those of Laplace.

Your successors, gentlemen, will witness the accomplishment of the great phenomena whose laws he discovered. They will observe in the motions of the Moon the changes which he predicted and of which he alone was able to assign the cause. The continual observation of Jupiter's satellites will perpetuate the memory of the inventor of the theorems which govern them in their courses. The great inequality of Jupiter and Saturn, running through their long periods, and giving to these bodies new situations will recall without ceasing one of his most astonishing discoveries. These are the titles of a true glory which nothing can extinguish. The spectacle of the heavens will be changed, but at these remote epochs the glory of the inventor will continue forever; the traces of his genius bear the seal of immortality.

I have presented to you, gentlemen, some characteristics of an illustrious life consecrated to the glory of the sciences; may your memories supplement my feeble language! May the voice of our country, may that of all humanity rise to celebrate the benefactors of nations, the only homage worthy of those who have been able, like Laplace, to enlarge the domain of thought, and bear witness to man of the dignity of his being, by unveiling to our contemplation all the majesty of the heavens!

PLATE III.

PROFESSOR BENJAMIN PEIRCE.

(1809-1880)

(From a Photograph taken in 1879.)

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THE SERVICES OF BENJAMIN PEIRCE TO AMERICAN MATHEMATICS AND ASTRONOMY.

T. J. J. SEE.

FOR POPULAR ASTRONOMY.

It is related that when Dr. Bowditch was preparing the manuscript of his translation of Laplace's *Mécanique Céleste* a lad by the name of Benjamin Peirce with a playmate happened to enter his library in Salem; while glancing over some of the pages which the illustrious translator had finished young Peirce exclaimed, "here is a mistake." The great mathematician examined the passage in question and found that his young friend had indeed detected a genuine error. This circumstance is said to have aroused Bowditch's interest in the brilliant young Peirce. It did not take Bowditch long to discover his extraordinary talents, and a casual acquaintance soon grew into an intimate and professional friendship. Peirce became the disciple, and subsequently the logical successor, of Bowditch, who predicted that his favorite student would some day be the foremost mathematician of his country.

Benjamin Peirce was born at Salem, Massachusetts, April 4th, 1809, the third of the four children of Benjamin Peirce and his wife, a sister of the Rev. Dr. Nichols of Portland, Maine. The elder Mr. Peirce graduated at Harvard in 1801, with the highest honors of his class. For a time he was a merchant in Salem but in later years inclined towards letters: from 1826 to 1831—the year of his death—he was college librarian and wrote a history of Harvard University from 1639 to the time of the American Revolution.

Benjamin Peirce Junior graduated from Harvard with the highest honors in 1829, and his college days thus brought him into contact with such classmates as Geo. T. Bigelow, W. H. Channing, B. R. Curtis, Oliver Wendell Holmes and James Freeman Clarke.

While still an undergraduate Peirce pursued his studies under the direction of Nathaniel Bowditch, and when the immortal translation of the *Mécanique Céleste* came to the press this de-

voted student rendered important services in critically reading and correcting the proofs. Those who have had occasion to make a profound examination of the work can bear witness to the very small number of errors which escaped both Bowditch and Peirce.

In 1831, after young Peirce had taught two years at Round Hill, Northampton, he was appointed tutor in mathematics at Harvard University; his connection with the college continued for the unprecedented period of forty-nine years. He became university professor of mathematics and natural philosophy in 1833 and during the next thirteen years issued a series of valuable text-books on geometry, algebra, trigonometry, and "curves, functions and forces," which were filled with new methods and have made a lasting impression upon the educational work of this country. In 1842 Peirce was appointed Perkins professor of mathematics and astronomy and continued to hold this position for thirty-eight years.

Professor Peirce's life may be said to have been as a whole remarkably fortunate and happy. He was married in 1833, and at the time of his death, his wife, a daughter and three distinguished sons survived him. His eldest son, James Mills Peirce, university professor of mathematics at Harvard, and dean of the graduate school, has long been celebrated as one of the most eminent mathematicians of the country; another son Dr. Chas. S. Peirce, who inherited his father's talents to a remarkable degree, has been professor in Harvard and Johns Hopkins, and now resides in Milford, Pa.; the third son, H. H. D. Peirce is a business man connected with a large firm in New York City.

Peirce's long career was one of great activity. In his first years as professor of mathematics he was busily occupied with the composition of mathematical text-books based upon new and original methods. In 1842 he entered somewhat more particularly into physical problems, and published papers on the motion of a top, Espy's theory of storms, and adopted the epicycles of Hipparchus to the analytical form now used by astronomers. The great comet of 1843, which was distinctly visible at midday, and attracted great popular attention, gave Professor Peirce the opportunity, by a few striking lectures in Boston, to arouse a public interest in astronomy which led to the founding of the Observatory at Cambridge. This was the first great and efficient Observatory to be established in the United States, and its value to American science may be judged from the fact that only a few years before Dr. Bowditch had declared that "America has as yet no

Observatory worthy of mention." The Observatory of Harvard College, equipped with a 15-inch refractor, (then the most powerful in the world) and other necessary apparatus, had also the good fortune to secure a Director of scientific training, and hence for many years Harvard College became the natural center of astronomical activity in the United States. Peirce's work on the orbit of this great comet brought him into close relations with the lamented Sears C. Walker, and prepared the way for their still more important researches on the orbit of Neptune. For several years after the discovery of this planet by Galle, the English and French were occupied with fierce contentions relative to the honors due to Adams and Leverrier and the discussion of the orbit was left almost entirely to Professor Peirce and Sears C. Walker. The latter boldly undertook the investigation of the orbit, and soon succeeded in tracing the planet's approximate path along the zodiac during the 18th century. He thus found that it had been previously observed by La Lande and Lemonnier as a fixed star. With the positions of Neptune thus secured at distant epochs, the orbit could be found with considerable precision, while without these data, accidentally recorded by the French, and brought to light by the genius of Walker, astronomers must have waited for half a century before getting even an approximate orbit of the planet. These important researches inspire the belief that American astronomy has seldom sustained a more serious loss than in the early death of Walker, whose career was cut short not unlike that of Tobias Mayer. The memory of this remarkable astronomer ought to be held in respect by all who desire to do justice to elevated character and to meritorious scientific work.

Peirce took up a theoretical examination of the processes by which Adams and Leverrier had predicted the place of the unknown body, and was led to the conclusion that while the processes involved had been exact, wonderfully laborious, and deserving of all honor, yet the result was not decisive, as two possible planets would have satisfied the conditions of the problem. The two bodies were, first, the theoretical planet predicted by Adams and Leverrier, having a larger mass at a greater distance (assumed to conform to Bode's law); and second, the actual Neptune discovered by Galle with an inferrior mass and revolving at a smaller distance. It is easy to see that the purturbations arising from two such bodies would be almost exactly the same during the interval since the discovery of Uranus, but this result could not hold true indefinitely. The two bodies were

in the same direction from the Sun only in 1846, and if Neptune had not been discovered at that epoch the calculations must have required sensible modifications in the course of a few years. This result is very obvious to one who is familiar with the theory of perturbations, but Peirce subjected the question to computation in order to leave no doubt of its entire rigor. When Peirce's results were announced to the American Academy they gave rise to much discussion and were even vigorously contested by several eminent mathematicians of Europe; but astronomers have since recognized the justice of his criticisms which were not meant to detract from, but rather to supplement the, glorious achievements of Adams and Leverrier. The reasoning of these two eminent astronomers had assigned a sufficient, but not the only possible explanation, of the purturbations of Uranus. Yet when Peirce reached this conclusion and showed that the discovery in 1846 was a "happy accident"—since the two bodies would be in the same longitude only at that time—President Edward Everett, of Harvard college, "hoped the announcement would not be made public; nothing could be more improbable than such a coincidence." "Yes," replied Peirce, "but it would be still more wonderful if there were an error in my calculations."

In 1849 Professor Peirce was made consulting geometer to the *American Ephemeris and Nautical Almanac*, which had just been established at Cambridge. The important services which he rendered to this publication during the next thirty years enabled it to take rank with the highest authorities in the world. Peirce's attention at this epoch was devoted almost exclusively to the science of astronomy. In 1852 he published, for the use of the *American Ephemeris*, lunar tables which came into general use among astronomers and for many years continued to give good positions of the Moon. From 1852 to 1855 he was occupied with an elaborate investigation of the constitution of Saturn's rings. Professor G. P. Bond had seen the rings apparently divide themselves and reunite, and had been led to the conclusion that they could not be solid, as Laplace had assumed in his theory of their constitution. Peirce investigated the question anew, and showed that an unsymmetrical solid ring, such as Laplace had postulated, could not be kept in stable equilibrium by the action of the planet and satellites; and that while fluid rings could not be sustained by the action of the planet alone, sufficiently large and numerous satellites might contribute the necessary conditions of stability. He therefore concluded that the rings of Saturn are thin masses of fluid kept in equilibrium by the action

of the planet and its eight satellites. We ought to add that while mathematicians now hold this conclusion to be dynamically unsound, it was at the time a very important step in advance of Laplace and doubtless stimulated the investigation of Clerk Maxwell in 1859, which revived the old theory of Cassini and showed that the rings must necessarily be of a meteoric character, made up of single discrete particles, each revolving as an independent satellite. Peirce's treatment of the occultations of the Pleiades is equally original and suggestive, and brings to light a method which enables us to deduce from such observations the figures of both Earth and Moon; in this respect his theory of the occultations of the Pleiades is closely analagous to the method developed by Laplace for deriving the figure of the Earth from the lunar perturbations depending upon the oblateness of the terrestrial spheroid. Peirce's influence as a man of science rapidly extended throughout the Republic, and in 1855 he was made one of the Scientific Council entrusted with the organ ization of the Dudley Observatory. Dr. Benjamin Apthorp Gould. who had qualified himself under the most eminent astronomers in Europe, and had been a favorite student of Gauss, at Göttingen, was called to the directorship and at once placed the Observatory in the very front ranks of scientific institutions. His establishment of the *Astronomical Journal* and the great eminence to which his name and reputation raised the Observatory, aroused the jealousy of certain trustees, a result which so often happens in a community dominated by ignoble and small-minded men! In the subsequent'controversy the Director maintained the unshaken support of the most illustrious scientific men of the country—Benjamin Peirce, Joseph Henry and A. D. Bache, who published a defense of his administration. Without commenting on the meritorious but unsuccessful effort of the Scientific Council—Peirce, Henry and Bache—to conserve the real interests of science, it may yet be permissible to remark that if the name of any one of these trustees shall escape the just fate of oblivion it will only be through that old injustice to Gould, whose incomparable scientific work must necessarily remain luminous through coming centuries.

Peirce was president of the American Association for the Advancement of Science at the Cleveland meeting in 1853. In 1847 he received the degree of LL. D. from the University of North Carolina and in 1867 from Harvard College. He was a member of the American Academy of Arts and Sciences, and of the American Philosophical Society, and was elected an associate of the Royal

Astronomical Society in 1847. In 1852 he was made an honorary fellow of the Royal Society of London. When the National Academy of Sciences was established in 1863, Peirce was one of the original members appointed by congress to organize the first scientific body of the United States, in which he exerted a strong and lasting influence for maintaining the highest standard of scientific excellence.

The attention of Peirce was for many years much devoted to Geodesy. He assisted greatly in the work of the coast survey while under the direction of Bache and in 1867 was himself appointed superintendent. Peirce held the office for seven years; his administration was distinguished by the introduction of improved scientific methods, by a wise selection of men for the working staff, and by remarkable practical executive ability. It has been said of him that he showed the applicability of the theory of the Stoics in our own time—the really great man shows himself great by any and every standard.

He detected important errors in former determinations of the force of gravity, due to defective supports of the pendulums employed, inspired the highest confidence in the work of the Survey, and exercised a powerful influence in impressing upon congress the dignity and importance of the work which had been undertaken by the government of the United States. He realized that our position among the nations of the earth required that a scientific work of national importance should be adequately supported by congress; and on the other hand when funds had been appropriated for the Survey he exercised a careful supervision to insure legitimate and proper expenditures.

Peirce presented many striking papers to various societies, academies and institutions, but unfortunately most of these have been published only in abstract. We may mention among these an investigation of the forms of stable equilibrium for a fluid mass enclosed in an extensible sack floating in another fluid, a problem of interest in connection with embryology. He treated also the discontinuous motion of a billiard ball, the motion of a sling, and showed in the vegetable world the demonstrable presence of an intellectual plan; that what is known as Phyllotaxis involved an algebraic idea since shown to be the solution of a physical problem and likely to prove of great importance to the future of botany and zoölogy.

In 1857 Peirce published his "System of Analytical Mechanics" including "the latest researches of the great geometers in their most exalted forms of thought," a work which took high rank

on both sides of the Atlantic. It was announced at the same time that the work was to be continued and that the subsequent volumes would be entitled respectively, "Celestial Mechanics," "Potential Physics" and "Analytic Morphology;" but unfortunately the plan was never carried out.

As one of the original members of the National Academy of Sciences Peirce from time to time communicated to that illustrious body various papers on pure mathematics and theoretical physics, especially on the heat of the Sun. In 1870 certain friends in the Coast Survey caused 100 copies of his communications to the National Academy on "Linear Associative Algebra" to be lithographed and brought out in a form convenient for the use of mathematicians. These researches are regarded as his most original and precious contributions and are justly famed in all the countries of the world. For the sake of brevity suffice it to say that the investigation deals with nearly a hundred possible forms of Algebra of which only three are in use, (1) Ordinary Algebra, (2) the Calculus of Newton and Leibnitz and (3) the Quaternions of Hamilton. He outlines the great features of these many new species of Algebra with scarcely a comment and only patient study enables the reader to comprehend the great depth of his author.

Succinctness of statement and brevity of exposition have always been a conspicuous trait of his genius, and this characteristic led not a few to complain of obscurity in his writings; the same complaint was made regarding his teaching, but such an obstacle to the student is inconsiderable in the presence of so amiable, so enthusiastic and so magnetic a teacher. Peirce, like many other great men, constantly over-rated the ability of his students; he assumed that they could follow wherever he led. But in spite of this difficulty so great was the devotion of his students in their admiration for his enthusiastic, generous and exalted character, that he was always the center of a great influence. The steadfast admiration felt towards him by his intimate friends and students was due to his moral as well as his intellectual character. Making the concession that he occasionally exhibited a touch of intolerance towards pretentious mediocrity, they would allow nothing in him to be aught else than of the highest quality. His great nobility of character frequently gave his students credit for what was in reality his own. He robbed himself of fame in two ways: by giving the credit of his discoveries to those who had merely suggested the line of thought, and by neglecting to write out and publish his own developments.

Peirce's students include the most eminent astronomers and mathematicians of our day, and his noble and generous influence on the development of American science can hardly be overestimated. Among those who came under his influence and guidance we may name: Dr. B. A. Gould, Profesor J. Winlock, Dr. S. C. Chandler, Professor S. Newcomb, Dr. G. W. Hill, Professor W. Ferrell, Professor G. W. Hough and Professor Van Vleck.

It will be seen from this array of illustrious names that Harvard was at one time the center of astronomical inspiration, and exercised a vast influence in the higher development of American Astronomy.

Well might Peirce's death be deeply lamented both in Europe and America and above all at Harvard College! Lord Kelvin in an address to the British Association referred to Peirce as the "Founder of higher mathematics in America," and Cayley styled him the "Father of American geometry."

His death was equally lamented by the public and given conspicuous notice by the leading journals of the country. The *Boston Daily Advertiser* of Oct. 7th, 1880, said editorially: "The death of Professor Benjamin Peirce is a great and a national loss; for he was the Nestor of American mathematicians, and the historic transition from the illustrious Nathaniel Bowditch to the present generation of mathematical minds." Similar press notices appeared in many other papers, and appropriate resolutions were adopted by the faculty of Harvard and by the president and fellows, lamenting the great and irreparable loss which had befallen the College.

It seems of interest to recall the circumstances of the last illness of the illustrious mathematician. Peirce had throughout life maintained a lively interest in every form of intellectual activity, and during the winter of 1879–80 was still active in many directions. His great powers seemed to be still employed in the study of cosmical physics, and he had announced a new course in that subject for the following year. With the assistance of a favorite student he resumed with unusual enthusiasm the study of the great comet of 1843, and undertook a complete inquiry into all its successive appearances from the beginning of astronomical records, incited thereto by hearing of the remarkable observation, strongly recalling that comet, made in South America by his life long friend Dr. Benjamin Apthorp Gould. But it is probable that his renewed activity was but the outward sign of a presentiment of the approaching end. His serious illness began on June 25th, 1880, and from this time his condition grew gradually worse. He passed away on Wednesday morning, Oct. 6th, 1880.

Distinguished throughout life by a stoicial view of death, which did not permit him to mourn when it came to others or to dread for himself, he maintained the same temper to the end.

The funeral took place at Appleton chapel, on Oct. 9th, and was the occasion of an impressive gathering of scholars who desired to pay a last tribute to this great and good man, justly ranked among the most illustrious of the Earth. The attendance included a full representation of the faculties and governing boards of the University; a large deputation of officers of the United States Coast and Geodetic Survey, headed by the Superintendent and Chief Assistant; delegations of eminent professors from Yale College and Johns Hopkins University; members of the class of 1829, and numerous other friends of the deceased.

The pall-bearers were: President Chas. W. Elliott; ex-president Thomas Hill, of Portland; Captain C. P. Patterson, superintendent of the U. S. Coast and Geodetic Survey; Professor J. J. Sylvester, of the Johns Hopkins University; Hon. J. Ingersoll Bowditch; Professor Simon Newcomb, superintendent of the *American Ephemeris and Nautical Almanac*; Dr. Oliver Wendell Holmes; Professor J. Lovering, and Dr. Morrill Wyman.

It was certainly fitting that one who had played so great and so noble a part in the advancement of American science should receive an appropriate homage from his grateful countrymen.

As Fourier exclaims in his eulogy on Laplace, "it is undoubtedly beautiful, it is glorious, it is worthy of a powerful nation, to ordain brilliant honors in memory of its celebrated men." Benjamin Peirce was a mathematician and astronomer of the highest type, worthy to be ranked with the greatest intellects of any age or country. We need not hesitate to compare his achievements with those of the greatest successors of the immortal Newton. It was not given to him to create entirely new sciences as did the early pioneers of modern thought; but the great scope of his work and above all the extraordinary originality displayed in his "Linear Associative Algebra" will transmit his name to the most remote ages. While on the other hand his noble work in the education and inspiration of astronomers and mathematicians and in behalf of the dignity of American science as a whole, will endear his memory to his grateful countrymen while the stars shall go through their wonted courses in the heavens.

As England has done honor to her Newton and Herschel, France to her Laplace and Lagrange, Germany to her Bessel and Gauss, so let America also be grateful for the priceless heritage she has received from her Bowditch and Peirce!

The University of Chicago, Sept. 18, 1895.

PHOTOGRAPHING THE MILKY WAY.

H. C. WILSON.

For Popular Astronomy.

The great irregular belt of light, stretching across the sky, which we call the galaxy or Milky Way, has always been an object of interest, of admiration and wonder, I suppose, to those whose thoughts at all turn to things above us. To the astronomer the structure of the galaxy is a problem of the deepest interest. Since the invention of the telescope it has been known that the light of the Milky Way, the greater part of it at least, comes from countless numbers of stars, individually invisible to the eye, but collectively giving a bright glow to the parts of the sky where they are crowded most thickly together.

The Milky Way is not a belt of uniform brightness, but there are irregular dark areas running all through it, breaking it up into bright patches or clouds, of various shapes and sizes. A number of attempts have been made to represent these irregularities of the galaxy upon the star maps, but generally with ill success. It is very difficult for a draftsman to represent the delicate shadings which are necessary.

In order to catch the fainter details the eye must be keenly sensitive, must have been in total darkness for some time. The drawing requires light, the presence of which destroys to a considerable degree the sensitiveness of the eye. It must therefore be done largely from memory. Either the work must be unreliable or an immense amount of time must be consumed in verifying every part of the details, by repeated comparison of the drawing with the sky, under the most favorable conditions. Even then personal bias is liable to affect the result. Details once seen are likely to be seen the same the next time by the same person, whether correct or not.

The most perfect pieces of work of this kind have been completed recently by Dr. Otto Boeddicker at Birr Castle, Parsontown, Ireland, and M. Eaton at Paris. Each of these gentlemen spent several years in the production of a map which should represent all that can be seen of the galaxy with the naked eye, and their results are far ahead of anything that has gone before. They are wonderful for the amount of detail which they show, but they do not agree with each other. The one shows much more detail than is ordinarily seen with the naked eye, the other less than is seen under the best conditions.

PLATE VII.

CAROLINE HERSCHEL.

ÆTAT 98.

PLATE VI

YERKES OBSERVATORY OF THE UNIVERSITY OF CHICAGO, WILLIAMS' BAY, WIS.

PLATE VII.

TELESCOPE OF YERKES OBSERVATORY (40 INCHES CLEAR APERTURE).

Photographed by E. E. Barnard.

PLATE VIII.

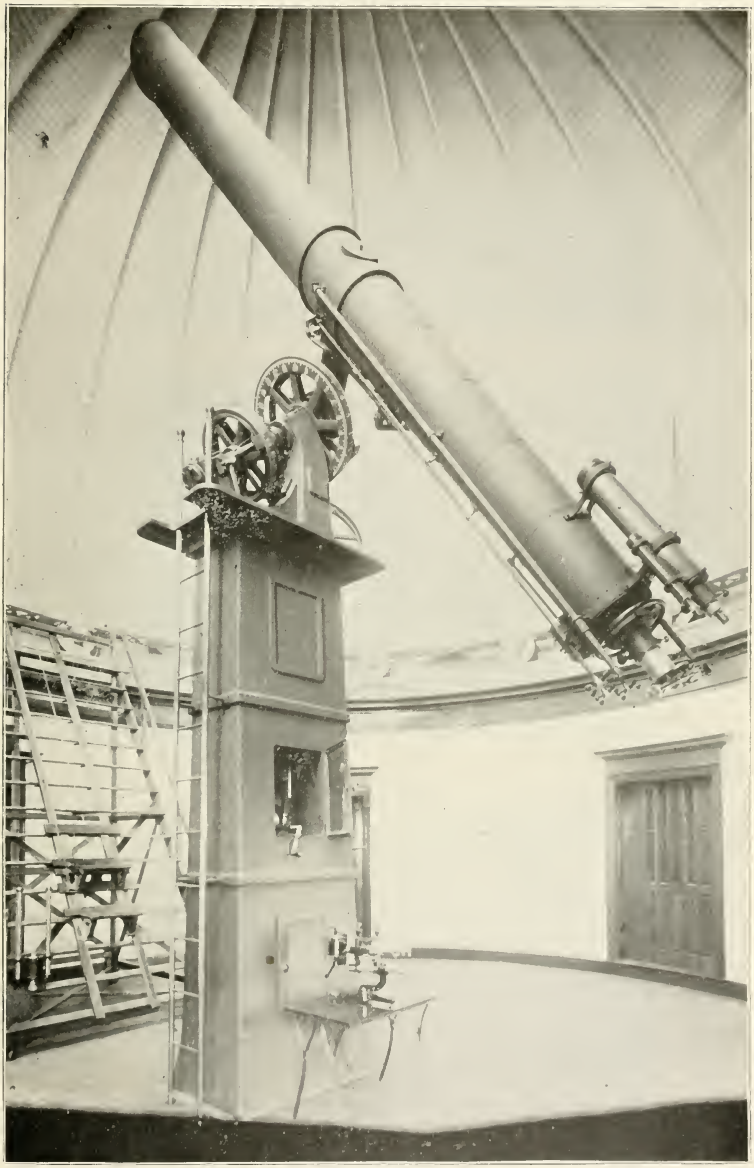
E. E. BARNARD'S HOUSE AND YERKES OBSERVATORY.

PLATE XI.

Dwelling House. Meridian Building. Equatorial Building.

FLOWER OBSERVATORY, UNIVERSITY OF PENNSYLVANIA.

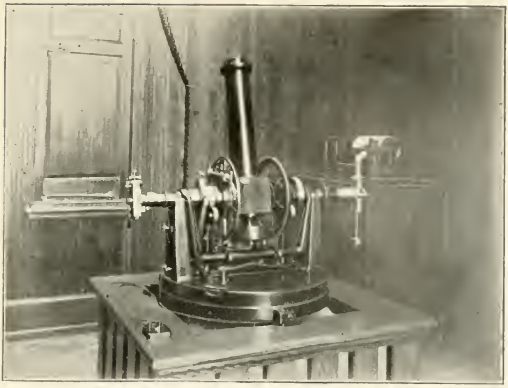
PLATE XII.

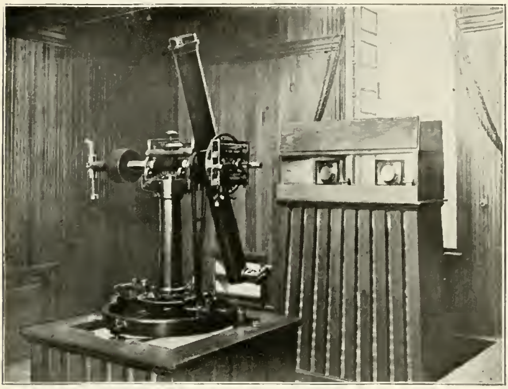


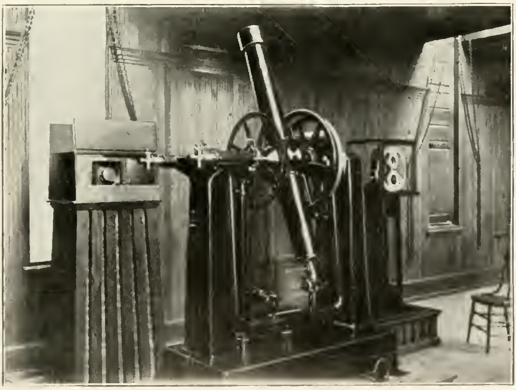
Equatorial Telescope of Flower Observatory.

18 inches Clear Aperture

PLATE XIII.







1. Uuiversal Instrument. 2. Zenith Telescope. 3. Meridian Circle. (Telescope 4 inches Aperture).

NEW INSTRUMENTS OF FLOWER OBSERVATORY, UNIVERSITY OF PENNSYLVANIA.

PLATE XXIX.

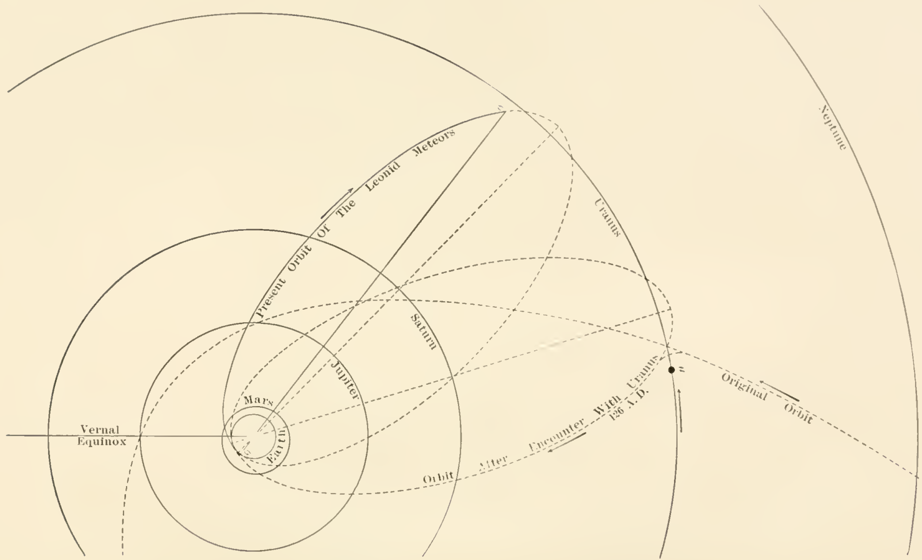
Popocatepetl From the West. (Amecameca).

PLATE VI.

FROM PHOTOGRAPH OF THE MOON BY LOEWY AND PUISEUX OF PARIS OBSERVATORY.

(Mare Crisium and Vicinity.)

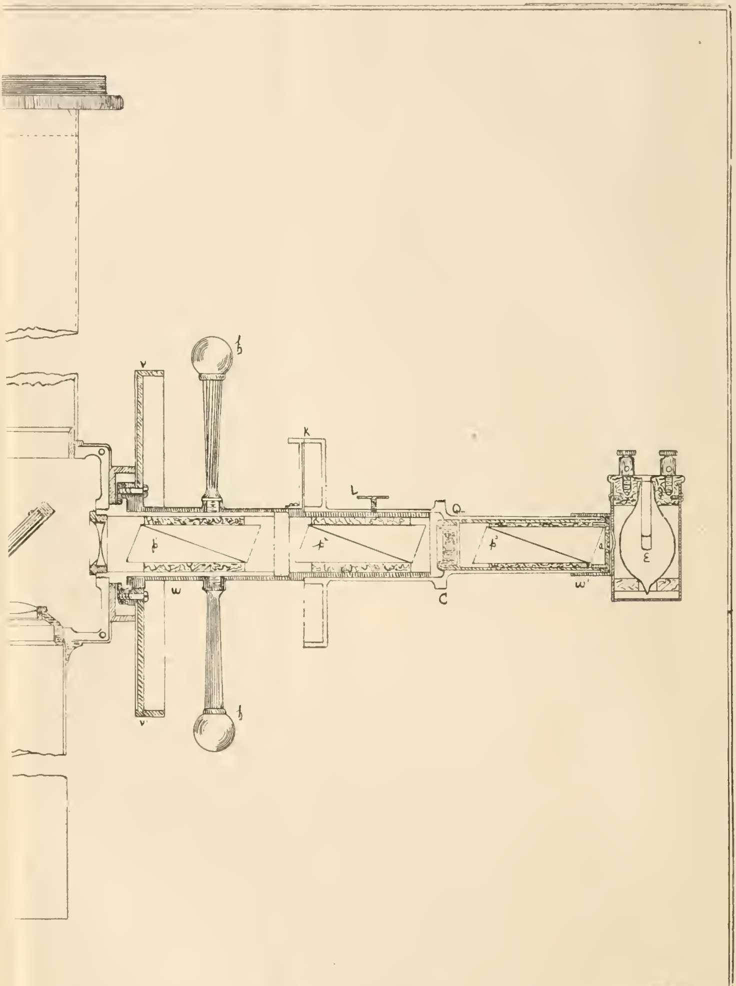
PLATE XLIX.



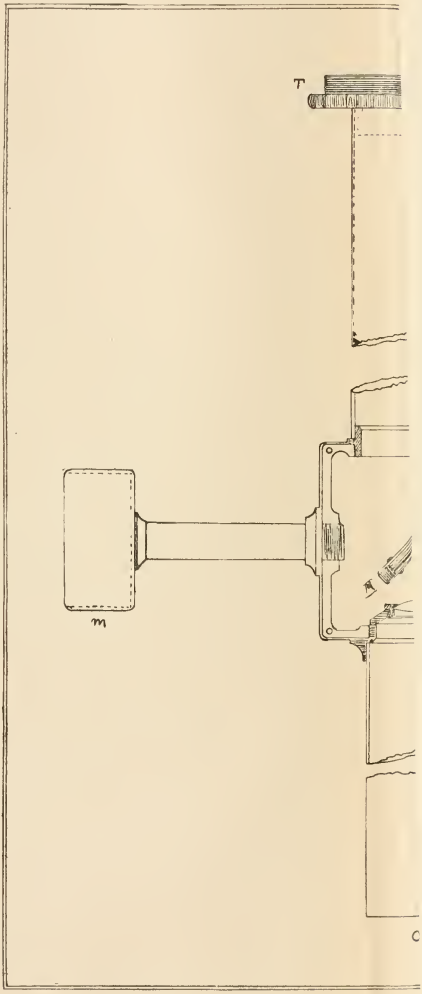
THE ORBIT OF THE LEONID METEORS AS CHANGED BY THE ATTRACTION OF URANUS

POPULAR ASTRONOMY, No. 39.

PLATE X.



TELLAR PHOTOMETER IN USE AT WASHBURN OBSERVATORY.



Detailed Drawing of the S

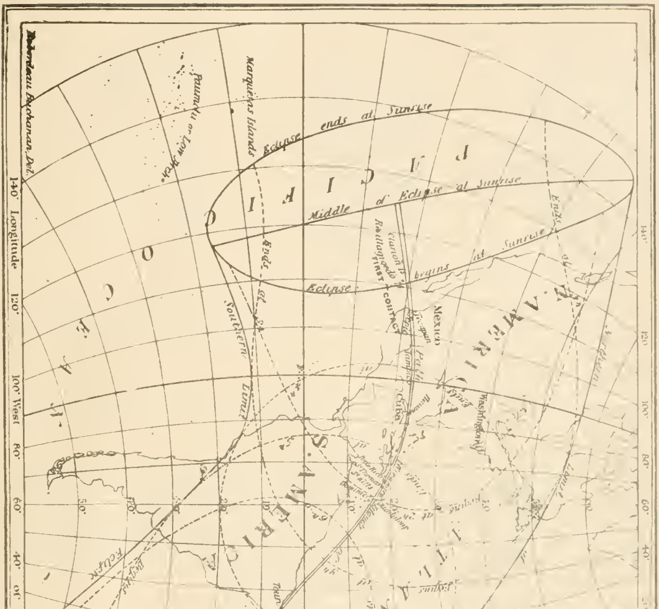


CHART OF THE ANNULAR ECLIPSE OF THE SUN JULY 29, 1897.

The hours of beginning and ending are expressed in Greenwich Mean Time.

Maxima and Minima of Long Period Variables.

1897 September.

MAXIMA.		Day	MAXIMA, CON'T.		Day	MAXIMA, CONT.		Day
110	S Tucanæ	8:	5617	U Libræ	17	8290	R Pegasi	22
1805	V Orionis	4:	5644	Z Libræ	21	8324	V Cassiopeæ	2
1981	S Camelopard.	19	5677	R Serpentis	21	8512	R Aquarii	19
2013	U Aurigæ	24	5761	Z Scorpii	4		MINIMA.	
2080	R Columbæ	6:	5795	W Scorpii	14			
2530	V Canis min.	10	6794	R Lyræ	11	906	R Trianguli	24
2676	U Monocerotis	4	6921	S Sagittarii	1	2583	L_2 Puppis	16
2684	S Canis min.	29	6943	T Sagittæ	29	2742	S Geminorum	9
2815	U Geminorum	17:	7106	S Vulpeculæ	19	5194	V Bootis	18
3128	R Pyxidis	22:	7155	RR Aquilæ	14:	7192	Z Cygni	24:
3184	T Hydræ	11	7234	R Capricorni	23:	7448	W Aquarii	6:
4896	T Centauri	7	7257	R Sagittæ	14	7577	X Capricorni	27:
5190	R Camelopard.	12*	7261	R Delphini	13	7754	W Cygni	2.

The above ephemeris was computed for POPULAR ASTRONOMY directly from the elements given in Chandler's Third Catalogue, with the following exceptions:

U Cephei. The revised elements, given in *Astronomical Journal* No. 396, have been used. By these the minima are about ¾ hour later than by the elements of the Third Catalogue.

U Ophiuchi. The period is so short that only every tenth minimum is given, with the length of period, so that the intermediate minima can be readily interpolated.

* *5190 R Camelopardalis.* In *Astronomical Journal* No. 400 Dr. Chandler has the following note on this star: "Observations within a few years appear to make certain, what I had previously suspected, that the period of this star has been slowly lengthening since its discovery in 1858, from about 265 days to nearly 275 days at the present time, at the average rate of about $0d.16$ from maximum to maximum. Therefore, for the elements given in the Third Catalogue, the following may be substituted with advantage:

$$1869 \text{ Sept. } 2\ (2403943) + 267.5\,E + 0d.08E^2$$

The individual phases are subject also to considerable irregularities." The date given in the table above is computed from the elements of the Third Catalogue. According to the new elements the maximum will occur Oct. 22.

U Geminorum. Rev. J. G Hagen has a note on this star in *Astronomical Journal* No. 400. He found it 9.0 mag. 1897 March 3, 7 and 10, while making the chart of the field for his Atlas of Variable Stars. This agrees well with Mr. Sperra's maximum. March 6, reported in the June number, page 49. At the close of the note Rev. Hagen says—"Counting from the last maximum, the next is due in the beginning of June and cannot be observed." This remark led me to look for the star every clear evening after May 1, and I was fortunate enough to find it bright, 9.6 mag., June 7, 15^h, Gr. M. T. June 11 the variable could not be seen, the limit being 10 mag. At the next maximum, August-October, it will be in the morning sky, and can probably be observed, though predictions in regard to this uncertain star can be better made after the fact.

Rev. Hagen states that U was invisible in the 12-inch refractor 1897 Feb. 28 and March 26. This seems to contradict the lower limit, 13 1 mag., usually set.

SS Cygni. It was quite a coincidence that this star, of the same type as the above, was also near a maximum June 7. Mr. Sperra sends me the following observed magnitudes :

1897 June 1, 11.3; June 4, 8.5; June 5, 8.2; June 9, 8.3. I observed it (magnitudes on a different light scale).